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Earthquake resistant design of structures (Subject Code: 06CV834)

UNIT 5 & Unit 6: Seismic lateral force analysis

Contents:

1 Introduction:

Apart from gravity loads, the structure will experience dominant lateral forces of considerable magnitude during earthquake shaking. It is essential to estimate and specify these lateral forces on the structure in order to design the structure to resist an earthquake. It is impossible to exactly determine the earthquake induced lateral forces that are expected to act on the structure during its lifetime. However, considering the consequential effects of earthquake due to eventual failure of the structure, it is important to estimate these forces in a rational and realistic manner.

The earthquake forces in a structure depend on a number of factors such as,

- Characteristics of the earthquake (Magnitude, intensity, duration, frequency, etc.)
- Distance from the fault
- Site geology
- Type of structure and its lateral load resisting system.

2 Earthquake Resistant Design Philosophy

Apart from the factors mentioned above, the consequences of failure of the structure may also be of concern in the reliable estimation of design lateral forces. Hence, it is important to include these factors in the lateral force estimation procedures.

Code of practice for earthquake resistant design of structures primarily aims at accomplishing two primary objectives; total safety against loss of life and minimization of economic loss. These objectives are fulfilled by design philosophy with following criteria,

- Resist minor earthquake shaking without damage

- Resist moderate earthquake shaking without structural damage but possibly with some damage to nonstructural members
- Resist major levels of earthquake shaking with both structural and nonstructural damage, but the building should not collapse thus endangerment of the lives of occupants is avoided.

Conceptual representation of the earthquake resistant design philosophy is depicted in Figure 1.

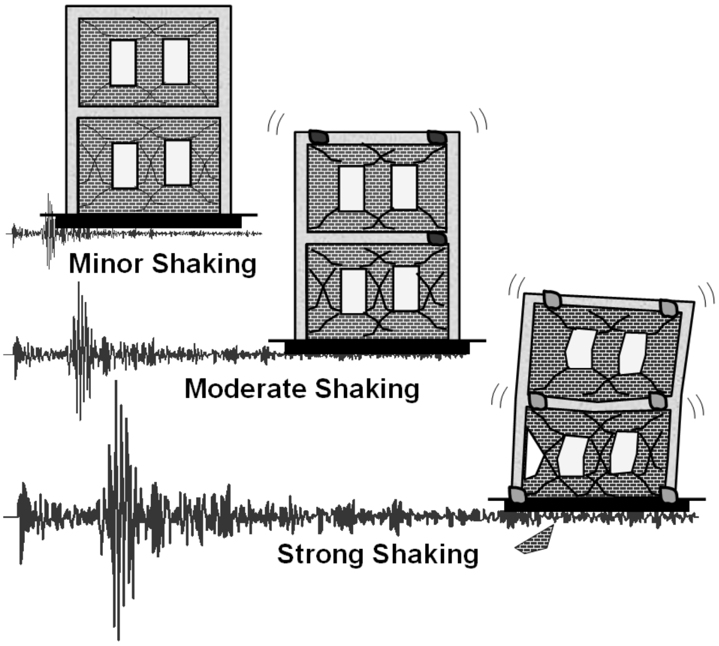


Figure 1: Schematic diagram depicting earthquake resistant design philosophy for different levels shaking [IITK-BMTPC (2004)]

The purpose of an earthquake-resistant design is to provide a structure with features, which will enable it to respond satisfactorily to seismic effects. These features are related to five major objectives, which are listed in order of importance:

- The likelihood of collapse after a very severe earthquake should be as low as possible.
- Damage to non-structural elements caused by moderate earthquakes should be kept within reasonable limits. Although substantial damage due to severe earthquakes, which have a low probability of occurrence is acceptable, such damage is unacceptable in the case of moderate tremors which are more likely to occur.

- Buildings in which many people are usually present should have deformability features which will enable occupants to remain calm even in the event of strong shocks.
- Personal injury should be avoided.
- Damage to neighboring buildings should be avoided

3 Guidelines for Earthquake Resistant Design

As mentioned above, the philosophy of earthquake design is to prevent non-structural damage in frequent minor ground shaking, is to prevent structural damage and minimize non-structural damage in occasional moderate ground shaking and to avoid collapse or serious damage in rare major ground shaking. In order to meet these requirements the code of practice for earthquake resistant design of structures generally prescribes guidelines with respect to following aspects,

- Intensity of shaking is prescribed based on zone factor depending upon seismic activity in the region of geographical location of the site
- Characteristics of the structures that affect its dynamic behaviour is accounted by prescribing appropriate natural period depending on distribution of mass and stiffness properties also, by considering type of soil beneath its foundation.
- Importance factor is assigned depending on occupancy type, functionality etc. of the structure
- Capability of a particular structure to resist lateral forces is incorporated by identifying its redundancy and ductility features through response modification factor.

When inertia of the structure offers resistance to ground motions, structure will experience earthquake forces. The relative movement between the ground and the structure induces a force dependent on the ground acceleration, mass and stiffness properties of the structure. The ground acceleration depends on the magnitude and intensity of the seismic event at a location. Based on seismic records, experience, and research, some areas of the country are determined to have a greater probability of earthquakes than others, and some areas have more severe earthquakes. This is taken into account by dividing the country into different zones that represent estimates of future earthquake occurrence and strength.

The magnitude of the seismic force also depends on the type of foundation soil under the building. Some soils tend to amplify seismic waves and can even tend to liquefy during an earthquake. Hence, it is important to suitably incorporate the effect of prevailing soil conditions in the procedures of evaluation of seismic forces on the structure.

Introduction of an occupancy importance factor to provide for more conservative design of important facilities is necessary such that the structure importance factor indirectly accounts for less risk, or better expected performance specified for important structures. Important structures are those

- Emergency facilities that are expected to remain functional after a severe earthquake such as hospitals, fire stations, etc.
- Buildings, whose failure may lead to other disasters, affecting people or environment, such as nuclear power plants, dams, petrochemical facilities, etc.
- Life-line facilities e.g. communication lines, pipelines, bridges, power stations, etc.
- Facilities for large number of people such as community centers, schools, etc.

Accordingly these structures are designed for higher lateral strength, and hence they are expected to sustain less damage under the design earthquake.

Finally, it is imperative to rationally incorporate means of reducing the required lateral strength in case of structures that are capable of withstanding extensive inelastic behaviour by virtue of their structural configuration and detailing. In this regard, generally provision is made in the code of practice by introducing the response modification factor. The response reduction factor essentially reduces the design lateral strength of the structure from required strength to resist the linear response to the strength that would be required to limit inelastic behaviour to acceptable levels. Response reduction factor magnitude mainly depends on the ductility characteristics of the structure under consideration. Structural systems deemed capable of withstanding extensive inelastic behavior are assigned relatively high response reduction factor values, permitting minimum design strength that is required for elastic response to the design ground motion. Systems deemed to be incapable of providing reliable inelastic behavior are assigned with low response reduction factor value that results in strength sufficient to resist design motion in a nearly elastic manner.

4 General Earthquake Resistant Design Principles of IS-1893 (2002)

Clause 6.1 of IS-1893 (2002) provides the following design principles,

- The random earthquake ground motions, which cause the structure to vibrate, can be resolved in any three mutually perpendicular directions. The predominant direction of ground vibration is usually horizontal.
- Earthquake-generated vertical inertia forces are to be considered in design unless checked and proven in specimen calculations to be not significant. Vertical acceleration should be considered in structures with large spans and those in which stability is a criterion for design. Reduction in gravity force due to vertical component of ground motions can be particularly detrimental in cases of prestressed horizontal members and of cantilevered members. Hence, special attention should be paid to the effect of vertical component of the ground motion on prestressed or cantilevered beams, girders and slabs.
- The response of a structure to ground vibration is a function of the nature of foundation soil: materials, form, size and mode of construction of structures and the duration and characteristics of ground motion. IS-1893 specifies design forces for structures standing on rocks or soils which do not settle or liquefy or slide due to loss of strength during ground vibrations.
- The design approach adopted in IS 1893 ensures that structures possess at least a minimum strength to withstand minor earthquakes of intensity less than DBE (Design Basis Earthquake) without damage; resist moderate earthquakes equal to DBE without significant structural damage though some non-structural damage may occur; and aims that structures withstand a major earthquake (Maximum Considered Earthquake - MCE) without collapse.
- Actual forces that appear on structures during earthquakes are much greater than the design forces specified in the code. However, ductility, arising from inelastic material behaviour and detailing, and over strength, arising from the additional reserve strength in structures over and above the design strength, are relied upon to account for this difference in actual and design lateral loads.
- The design lateral force specified in this standard shall be considered in each of the two orthogonal horizontal directions of the structure. For structures which have lateral force resisting elements in the two orthogonal directions only, the design lateral force shall be

considered along one direction at a time, and not in both directions simultaneously. Structures, having lateral force resisting elements (for example frames, shear walls) in directions other than the two orthogonal directions, shall be analysed considering the load combinations specified in Clause: 6.3.2 [IS-1893 (2002)]. Where both horizontal and vertical seismic forces are taken into account, load combinations specified in Clause: 6.3.3 [IS-1893 (2002)] shall be considered. (Refer to equation (3) & (4) for load combinations specified in IS-1893)

5 Assumptions Made Earthquake Resistant Design of Structures

The following assumptions are made in IS-1893 (2002) for earthquake resistant design of structures (Clause: 6.2, IS 1893-2002):

- Earthquake causes impulsive ground motions, which are complex and irregular in character, changing in period and amplitude each lasting for a small duration. Therefore, resonance of the type as visualised under steady-state sinusoidal excitations, will not occur as it would need time to build up such amplitudes
- Earthquake is not likely to occur simultaneously with wind or maximum flood or maximum sea waves.
- The value of elastic modulus of materials, wherever required, may be taken as for static analysis unless a more definite value is available for use in such condition

6 Load combinations

Clause: 6.3 of IS-1893 (2002) specifies following load combinations

- In the plastic design of steel structures, the following load combinations shall be accounted for:

$$\begin{aligned} &1)\ 1.7(DL+IL) \\ &2)\ 1.7(DL\pm IL) \qquad (1) \\ &3)\ 1.3(DL+IL\pm EL) \end{aligned}$$

- In the limit state design of reinforced and prestressed concrete structures, the following load combinations shall be accounted for:

1) $1.5(DL + IL)$
2) $1.2(DL + IL \pm EL)$
3) $1.5(DL \pm IL)$
4) $0.9DL \pm 1.5EL$

(2)

Where DL, IL and EL denote dead load, imposed load and earthquake load respectively.

- Design Horizontal Earthquake Load:
  - When the lateral load resisting elements are oriented along orthogonal horizontal direction, the structure shall be designed for the effects due to full design earthquake load in one horizontal direction at time.
  - When the lateral load resisting elements are not oriented along the orthogonal horizontal directions, the structure shall be designed for the effects due to full design earthquake load in one horizontal direction plus 30 percent of the design earthquake load in the other direction
- Design Vertical Earthquake Load: When effects due to vertical earthquake loads are to be considered, the design vertical force shall be calculated in accordance with Clause: 6.4.5 of IS-1893 (2002). (i.e., the design acceleration spectrum for vertical motions may be taken as two-thirds of the design horizontal acceleration spectrum)
- Combination for Two or Three Component Motion: When responses from the three earthquake components are to be considered, the responses due to each component may be combined using the assumption that when the maximum response from one component occurs, the responses from the other two components are 30 percent of their maximum. All possible combinations of the three components (EL_x, EL_y and EL_z where x and y are two orthogonal directions and z is vertical direction) including variations in sign (plus or minus) shall be considered. Thus, the response due earthquake force (EL) is the maximum of the following three cases (Clause: 6.3.4.1, IS 1893-2002)

1) $\pm EL_x \pm 0.3EL_y \pm 0.3EL_z$
2) $\pm 0.3EL_x \pm EL_y \pm 0.3EL_z$
3) $\pm 0.3EL_x \pm 0.3EL_y \pm EL_z$

(3)

Or as an alternative to the procedure mentioned above, the response *(EL)* due to the combined effect of the three components can be obtained (Clause: 6.3.4.2, IS 1893-2002) on the basis SRSS that is,

$$EL = \sqrt{(EL_x)^2 + (EL_y)^2 + (EL_z)^2} \tag{4}$$

7 Design Spectrum

- For the purpose of determining seismic forces, the country is classified into four seismic zones as shown in Figure 2.

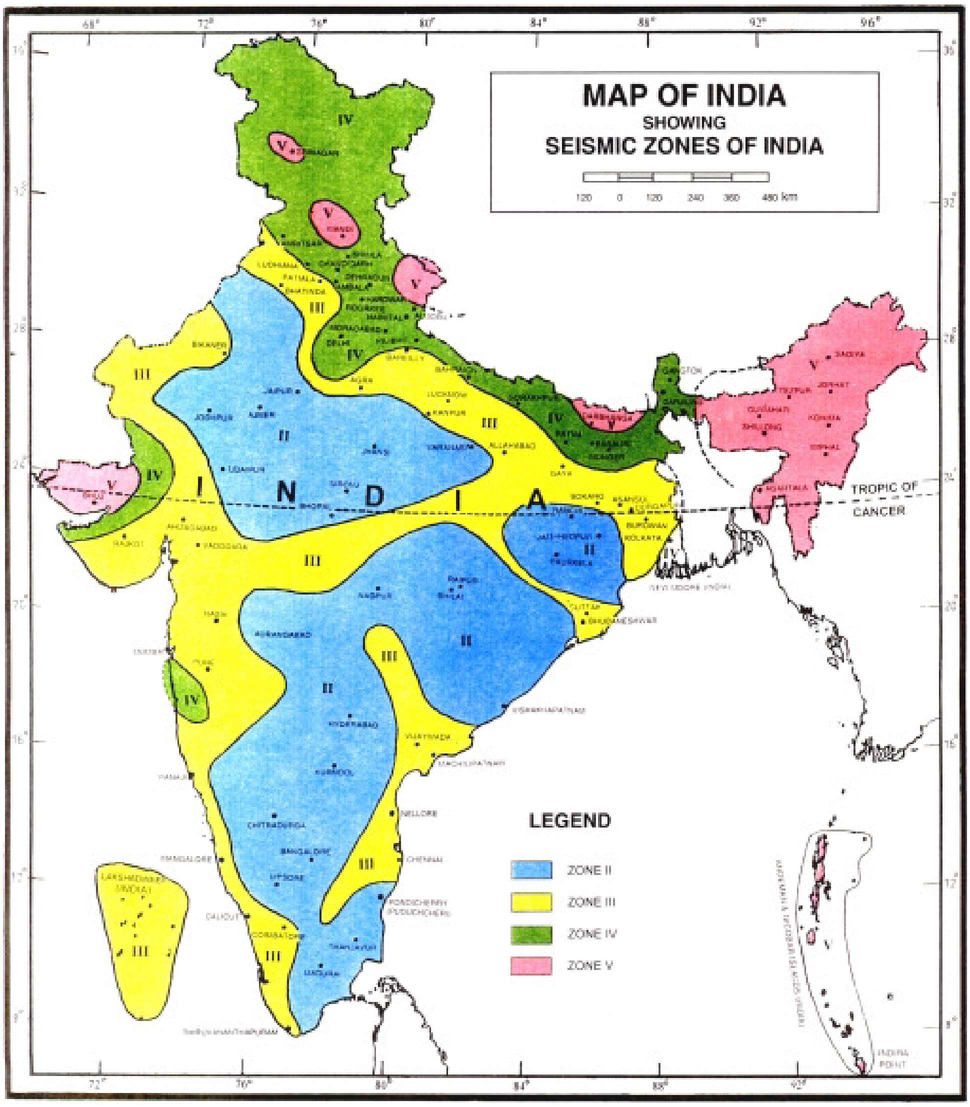


Figure 2: Seismic zones of India [Fig. 1, IS-1893 (2002)]

- The design horizontal seismic coefficient for a structure shall be determined by the following expression (Clause: 6.4.2.1, IS 1893-2002)

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} \qquad (5)$$

Provided that for any structure with T ≤ 0.1 s, the value of A_h will not be taken less than *Z/2* whatever be the value of *I/R*. Where,

Z = Zone factor given in Table 1, is for the Maximum Considered Earthquake (MCE) and service life of structure in a zone. The factor 2 in the denominator of Z is used so as to reduce the Maximum Considered Earthquake (MCE) zone factor to the factor for Design Basis Earthquake (DBE).

I = Importance factor, depending upon the functional use of the structures, characterised by hazardous consequences of its failure, post earthquake functional needs, historical value, or economic importance (Table 2).

(S_a / g) = Average response acceleration coefficient for rock or soil sites as given by Figure 3 (or from table adjacent to the Figure 3) based on appropriate natural periods and damping of the structure. These curves represent free field ground motion. Figure 3 shows the proposed 5% spectra for rocky and soils sites and Table 3 gives the multiplying factors for obtaining spectral values for various other damping.

R = Response reduction factor, depending on the perceived seismic damage performance of the structure, characterised by ductile or brittle deformations. However, the ratio *(I/R)* shall not be greater than 1.0. The values of *R* for buildings are given in Table 4.

- Where a number of modes are to be considered for dynamic analysis, the value of A_h as defined in equation (5), for each mode shall be determined using the natural period of vibration of that mode.
- For underground structures and foundations at depths of 30 m or below, the design horizontal acceleration spectrum value shall be taken as half the value obtained from equation (5). For structures and foundations placed between the ground level and 30 m depth, the design horizontal acceleration spectrum value shall be linearly interpolated between A_h and $0.5A_h$ where A_h is as specified in equation (5)

Table 1: Zone factor (Z) [Table 2, IS-1893 (2002)]

Seismic Zone	II	III	IV	V
Seismic Intensity	Low	Moderate	Severe	Very Severe
Z	0.10	0.16	0.24	0.36

Table 2: Importance factor (I) [Table 6, IS-1893 (2002)]

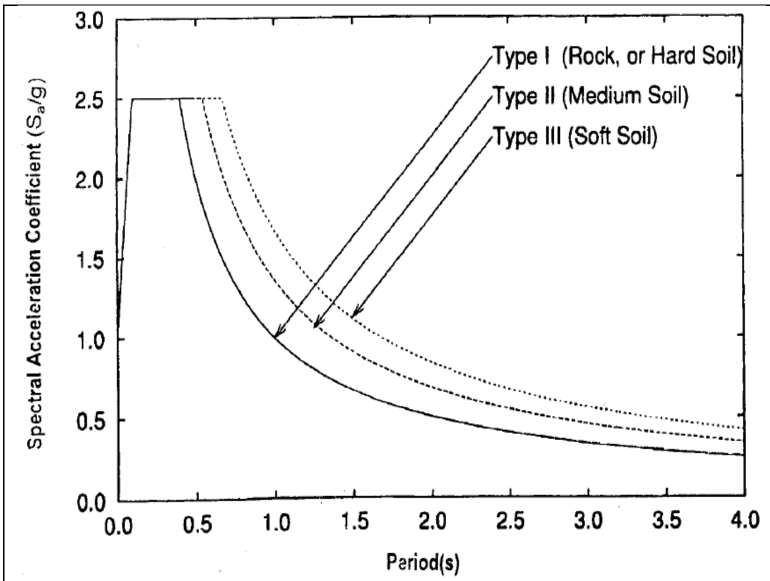
SI. No. (1)	Structure (2)	Importance Factor (3)
i)	Important service and community buildings, such as hospitals; schools; monumental structures; emergency buildings like telephone exchange, television stations, radio stations, railway stations, tire station buildings; large community halls like cinemas, assembly halls and subway stations, power stations	1.5
ii)	All other buildings	1.0

NOTES

1. The design engineer may choose values of importance factor I greater than those mentioned above.

2. Buildings not covered in SI. No. (i) and (ii) above may be designed for higher value of I. depending on economy, strategy considerations like multi-storey buildings having several residential units

3. This docs not apply to temporary structures like excavations, scaffolding etc of short duration.



For rocky, or hard soil sites

$$\frac{S_a}{g} = \begin{cases} 1+15T & 0.00 \le T \le 0.10 \\ 2.50 & 0.10 \le T \le 0.40 \\ 1.00/T & 0.40 \le T \le 4.00 \end{cases}$$

For medium soil sites

$$\frac{S_a}{g} = \begin{cases} 1+15T & 0.00 \le T \le 0.10 \\ 2.50 & 0.10 \le T \le 0.55 \\ 1.36/T & 0.55 \le T \le 4.00 \end{cases}$$

For soft soil sites

$$\frac{S_a}{g} = \begin{cases} 1+15T & 0.00 \le T \le 0.10 \\ 2.50 & 0.10 \le T \le 0.67 \\ 1.67/T & 0.67 \le T \le 4.00 \end{cases}$$

Figure 3: Response spectra for rock and soil sites for 5% damping [Fig. 2, IS-1893 (2002)]

Table 3 Multiplying factors for damping other than 5% [Table 3, IS-1893 (2002)]

Damping Percent	0	2	5	7	10	15	20	25	30
Factors	3.20	1.40	1.00	0.90	0.80	0.70	0.60	0.55	0.50

Table 4: Response reduction factor (*R*) for building systems [Table 7, IS-1893 (2002)]

SI. No.	Lateral Load Resisting System	*R*
(1)	(2)	(3)
	Building Frame Systems	
(i)	Ordinary RC moment-resisting frame (OMRF)[2)]	3.0
(ii)	Special RC moment-resisting frame (SMRF)[3)]	5.0
(iii)	Steel frame with	
	a) Concentric braces	4.0
	b) Eccentric braces	5.0
(iv)	Steel moment resisting frame designed as per SP 6 (6)	5.0
	Building with Shear Walls[4)]	
(v)	Load bearing masonry wall buildings[5)]	
	a) Unreinforced	1.5
	b) Reinforced with horizontal RC bands	2.5
	c) Reinforced with horizontal RC bands and vertical bars at comers of rooms and	3.0
(vi)	Ordinary reinforced concrete shear walls[6)]	3.0
(vii)	Ductile shear walls[7)]	4.0
	Buildings with Dual System[8)]	
(viii)	Ordinary shear wall with OMRF	3.0
(ix)	Ordinary shear wall with SMRF	4.0
(x)	Ductile shear wall with OMRF	4.5
(xi)	Ductile shear wall with SMRF	5.0

1) The values of response reduction factors are to be used for buildings with lateral load resisting elements, and not just for the lateral load resisting elements built in isolation.
2) OMRF are those designed and detailed as per IS 456 or IS 800 but not mooting ductile detailing requirement as per IS 13920 or SP 6 (6) respectively.
3) SMRF defined in 4.15.2.
4) Buildings with shear walls also include buildings having shear walls and frames, but where:

a) frames are not designed to carry lateral loads, or
b) frames are designed to carry lateral loads but do not fulfil the requirements of 'dual systems'.

5) Reinforcement should be as per IS 4326.
6) Prohibited in zones IV and V.
7) Ductile shear walls are those designed and detailed as per IS 13920.
8) Buildings with dual systems consist of shear walls (or braced frames) and moment resisting frames such that:

a) the two systems arc designed to resist the total design force in proportion to their lateral stiffness considering the interaction of the dual system at all floor levels; and
b) the moment resisting frames are designed to independently resist at least 25 percent of the design seismic base shear.

8 Design imposed loads for earthquake force calculation [Clause 7.3, IS 1893 (2002)]

- For various loading classes as specified in IS 875 (Part 2), the earthquake force shall be calculated for the full dead load plus the percentage of imposed load as given in Table 5.

Table 5: Percentage of imposed load to be considered in seismic weight calculation [Table 8, IS-1893 (2002)]

Imposed Uniformly Distributed Floor Loads (kN/ m^2)	Percentage of Imposed Load
Up to and including 3.0	25
Above 3.0	50

- For calculating the design seismic forces of the structure, the imposed load on roof need not be considered.
- The percentage of imposed loads given above shall also be used for 'Whole frame loaded' condition in the load combinations specified in equation (2) and equation (3) where the gravity loads are combined with the earthquake loads. No further reduction in the imposed load will be used as envisaged in IS 875 (Part 2) for number of storeys above the one under consideration or for large spans of beams or floors.
- The proportions of imposed load indicated above for calculating the lateral design forces for earthquakes are applicable to average conditions. Where the probable loads at the time of earthquake are more accurately assessed, the designer may alter the proportions indicated or even replace the entire imposed load proportions by the actual assessed load. In such cases, where the imposed load is not assessed as mentioned above only that part of imposed load, which possesses mass, shall be considered. Lateral design force for earthquakes shall not be calculated on contribution of impact effects from imposed loads.
- Other loads apart from those given above (for example snow and permanent equipment) shall be considered as appropriate.

9 Design lateral forces

Design Seismic Base Shear: The total design lateral force or design seismic base shear (V_B) along any principal direction shall be determined by the following expression: [Clause 7.5.3, IS-1893 (2002)]

$$V_B = A_h W \tag{6}$$

Where

A_h = Design horizontal acceleration spectrum value as per equation (5), using the fundamental natural period T_a as per equation (7) or (8) in the considered direction of vibration; and

W = Seismic weight of the building is computed as given below [Clauses 7.4.2 & 7.4.3, IS-1893 (2002)]

- *Seismic Weight of floors:* The seismic weight of each floor is its full dead load plus appropriate amount of imposed load. While computing the seismic weight of each floor, the weight of columns and walls in any storey shall be equally distributed to the floors above and below the storey.
- *Seismic Weight of Building:*
  - The seismic weight of the whole building is the sum of the seismic weights of all the floors.
  - Any weight supported in between storeys shall be distributed to the floors above and below in inverse proportion to its distance from the floors.

10 Fundamental period

- The approximate fundamental natural period of vibration (T_a), in seconds, of a moment-resisting frame building without brick infill panels may be estimated by the empirical expression: [Clause 7.6.1, IS-1893 (2002)]

$$\begin{aligned} T_a &= 0.075h^{0.75} \text{....(for RC frame building)} \\ T_a &= 0.085h^{0.75} \text{....(for steel frame building)} \end{aligned} \tag{7}$$

- The approximate fundamental natural period of vibration (T_a), in seconds, of all other buildings, including moment-resisting frame buildings with brick infill panels, may be estimated by the empirical expression: [Clause 7.6.2, IS-1893 (2002)]

$$T_a = \frac{0.09h}{\sqrt{d}} \tag{8}$$

where,

h = Height of building, in m. This excludes the basement storeys, where basement walls are connected with the ground floor deck or fitted between the building columns. But, it includes the basement storeys, when they are not so connected.

d = Base dimension of the building at the plinth level, in m, along the considered direction of the lateral force.

11 Earthquake Lateral Force Analysis

The design lateral force shall first be computed for the building as a whole. This design lateral force shall then be distributed to the various floor levels. The overall design seismic force thus obtained at each floor level shall then be distributed to individual lateral load resisting elements depending on the floor diaphragm action. There are two commonly used procedures for specifying seismic design lateral forces:

1. Equivalent static force analysis
2. Dynamic analysis

12 Equivalent static force analysis

The equivalent lateral force for an earthquake is a unique concept used in earthquake engineering. The concept is attractive because it converts a dynamic analysis into partly dynamic and partly static analyses for finding the maximum displacement (or stresses) induced in the structure due to earthquake excitation. For seismic resistant design of structures, only these maximum stresses are of interest, not the time history of stresses. The equivalent lateral force for an earthquake is defined as a set of lateral static forces which will produce the same peak response of the structure as that obtained by the dynamic analysis of the structure under the same earthquake. This equivalence is restricted only to a single mode of vibration of the structure. Inherently, equivalent static lateral force analysis is based on the following assumptions,

- Assume that structure is rigid.
- Assume perfect fixity between structure and foundation.
- During ground motion every point on the structure experience same accelerations
- Dominant effect of earthquake is equivalent to horizontal force of varying magnitude over the height.
- Approximately determines the total horizontal force (Base shear) on the structure

However, during an earthquake structure does not remain rigid, it deflects, and thus base shear is disturbed along the height.

The limitations of equivalent static lateral force analysis may be summarised as follows,

- In the equivalent static force procedure, empirical relationships are used to specify dynamic inertial forces as static forces.
- These empirical formulas do not explicitly account for the dynamic characteristics of the particular structure being designed or analyzed.
- These formulas were developed to approximately represent the dynamic behavior of what are called *regular* structures (Structures which have a reasonably uniform distribution of mass and stiffness). For such structures, the equivalent static force procedure is most often adequate.
- Structures that are classified as *irregular* violate the assumptions on which the empirical formulas, used in the equivalent static force procedure, are developed. Common types of irregularities in a structure include large floor-to-floor variation in mass or center of mass and soft stories etc. Therefore in such cases, use of equivalent static force procedure may lead to erroneous results. In these cases, a dynamic analysis should be used to specify and distribute the seismic design forces.

13 Step by step procedure for Equivalent static force analysis

Step-1: Depending on the location of the building site, identify the seismic zone and assign Zone factor (*Z*)

- Use Table 2 along with Seismic zones map or Annex of IS-1893 (2002)

Step-2: Compute the seismic weight of the building (*W*)

- As per Clause 7.4.2, IS-1893 (2002) – Seismic weight of floors
- As per Clause 7.4.3, IS-1893 (2002) – Seismic weight of the building

Step-3: Compute the natural period of the building (T_a)

- As per Clause 7.6.1 or Clause 7.6.2, IS-1893 (2002), as the case may be.

Step-4: Obtain the data pertaining to type of soil conditions of foundation of the building

- Assign type, I for hard soil, II for medium soil & III for soft soil

Step-5: Using T_a and soil type (I / II / III), compute the average spectral acceleration $\left(\frac{S_a}{g}\right)$

- Use Figure 2 or corresponding table of IS-1893 (2002), to compute S_a/g

Step-6: Assign the value of importance factor (I) depending on occupancy and/or functionality of structure

- As per Clause 7.2 and Table 6 of IS-1893 (2002),

Step-7: Assign the values of response reduction factor (R) depending on type of structure

- As per Clause 7.2 and Table 7 of IS-1893 (2002)

Step-8: Knowing Z, S_a/g, R and I compute design horizontal acceleration coefficient (A_h) using the relationship, $A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R}$ [Clause 6.4.2, IS-1893 (2002)]

Step-9: Using A_h and W compute design seismic base shear (V_B), from $V_B = A_h W$ [Clause 7.5.3, IS-1893 (2002)]

Step-10: Compute design lateral force (Q_i) of i^{th} floor by distributing the design seismic base shear (V_B) as per the expression, $Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2}$ [Clause 7.7.1, IS-1893 (2002)]

14 <u>Dynamic Analysis</u>

- Dynamic analysis is classified into two types, namely, *<u>Response spectrum method</u>* and *<u>Time history method</u>*
- Dynamic analysis shall be performed to obtain the design seismic force, and its distribution to different levels along the height of the building and to the various lateral load resisting elements, for the following buildings:

a) *Regular buildings* — Those greater than 40 m in height in Zones IV and V, and those greater than 90 m in height in Zones II and III.
b) *Irregular buildings* — All framed buildings higher than 12 m in Zones IV and V, and those greater than 40 m in height in Zones II and III.

- *Time History Method:* Time history method of analysis, when used, shall be based on an appropriate ground motion and shall be performed using accepted principles of dynamics.
- *Response Spectrum Method:* Response spectrum method of analysis shall be performed using the design spectrum specified in Clause 6.4.2 or by a site specific design, spectrum mentioned in Clause 6.4.6 of IS 1893 (2002)
- When dynamic analysis is carried out either by the Time History Method or by the Response Spectrum Method, the design base shear computed from dynamic analysis (V_B) shall be compared with a base shear calculated using a fundamental period T_a ($\bar{V}_B$), where T_a is as per Clause 7.6. If base shear obtained from dynamic analysis (V_B) is less than base shear computed from equivalent static load method ($\bar{V}_B$ *i.e., using* T_a *as per Clause 7.6*), then as per Clause 7.8.2, all the response quantities (for example member forces, displacements, storey forces, storey shears and base reactions) shall be multiplied by ratio $\dfrac{\bar{V}_B}{V_B}$.
- *Free Vibration Analysis:* Undamped free vibration analysis of the entire building shall be performed as per established methods of mechanics using the appropriate masses and elastic stiffness of the structural system, to obtain natural periods (T) and mode shapes (φ) of those of its modes of vibration that need to be considered.
- *Modes to be considered:* The number of modes to be used in the analysis should be such that the sum total of modal masses of all modes considered is at least 90%. If modes with natural frequency beyond 33 *Hz* are to be considered, modal combination shall be carried out only for modes up to 33 *Hz*. The effect of modes with natural frequency beyond 33 *Hz* be included by considering missing mass correction following well established procedures.

- *Modal combination:* The peak response quantities (for example, member forces, displacements, storey forces, storey shears and base reactions) shall be combined as per Complete Quadratic Combination (CQC) method or alternatively, when building does not have closely spaced modes then Square Root of Square Sum (SRSS) method may be employed.

CQC method: $$\lambda = \sqrt{\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i \rho_{ij} \lambda_j} \qquad (9)$$

Where,

r = Number of modes being considered,

$$\rho_{ij} = \frac{8\varsigma^2(1+\beta_{ij})\beta_{ij}^{1.5}}{(1-\beta_{ij}^2)^2 + 4\varsigma^2\beta_{ij}(1+\beta_{ij})^2}$$ = Cross-modal coefficient,

λ_i = Response quantity in mode i(including sign),

λ_j = Response quantity in mode j(including sign),

β_{ij} = frequency ratio between i^{th} and the j^{th} mode is, $\beta_{ij} = \frac{\omega_j}{\omega_i} = \frac{T_j}{T_i}$

SRSS method: $$\lambda = \sqrt{\sum_{k=1}^{r}\lambda_k^2} \qquad (10)$$

λ_k = Absolute value of response quantity in mode k

15 <u>Step by step procedure for Response spectrum method</u>

Step-1: Depending on the location of the building site, identify the seismic zone and assign Zone factor (Z)

- Use Table 2 along with Seismic zones map or Annex of IS-1893 (2002)

Step-2: Compute the seismic weight of the building (W)

- As per Clause 7.4.2, IS-1893 (2002) – Seismic weight of floors (W_i)

Step-3: Establish mass *[M]* and stiffness *[K]* matrices of the building using system of masses lumped at the floor levels with each mass having one degree of freedom, that

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of lateral displacement in the direction under consideration. Accordingly, to develop stiffness matrix effective stiffness of each floor is computed using the lateral stiffness coefficients of columns and infill walls. Usually floor slab is assumed to be infinitely stiff.

Step-4: Using *[M]* and *[K]* of previous step and employing the principles of dynamics compute the modal frequencies, $\{\omega\}$ and corresponding mode shapes, $[\varphi]$.

Step-5: Compute modal mass M_k of mode k using the following relationship with n being number of modes considered

$$M_k = \frac{\left[\sum_{i=1}^{n} W_i \phi_{ik}\right]^2}{g \sum_{i=1}^{n} W_i \phi_{ik}^2} \quad \text{[Clause 7.8.4.5a of IS 1893 (2002)]} \qquad (11)$$

Step-6: Compute modal participation factors P_k of mode k using the following relationship with n being number of modes considered

$$P_k = \frac{\sum_{i=1}^{n} W_i \phi_{ik}}{\sum_{i=1}^{n} W_i \phi_{ik}^2} \quad \text{[Clause 7.8.4.5b of IS 1893 (2002)]} \qquad (12)$$

Step-7: Compute design lateral force (Q_{ik}) at each floor in each mode (i.e., for i^{th} floor in mode k) using the following relationship,

$$Q_{ik} = A_{h(k)}\ \phi_{ik}\ P_k\ W_i \quad \text{[Clause 7.8.4.5c of IS 1893 (2002)]} \qquad (13)$$

$A_{h(k)}$ = Design horizontal acceleration spectrum value as per Clause 6.4.2 of IS 1893 using the natural period $\left(T_k = \dfrac{2\pi}{\omega_k}\right)$ of vibration of mode *k*.

Step-8: Compute storey shear forces in each mode (V_{ik}) acting in storey *i* in mode *k* as given by,

$$V_{ik} = \sum_{i+1}^{n} Q_{ik} \quad \text{[Clause 7.8.4.5d of IS 1893 (2002)]} \qquad (14)$$

Step-9: Compute storey shear forces due to all modes considered, V_i in storey *i,* by combining shear forces due to each mode in accordance with Clause 7.8.4.4 of IS 1893 (2002). i.e., either CQC or SRSS modal combination methods are used.

Step-10: Finally compute design lateral forces at each storey as,

$$F_{roof} = V_{roof} \text{ and}$$
$$F_i = V_i - V_{i+1}$$
[Clause 7.8.4.5f of IS 1893 (2002)] (15)

EXAMPLE: 1

Plan and elevation of a four-storey reinforced concrete office building is shown in Fig. 1.1. The details of the building are as follows.

Number of Storey = 4

Zone = III

Live Load = 3 kN/m^2

Columns = 450 x 450 mm

Beams = 250 x 400 mm

Thickness of Slab = 150 mm

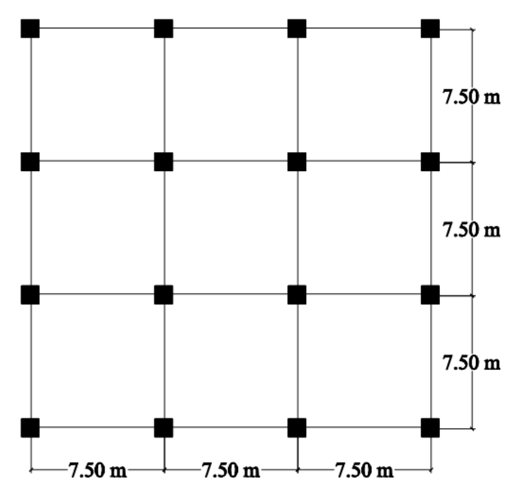
Thickness of Wall = 120 mm

Importance factor = 1.0

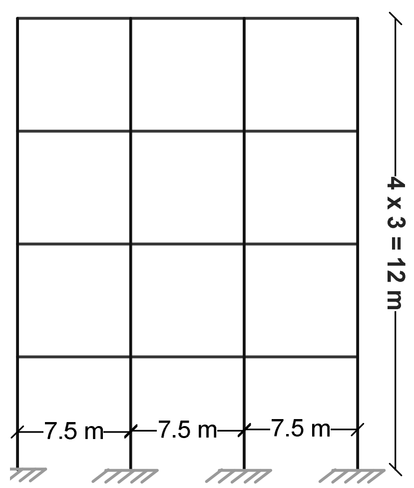
Structure type = OMRF Building

Determine design seismic lateral load and storey shear force distribution.

Plan



Elevation



Solution: Analysis considering stiffness of infill masonry

1. Computation of Seismic weights

(Assuming unit weight of concrete as 25 kN/m^3 & 22.5 kN/m^3 for masonry)

1) Slab:

   DL due to self weight of slab = (22.5 x 22.5 x 0.15) x 25 = 1898.40 kN

2) Beams:

   Self weight of beam per unit length = 0.25 x 0.4 x 25 = 2.5 kN/m

   Total length = 4x22.5x2 = 180 m

   DL due to self weight of beams = (2.5 x 22.5) x 4 x 2 = 450 kN

3) Columns:

   Self weight of column per unit length = 0.45 x 0.45 x 25 = 5.0625 kN/m

   DL due to self weight of columns (16 No.s) = 16 x 5.0625 x 3.0 = 243 kN

4) Walls:

   Self weight of wall per unit length = 0.12 x 3 x 20 = 7.2 kN/m

   Total length = 4x22.5x2 = 180 m

   DL due to self weight of Walls = 7.2 x 22.5 x 4 = 648 kN

5) Live Load [Imposed load] (25 %) = (0.25 x 3) x 22.5 x 22.5 = 380 kN

Load on all floors:

W1=W2=W3= 1898 + 380 + 450 + 243 + 648 = **3619 kN**

Load on roof slab (Live load on slab is zero)

W4 = 1898 + 0+450 + (243/2) + (648/2) = **2793.5 kN**

Total Seismic weight, W = (3619 x 3) + 2793.5 = **13650.5 kN**

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Fundamental period:

Natural period, $T_a = 0.09\frac{h}{\sqrt{d}} = 0.09\frac{12}{\sqrt{22.5}} = 0.2277$

(Moment resisting frame with in-fill walls)

Spectral acceleration:

Type of soil: Medium Soil

For T_a=0.2277 s

Sa/g = 2.5

Zone factor: For Zone III, Z = 0.16

Importance Factor: I = 1.0

Response Reduction Factor: R = 3.0 (OMRF)

Horizontal acceleration coefficient ($\mathbf{A_h}$) :

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.16}{2}(2.5)\left(\frac{1}{3}\right)$$

$$A_h = 0.0667$$

Base shear ($\mathbf{V_B}$) :

$$V_B = A_h W = 0.0667 \times 13650.50$$

$$V_B = 910.0333\ kN$$

Storey lateral forces and shear forces are calculated and tabulated in the following table.

Floor level (i)	$W_i\,(kN)$	$h_i\,(m)$	$W_i\,h_i^2$ (kN-$m2$)	Storey forces $Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2}$	Storey shear forces [V_i] (Cumulative sum) (kN)
4	2793.5	12.0	402264	426.53	426.53
3	3619	9.0	293139	310.83	737.35
2	3619	6.0	130284	138.14	875.50
1	3619	3.0	32571	34.54	910.03

Storey shear forces are calculated as follows (last column of the table),

$$V_4 = Q_4 = 426.53 \ kN$$
$$V_3 = V_4 + Q_3 = 426.53+310.82=737.35 \ kN$$
$$V_2 = V_3 + Q_2 = 737.35+138.14=875.50 \ kN$$
$$V_1 = V_2 + Q_1 = 875.50+34.54= 910.03 \ kN = V_B$$

Later force and shear force distribution is shown in the Figure-EX1.

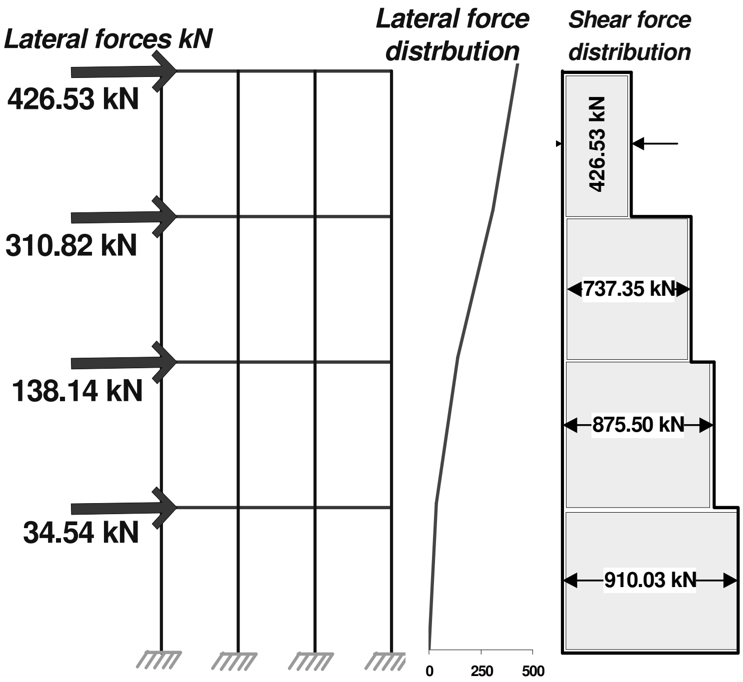

Figure – EX1: Lateral and Shear Force distribution along the height of the structure

Solution: Analysis without considering stiffness of infill masonry

Fundamental period:

Natural period, $T_a = 0.075h^{0.75} = 0.075 \times 12^{0.75} = 0.4836$
(Moment resisting frame without in-fill walls)

Spectral acceleration:

Type of soil: Medium Soil

For $T_a = 0.4836$ s

Sa/g = 2.5 (because, $T_a = 0.4836$ s, i.e., $0.10 \leq T_a \leq 0.55$)

Zone factor: For Zone III, Z = 0.16

Importance Factor: I = 1.0

Response Reduction Factor: R = 3.0 (OMRF)

Horizontal acceleration coefficient (A_h) :

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.16}{2}(2.5)\left(\frac{1}{3}\right)$$

$$A_h = 0.0667$$

Base shear (V_B) :

$$V_B = A_h W = 0.0667 \times 13650.50$$

$$V_B = 910.0333 \ kN$$

Since, base shear V_B is same as in case of considering stiffness of infill walls, the storey lateral forces and shear forces are same as in the previous case. Therefore, Lateral and Shear Force distribution along the height of the structure shown in Figure-EX1 is valid. That is, for the structure under consideration, the lateral force and shear force distribution is unaltered irrespective of stiffness of infill walls is included or not in the analysis.

<u>EXAMPLE: 2</u>

Analyse the building frame considered in Example-1 using response spectrum method (Dynamic analysis) with all other data being same.

Solution:

Note: In plan structure is symmetrical about both X and Y directions)

1) Seismic weights:

   W1=W2=W3= 1898 + 380 + 450 + 243 + 648 = **3619 kN**

   W4 = 1898 + 0+450 + (243/2) + (648/2) = **2793.5 kN**

   Therefore, seismic masses are,

   M1 = M2 = M3 = 368.91 x 10^3 kg.

   M4 = 284.76 x 10^3 kg

2) Floor stiffness (Without considering stiffness of infill wall):

   MI of columns, $I_C = (0.45)^4/12 = 3.1417875 \times 10^{-3}$ m^4

Young's Modulus, $E_C = 5000(f_{ck})^{0.5} = 25000$ MPa = 25×10^9 N/m^2

(Assuming M25 concrete for columns)

$K1 = K2 = K3 = K4 = 16 \times (12 \times 25 \times 10^9 \times 3.1417875 \times 10^{-3})/(3^3) = 0.6075 \times 10^9$ N/m

3) Natural frequencies and Mode shapes:

Mass matrix,

$$M = \begin{bmatrix} M1 & 0 & 0 & 0 \\ 0 & M2 & 0 & 0 \\ 0 & 0 & M3 & 0 \\ 0 & 0 & 0 & M4 \end{bmatrix} = \begin{bmatrix} 368.91 & 0 & 0 & 0 \\ 0 & 368.91 & 0 & 0 \\ 0 & 0 & 368.91 & 0 \\ 0 & 0 & 0 & 284.76 \end{bmatrix} \times 10^3 kg$$

Stiffness Matrix,

$$K = \begin{bmatrix} K1+K2 & -K2 & 0 & 0 \\ -K2 & K2+K3 & -K3 & 0 \\ 0 & -K3 & K3+K4 & -K4 \\ 0 & 0 & -K4 & -K4 \end{bmatrix} = \begin{bmatrix} 1.215 & -0.6075 & 0 & 0 \\ -0.6075 & 1.215 & -0.6075 & 0 \\ 0 & -0.6075 & 1.215 & -0.6075 \\ 0 & 0 & -0.6075 & 0.6075 \end{bmatrix} \times 10^9 \; N/m$$

Solving the Eigen equation, $\left|K - M\omega^2\right| = 0$, we get Eigen value and corresponding Eigen vectors as,

Eigen values, $\omega^2 = \begin{Bmatrix} 219.9 \\ 1793.2 \\ 4079.8 \\ 5920.9 \end{Bmatrix}$ $\therefore$ the natural frequencies are, $\omega = \begin{Bmatrix} 14.83 \\ 42.35 \\ 63.87 \\ 76.95 \end{Bmatrix}$ rad/s

The mode shapes are,

$$\phi_1 = \begin{Bmatrix} 1.00 \\ 1.87 \\ 2.48 \\ 2.77 \end{Bmatrix}, \; \phi_2 = \begin{Bmatrix} 1.00 \\ 0.91 \\ -0.17 \\ -1.07 \end{Bmatrix}, \; \phi_3 = \begin{Bmatrix} 1.00 \\ -0.48 \\ -0.77 \\ 0.85 \end{Bmatrix}, \; \& \; \phi_4 = \begin{Bmatrix} 1.00 \\ -1.60 \\ 1.55 \\ -0.87 \end{Bmatrix}$$

The natural periods are, $T = 2\pi / \{\omega\} = \begin{Bmatrix} 0.424 \\ 0.148 \\ 0.098 \\ 0.082 \end{Bmatrix}$ seconds

Calculation of modal participation factor

Storey Level	Seismic weight (W_i), kN	MODE-1		
		ϕ_{i1}	$W_i\ \phi_{i1}$	$W_i\ \phi_{i1}^{\ 2}$
4	2793.5	2.77	7737.995	21434.25
3	3619	2.48	8975.12	22258.3
2	3619	1.87	6767.53	12655.28
1	3619	1.00	3619.00	3619.00
Σ	**13650.5**		**27099.65**	**59966.82**
Modal mass $M_1 = \dfrac{\left[\sum W_i\phi_{i1}\right]^2}{g\sum W_i\phi_{i1}^2}$		$= \dfrac{27099.65^2}{59966.82g} =$ 12246.62 kN/g		
% of Total weight		89.72 %		
Modal participation factor, $P_1 = \dfrac{\sum W_i\phi_{i1}}{\sum W_i\phi_{i1}^2}$		$= \dfrac{27099.65}{59966.82} =$ **0.452**		

Storey Level	Seismic weight (W_i), kN	MODE-2		
		ϕ_{i2}	$W_i\ \phi_{i2}$	$W_i\ \phi_{i2}^{\ 2}$
4	2793.5	-1.07	-2989.05	3198.278
3	3619	-0.17	-615.23	104.5891
2	3619	0.91	3293.29	2996.894
1	3619	1.00	3619.00	3619.00
Σ	**13650.5**		**3308.015**	**9918.761**
Modal mass, $M_2 = \dfrac{\left[\sum W_i\phi_{i2}\right]^2}{g\sum W_i\phi_{i2}^2}$		$= \dfrac{3308.015^2}{9918.761g} =$ 1103.26 kN/g		
% of Total weight		8.08 %		
Modal participation factor, $P_2 = \dfrac{\sum W_i\phi_{i2}}{\sum W_i\phi_{i2}^2}$		$= \dfrac{3308.015}{9918.761} =$ **0.334**		

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Storey Level	Seismic weight (W_i), kN	MODE-3		
		ϕ_{i3}	$W_i\ \phi_{i3}$	$W_i\ \phi_{i3}^{\ 2}$
4	2793.5	0.85	2374.475	2018.304
3	3619	-0.77	-2786.63	2145.705
2	3619	-0.48	-1737.12	833.8176
1	3619	1.00	3619.00	3619.00
Σ	**13650.5**		**1469.725**	**8616.826**
Modal mass, $M_3 = \dfrac{\left[\sum W_i\phi_{i3}\right]^2}{g\sum W_i\phi_{i3}^2}$		$= \dfrac{1469.725^2}{8616.826g} =$ 250.683 kN/g		
% of Total weight		1.84 %		
Modal participation factor, $P_3 = \dfrac{\sum W_i\phi_{i3}}{\sum W_i\phi_{i3}^2}$		$= \dfrac{1469.725}{8616.826} =$ **0.171**		

Storey Level	Seismic weight (W_i), kN	MODE-4		
		ϕ_{i4}	$W_i\ \phi_{i4}$	$W_i\ \phi_{i4}^{\ 2}$
4	2793.5	-0.87	-2430.35	2114.4
3	3619	1.55	5609.45	8694.648
2	3619	-1.60	-5790.4	9264.64
1	3619	1.00	3619.00	3619.00
Σ	**13650.5**		**1007.705**	**23692.69**
Modal mass, $M_4 = \dfrac{\left[\sum W_i\phi_{i4}\right]^2}{g\sum W_i\phi_{i4}^2}$		$= \dfrac{1007.705^2}{23692.69g} =$ 42.86 kN/g		
% of Total weight		0.314 %		
Modal participation factor, $P_4 = \dfrac{\sum W_i\phi_{i4}}{\sum W_i\phi_{i4}^2}$		$= \dfrac{1007.705}{23692.69} =$ **0.043**		

The lateral load Q_{ik} acting at i^{th} floor in the k^{th} mode is,

$Q_{ik} = A_{h(k)}\ \phi_{ik}\ P_k\ W_i$ ……… (Clause 7.8.4.5c of IS: 1893 Part 1)

The value of $A_{h(k)}$ for different modes is obtained from clause 6.4.2.

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MODE-1:

$$T_1 = 0.424 \ s$$

$$\frac{S_a}{g} = 2.5 \text{....}(0.10 \le T_1 \le 0.55 - \text{Medium soil})$$

$$A_{h(1)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.16}{2}(2.5)\frac{1}{3} = 0.0667$$

$$Q_{i1} = A_{h(1)} \ \phi_{i1} \ P_1 \ W_i = 0.0667 \times 0.452 \times (\phi_{i1} \ \ W_i) = 0.03015(\phi_{i1} \ W_i)$$

MODE-2:

$$T_2 = 0.148 \ s$$

$$\frac{S_a}{g} = 2.5 \text{....}(0.10 \le T_2 \le 0.55 - \text{Medium soil})$$

$$A_{h(2)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.16}{2}(2.5)\frac{1}{3} = 0.0667$$

$$Q_{i2} = A_{h(2)} \ \phi_{i2} \ P_2 \ W_i = 0.0667 \times 0.334 \times (\phi_{i2} \ \ W_i) = 0.0223(\phi_{i2} \ W_i)$$

MODE-3:

$$T_3 = 0.098 \ s$$

$$\frac{S_a}{g} = 1 + 15T_3 = 2.47 \text{....}(0.00 \le T_3 \le 0.10 - \text{Medium soil})$$

$$A_{h(3)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.16}{2}(2.47)\frac{1}{3} = 0.0659, \text{But, } T_3 \le 0.10,$$

$$\therefore A_{h(3)} = \frac{Z}{2} = 0.08 > 0.0659$$

$$Q_{i3} = A_{h(3)} \ \phi_{i3} \ P_4 \ W_i = 0.08 \times 0.171 \times (\phi_{i3} \ W_i) = 0.01368(\phi_{i3} \ W_i)$$

MODE-4:

$$T_4 = 0.082 \ s$$

$$\frac{S_a}{g} = 1 + 15T_4 = 2.23 \text{....}(0.00 \le T_4 \le 0.10 - \text{Medium soil})$$

$$A_{h(4)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.16}{2}(2.23)\frac{1}{3} = 0.0595, \text{But, } T_3 \le 0.10,$$

$$\therefore A_{h(4)} = \frac{Z}{2} = 0.08 > 0.0595$$

$$Q_{i4} = A_{h(4)} \ \phi_{i4} \ P_4 \ W_i = 0.08 \times 0.043 \times (\phi_{i4} \ W_i) = 0.00344(\phi_{i4} \ W_i)$$

Lateral load calculation by modal analysis – SRSS method

Storey level	Weight Wi (kN)	**Mode – 1** [$Q_{i1} = 0.03015(\phi_{i1}\ W_i)$]		
		ϕ_{i1}	Q_{i1}	V_{i1}
4	2793.5	2.77	233.30	**233.30**
3	3619	2.48	270.60	**503.90**
2	3619	1.87	204.04	**707.94**
1	3619	1.00	109.11	**817.05**

Storey level	Weight Wi (kN)	**Mode – 2** [$Q_{i2} = 0.0223(\phi_{i2}\ W_i)$]		
		ϕ_{i2}	Q_{i2}	V_{i2}
4	2793.5	-1.07	-66.66	**-66.66**
3	3619	-0.17	-13.72	**-80.38**
2	3619	0.91	73.44	**-6.93**
1	3619	1.00	80.70	**73.77**

Storey level	Weight Wi (kN)	**Mode – 3** [$Q_{i3} = 0.01368(\phi_{i3}\ W_i)$]		
		ϕ_{i3}	Q_{i3}	V_{i3}
4	2793.5	0.85	32.48	**32.48**
3	3619	-0.77	-38.12	**-5.64**
2	3619	-0.48	-23.76	**-29.40**
1	3619	1.00	49.51	**20.11**

Storey level	Weight Wi (kN)	**Mode – 4** [$Q_{i4} = 0.0034(\phi_{i4}\ W_i)$]		
		ϕ_{i4}	Q_{i4}	V_{i4}
4	2793.5	-0.87	-8.26	**-8.26**
3	3619	1.55	19.07	**10.81**
2	3619	-1.6	-19.69	**-8.88**
1	3619	1.00	12.30	**3.43**

SRSS method (Clause 7.8.4.4 – IS1893-2002):

The contribution of different modes are combined by Square Root of the Sum of the Squares (SRSS) using the following relationship, $V_i = \sqrt{V_{i1}^2 + V_{i2}^2 + V_{i3}^2 + V_{i4}^2}$

Then, storey lateral forces are calculated by, $F_i = V_i - V_{i+1}$.

The results obtained are tabulated in the following table.

Storey level	V_{i1}	V_{i2}	V_{i3}	V_{i4}	**Combined shear force (SRSS) V_i (kN)**	**Combined lateral force (SRSS) F_i (kN)**
4	**233.30**	**-66.66**	**32.48**	**-8.26**	**244.94**	**244.94**
3	**503.90**	**-80.38**	**-5.64**	**10.81**	**510.42**	**265.48**
2	**707.94**	**-6.93**	**-29.40**	**-8.88**	**708.64**	**198.22**
1	**817.05**	**73.77**	**20.11**	**3.43**	**820.63**	**111.99**

CQC method (Clause 7.8.4.4 – IS1893-2002):

(Important note: *Since modal frequencies are well separated in this example, the SRSS modal combination method is sufficient to combine contribution of each mode. For the purpose of demonstration CQC method of modal combination and to compare SRSS and CQC methods following calculations are carried out. However, CQC method is preferred when modal frequencies are closely spaced*)

The contributions of different modes are combined by Complete Quadratic Combination (CQC) method as demonstrated in the following calculations. Shear force quantities in each of the four modes can be expressed as,

$$\lambda_4 = \{V_{41} \quad V_{42} \quad V_{43} \quad V_{44}\} = \{233.30 \quad -66.66 \quad 32.48 \quad -8.26\}$$
$$\lambda_3 = \{V_{31} \quad V_{32} \quad V_{33} \quad V_{34}\} = \{503.90 \quad -80.38 \quad -5.64 \quad 10.81\}$$
$$\lambda_2 = \{V_{21} \quad V_{22} \quad V_{23} \quad V_{24}\} = \{707.94 \quad -6.93 \quad -29.40 \quad -8.88\}$$
$$\lambda_1 = \{V_{11} \quad V_{12} \quad V_{13} \quad V_{14}\} = \{817.05 \quad 73.77 \quad 20.11 \quad 3.43\}$$

Where λ_i is the shear force in the i^{th} mode.

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β_{ij} is the frequency ratio between i^{th} and the j^{th} mode is, $\beta_{ij} = \frac{\omega_j}{\omega_i} = \frac{T_j}{T_i}$, hence considering all the four modes, β_{ij} may be expressed in matrix form as,

$$\beta_{ij} = \begin{bmatrix} T_1/T_1 & T_2/T_1 & T_3/T_1 & T_4/T_1 \\ T_1/T_2 & T_2/T_2 & T_3/T_2 & T_4/T_2 \\ T_1/T_3 & T_2/T_3 & T_3/T_3 & T_4/T_3 \\ T_1/T_4 & T_2/T_4 & T_3/T_4 & T_4/T_4 \end{bmatrix} = \begin{bmatrix} 1.00 & 0.35 & 0.23 & 0.19 \\ 2.86 & 1.00 & 0.66 & 0.55 \\ 4.33 & 1.51 & 1.00 & 0.84 \\ 5.17 & 1.80 & 1.20 & 1.00 \end{bmatrix}$$

Where natural periods of different modes are, $T = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 0.424 \\ 0.148 \\ 0.098 \\ 0.082 \end{Bmatrix}$ sec

Now calculate cross modal coefficient ρ_{ij},

$$\rho_{ij} = \frac{8\varsigma^2 (1+\beta_{ij})\beta_{ij}^{1.5}}{(1-\beta_{ij}^2)^2 + 4\varsigma^2 \beta_{ij} (1+\beta_{ij})^2}$$

Taking damping ratio, $\varsigma = 0.05$ and β_{ij} values computed above, cross modal coefficient ρ_{ij} may be computed and expressed in matrix form as,

$$\rho_{ij} = \begin{bmatrix} 1.0000 & 0.01294 & 0.00460 & 0.00311 \\ 0.09597 & 1.0000 & 0.13526 & 0.06039 \\ 0.07799 & 0.26212 & 1.0000 & 0.51229 \\ 0.07494 & 0.17222 & 0.59962 & 1.0000 \end{bmatrix}$$

For example in the above matrix ρ_{12} & ρ_{34} are computed as,

$$\rho_{12} = \frac{8(0.05)^2 (1+0.35)0.35^{1.5}}{(1-0.35^2)^2 + 4(0.05)^2 (0.35)(1+0.35)^2} = 0.01294$$

$$\rho_{34} = \frac{8(0.05)^2 (1+0.84)0.84^{1.5}}{(1-0.84^2)^2 + 4(0.05)^2 (0.35)(1+0.84)^2} = 0.51229$$

Storey shear forces are computed by combining shear forces of different modes as follows,

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$$V_4 = \sqrt{\{\lambda_4\}\left[\rho_{ij}\right]\{\lambda_4\}^T}$$

$$\lambda_4 = \{233.30 \quad -66.66 \quad 32.48 \quad -8.26\}$$

$$\{\lambda_4\}^T = \{233.30 \quad -66.66 \quad 32.48 \quad -8.26\}^T$$

$$\therefore V_4 = \sqrt{\{\lambda_4\}\begin{bmatrix} 1.0000 & 0.01294 & 0.00460 & 0.00311 \\ 0.09597 & 1.0000 & 0.13526 & 0.06039 \\ 0.07799 & 0.26212 & 1.0000 & 0.51229 \\ 0.07494 & 0.17222 & 0.59962 & 1.0000 \end{bmatrix}\{\lambda_4\}^T}$$

$$V_4 = \sqrt{57747}$$

$$V_4 = 240.31 \ kN$$

Simillarly,

$$\mathrm{V}_3 = 532.85 \ kN$$

$$\mathrm{V}_2 = 706.97 \ kN$$

$$\mathrm{V}_1 = V_{Base} = 826.01 \ kN$$

Now, storey lateral forces are computed from storey shear forces

$$Q_4 = V_4 = 240.31 \ kN$$

$$Q_3 = V_3 - V_4 = 532.85 - 240.31 = 292.31 \ kN$$

$$Q_2 = V_2 - V_3 = 706.97 - 532.85 = 174.12 \ kN$$

$$Q_1 = V_1 - V_2 = 826.01 - 706.97 = 119.04 \ kN$$

Table: Summary of results from different methods of analyses

Storey level	Equivalent static Method		CQC Method		SRSS Method	
	Shear force V_i (kN)	Lateral force F_i (kN)	Shear force V_i (kN)	Lateral force F_i (kN)	Shear force V_i (kN)	Lateral force F_i (kN)
4	**426.53**	**426.53**	**240.31**	**240.31**	**244.94**	**244.94**
3	**737.35**	**310.83**	**532.85**	**292.31**	**510.42**	**265.48**
2	**875.50**	**138.14**	**706.97**	**174.12**	**708.64**	**198.22**
1	**910.03**	**34.54**	**826.01**	**119.04**	**820.63**	**111.99**

The storey lateral forces and shear forces computed from equivalent static method and response spectrum method (dynamic analysis) are compared in the above table. In case of dynamic analysis the responses computed from both CQC and SRSS are tabulated for purpose of comparing these two methods of combining the individual modal response contributions. Comparison clearly indicates that both CQC and SRSS techniques consistently yield comparable shear and lateral force distribution with insignificantly small variation. It is important to note that in this particular example the modal frequencies are well separated. In case of such a situation, it sufficient to implement SRSS combination rule because CQC is a comparatively tedious when calculations are carried out manually.

Note: In the previous example, the building considered herein was analysed using equivalent static load method, wherein the fundamental period (T_a) of the structure is obtained using equation given under Clause 7.6 of IS-1893 (2002). When dynamic analysis is carried out either by the Time History Method or by the Response Spectrum Method, the design base shear computed from dynamic analysis (V_B) shall be compared with a base shear calculated using a fundamental period T_a ($\bar{V}_B$), where T_a is as per Clause 7.6. If base shear obtained from dynamic analysis (V_B) is less than base shear computed from equivalent static load method ($\bar{V}_B$ *i.e., using T_a as per Clause 7.6*), then as per Clause 7.8.2, all the response quantities (for example member forces, displacements, storey forces, storey shears and base reactions) shall be multiplied by ratio $\dfrac{\bar{V}_B}{V_B}$.

In the above example, $\bar{V}_B = 910.03\ kN$

Base shear as calculated by response spectrum method (SRSS) is, $V_B = 820.63\ kN$

$$\therefore\ \frac{\bar{V}_B}{V_B} = \frac{910.03}{820.63} = 1.109$$

Thus, the seismic forces obtained above by dynamic analysis should be scaled up as follows:

$$Q_4 = 244.94 \times 1.109 = 271.64\ kN$$
$$Q_3 = 265.48 \times 1.109 = 294.42\ kN$$
$$Q_2 = 198.22 \times 1.109 = 219.83\ kN$$
$$Q_1 = 111.99 \times 1.109 = 124.20\ kN$$

<u>EXAMPLE: 3</u> (Problem from VTU Question Paper)

<u>Subject code: 06CV834, Exam: December 2010</u>

For a four storeyed RCC office building located in zone V and resting on hard rock, compute the seismic forces as per IS-1893-2002 equivalent static procedure. Height of first is 4.2 m and the remaining three stories are of height 3.2 m each. Plan dimensions (length and width) of the structure are 15 m x 20 m. The RCC frames are infilled with brick masonry.

Dead load on floor 12 kN/m^2 on floors and 10 kN/m^2 on roof. Live = 4 kN/m^2 on floors and 1.5 kN/m^2 on roof.

Also compute the base shear, neglecting the stiffness of infill walls. Compare the base shears for the two cases and comment on the result. **(20 Marks)**

<u>Solution</u>

<u>Given data</u>

Floor area = 15×20=300 m^2

Dead load: on floor = 12 kN/m^2

On roof = 10 kN/m^2

Live load: on floor = 4 kN/m^2

On roof = 1.5 kN/m^2

Note: Only 50% of the live load is lumped at the floors. At roof, no live load is to be lumped

Zone V, Z = 0.36

Assume SMRF thus, R = 5, Soil type = Hard Rock (Type-I)

<u>Load at floor levels:</u>

$$\text{Floors}: W1 = W2 = W3 = [12 + (0.5 \times 4)] \times 300 = 4200 \text{ kN}$$
$$\text{Roof}: W4 = 10 \times 300 = 3000 \text{ kN}$$

Total seismic weight:
$$W = W1 + W2 + W3 + W4 = (3 \times 4200) + 3000 = \mathbf{15600\ kN}$$
Total height of the building:
$$h = (3.2 \times 3) + 4.2 = \mathbf{13.8\ m}$$

<u>CASE – 1: With infill walls</u>

Fundamental natural period :

(Moment resisting frame with in - fill walls)

a) Along 20 m direction :

$$T_a = 0.09\frac{h}{\sqrt{d}} = 0.09\frac{13.8}{\sqrt{20.0}} = 0.2777$$

b) Along 15 m direction :

$$T_a = 0.09\frac{h}{\sqrt{d}} = 0.09\frac{13.8}{\sqrt{15.0}} = 0.3207$$

<u>Along 20 m direction</u>

(Hard Rock),

For Ta=0.2777 s Sa/g = 2.5

For Zone V, Z = 0.36

Importance Factor, I = 1.0

Response Reduction Factor, R = 5.0 (SMRF)

<u>Along 15 m direction</u>

(Hard Rock),

For Ta=0.3207 s Sa/g = 2.5

For Zone V, Z = 0.36

Importance Factor, I = 1.0

Response Reduction Factor, R = 5.0 (SMRF)

Note: Since, Sa/g, Z, I and R values are same for both principal directions, it is sufficient calculate lateral forces in any one the principal axis.

<u>Calculation of Base shear</u>

a) A_h & V_B Along 20 m direction:

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(2.5)\left(\frac{1}{5}\right) = 0.09$$

$$V_B = A_h W = 0.09 \times 15600 = \mathbf{1404.00\ kN}$$

b) A_h & V_B Along 15 m direction:

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(2.5)\left(\frac{1}{5}\right) = 0.09$$

$$V_B = A_h W = 0.09 \times 15600 = \mathbf{1404.00\ kN}$$

Storey lateral forces and shear forces are calculated and tabulated in the following table.

Floor level (i)	W_i (kN)	h_i (m)	$W_i h_i^2$ (kN-m^2)	Storey forces $Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2}$	Storey shear forces [V_i] (Cumulative sum) (kN) Along 20 m Direction	Along 15 m Direction
4	3000	13.8	571320	595.36	595.36	595.36
3	4200	10.6	471912	491.77	1087.13	1087.13
2	4200	7.4	229992	239.67	1326.79	1326.79
1	4200	4.2	74088	77.21	**1404.00**	**1404.00**

CASE – 2: Without infill walls

Fundamental Natural period :
(Moment resisting frame without in-fill walls)
a) Along 20 m & 15 m directions:

$$T_a = 0.075h^{0.75} = 0.075 \times 13.8^{.75} = 0.537 \text{ sec}$$

Along 20 m & 15 m directions

(Hard Rock),

For Ta=0.537 s Sa/g = 1/T = 1/0.537= 1.862

For Zone V, Z = 0.36

Importance Factor, I = 1.0

Response Reduction Factor, R = 5.0 (SMRF)

Base shear:

a) A_h & V_B Along 20 m & 15 m directions:

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(1.862)\left(\frac{1}{5}\right) = 0.06704$$

$$V_B = A_h W = 0.06704 \times 15600 = \mathbf{1045.81\ kN}$$

Storey lateral forces and shear forces are calculated and tabulated in the following table.

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Floor level (*i*)	W_i (*kN*)	h_i (*m*)	$W_i h_i^2$ (*kN-m*^{2})	Storey forces $Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2}$	Storey shear forces [V_i] (Cumulative sum) (kN	
					Along 20 m Direction	Along 15 m Direction
4	3000	13.8	571320	443.47	443.47	443.47
3	4200	10.6	471912	366.31	809.78	809.78
2	4200	7.4	229992	178.52	988.30	988.30
1	4200	4.2	74088	57.51	**1045.81**	**1045.81**

EXAMPLE: 4 (Problem from VTU Question Paper)

Subject code: 06CV834, Exam: June/July 2011

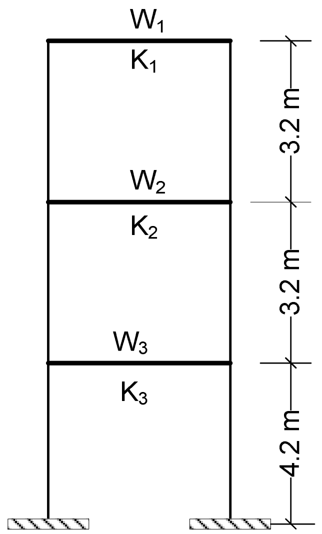
For the residential RCC (SMRF) building founded on soft soil and situated in zone V shown in figure. Compute the seismic forces for each storey using dynamic analysis procedure. Given the free vibration analysis results as follows,

Frequency: $\{\omega\} = \{47.832 \ 120.155 \ 167.00\} \ rad/\sec$

Modes: $\{\phi_1\} = \begin{Bmatrix} 1.00 \\ 0.759 \\ 0.336 \end{Bmatrix}$ $\{\phi_2\} = \begin{Bmatrix} 1.00 \\ -0.805 \\ -1.157 \end{Bmatrix}$ $\{\phi_3\} = \begin{Bmatrix} 1.00 \\ -2.427 \\ 0.075 \end{Bmatrix}$

Seismic weights: $W_1 = W_2 = W_3 = 1962 \ kN$

Stiffness: $k_1 = k_2 = 160 \times 10^3 \ kN/m$ and $k_3 = 240 \times 10^3 \ kN/m$



Solution

Given data

W3=W2=W1=1962 kN,

Frame: SMRF, R = 5,

Zone V: Z = 0.36

Soil type: Soft soil (Type-III)

Structure type: Residential, I=1

Free vibration characteristics:

Modes	Natural Period (sec)	Mode shapes		
		3rd Floor	2nd Floor	1st Floor
Mode 1	0.131	1.000	0.759	0.336
Mode 2	0.052	1.000	-0.805	-1.157
Mode 3	0.038	1.000	-2.427	0.075

Calculation of modal mass and modal participation factor (clause 7.8.4.5):

Storey Level	Seismic weight (W_i), kN	MODE-1		
		ϕ_{i1}	$W_i\ \phi_{i1}$	$W_i\ \phi_{i1}^{\ 2}$
3	1962.00	1.000	1962	1962.00
2	1962.00	0.759	1489.16	1130.27
1	1962.00	0.336	659.23	221.50
Σ	**5886.00**		4110.39	3313.77
Modal mass $M_1 = \frac{\left[\sum W_i\phi_{i1}\right]^2}{g\sum W_i\phi_{i1}^2}$		$= \frac{3313.77^2}{4110.39g} =$ 5098.512 kN/g		
% of Total weight		86.62 %		
Modal participation factor, $P_1 = \frac{\sum W_i\phi_{i1}}{\sum W_i\phi_{i1}^2}$		$= \frac{4110.39}{3313.77} =$ **1.2404**		

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<table>
<tr><th rowspan="2">Storey Level</th><th rowspan="2">Seismic weight (W_i), kN</th><th colspan="3">MODE-2</th></tr>
<tr><th>ϕ_{i2}</th><th>$W_i\ \phi_{i2}$</th><th>$W_i\ \phi_{i2}^{\ 2}$</th></tr>
<tr><td>3</td><td>1962.00</td><td>1.000</td><td>1962.00</td><td>1962.00</td></tr>
<tr><td>2</td><td>1962.00</td><td>-.805</td><td>-1579.41</td><td>1130.27</td></tr>
<tr><td>1</td><td>1962.00</td><td>-1.157</td><td>-2270.03</td><td>221.50</td></tr>
<tr><td>Σ</td><td>5886.00</td><td></td><td>-1887.44</td><td>5859.85</td></tr>
<tr><td colspan="2">Modal mass, $M_2 = \dfrac{\left[\sum W_i\phi_{i2}\right]^2}{g\sum W_i\phi_{i2}^2}$</td><td colspan="3">$= \dfrac{-1887.44^2}{5859.85g} =$ 607.94 kN/g</td></tr>
<tr><td colspan="2">% of Total weight</td><td colspan="3">10.34 %</td></tr>
<tr><td colspan="2">Modal participation factor, $P_2 = \dfrac{\sum W_i\phi_{i2}}{\sum W_i\phi_{i2}^2}$</td><td colspan="3">$= \dfrac{-1887.44}{5859.85} =$ -0.322</td></tr>
</table>

<table>
<tr><th rowspan="2">Storey Level</th><th rowspan="2">Seismic weight (W_i), kN</th><th colspan="3">MODE-3</th></tr>
<tr><th>ϕ_{i3}</th><th>$W_i\ \phi_{i3}$</th><th>$W_i\ \phi_{i3}^{\ 2}$</th></tr>
<tr><td>3</td><td>1962.00</td><td>1.000</td><td>1962</td><td>1962</td></tr>
<tr><td>2</td><td>1962.00</td><td>-2.247</td><td>-4761.77</td><td>11556.83</td></tr>
<tr><td>1</td><td>1962.00</td><td>0.075</td><td>147.15</td><td>11.03625</td></tr>
<tr><td>Σ</td><td>5886.00</td><td></td><td>-2652.62</td><td>13529.86</td></tr>
<tr><td colspan="2">Modal mass, $M_3 = \dfrac{\left[\sum W_i\phi_{i3}\right]^2}{g\sum W_i\phi_{i3}^2}$</td><td colspan="3">$= \dfrac{-2652.62^2}{13829.56g} =$ 520.066 kN/g</td></tr>
<tr><td colspan="2">% of Total weight</td><td colspan="3">8.84 %</td></tr>
<tr><td colspan="2">Modal participation factor, $P_3 = \dfrac{\sum W_i\phi_{i3}}{\sum W_i\phi_{i3}^2}$</td><td colspan="3">$= \dfrac{-2652.62}{13529.86} =$ -0.196</td></tr>
</table>

<u>MODE-1:</u>

$$T_1 = 0.131\ s$$

$$\frac{S_a}{g} = 2.5....(0.10 \le T_1 \le 0.67 - \text{Soft soil})$$

$$A_{h(1)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(2.5)\frac{1}{5} = 0.09$$

$$Q_{i1} = A_{h(1)}\ \phi_{i1}\ P_1\ W_i = 0.09 \times 1.2404 \times (\phi_{i1}\ \ W_i) = 0.11164(\phi_{i1}\ \ W_i)$$

MODE-2:

$T_2 = 0.052\ s\ (T_2 \le 0.10)$

$\frac{S_a}{g} = 1 + (15 \times 0.052) = 1.78\ \ .....\ (0 \le T_2 \le 0.10,\ Soft\ soil)$

$A_{h(2)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(1.78)\frac{1}{5} = 0.0641$

$\therefore A_{h(2)} = \frac{Z}{2} = \frac{0.36}{2} = 0.18 > 0.0641$

$Q_{i2} = A_{h(2)}\ \phi_{i2}\ P_2\ W_i = 0.18 \times -0.322 \times (\phi_{i2} W_i) = -0.05796(\phi_{i2}\ W_i)$

MODE-3:

$T_3 = 0.038\ s\ (T_2 \le 0.10)$

$\frac{S_a}{g} = 1 + (15 \times 0.052) = 1.57\ \ .....\ (0 \le T_3 \le 0.10,\ Soft\ soil)$

$A_{h(3)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(1.57)\frac{1}{5} = 0.0565$

$\therefore A_{h(3)} = \frac{Z}{2} = \frac{0.36}{2} = 0.18 > 0.0565$

$Q_{i3} = A_{h(3)}\ \phi_{i3}\ P_3\ W_i = 0.18 \times -0.196 \times (\phi_{i3} W_i) = -0.03528(\phi_{i3}\ W_i)$

Lateral load calculation by modal analysis – SRSS method

Storey level	Weight Wi (kN)	**Mode – 1** [$Q_{i1} = 0.11164(\phi_{i1}\ W_i)$]		
		ϕ_{i1}	Q_{i1}	V_{i1}
3	1962.00	1.000	219.04	**219.04**
2	1962.00	0.759	166.25	**385.29**
1	1962.00	0.336	73.60	**458.88**

Storey level	Weight Wi (kN)	**Mode – 2** [$Q_{i2} = -0.05796(\phi_{i2}\ W_i)$]		
		ϕ_{i2}	Q_{i2}	V_{i2}
3	1962.00	1.000	-113.72	**-113.72**
2	1962.00	-.805	91.54	**-22.18**
1	1962.00	-1.157	131.57	**109.39**

Storey level	Weight W_i (kN)	**Mode – 3** [$Q_{i3} = -0.03528(\phi_{i3}\ W_i)$]		
		ϕ_{i3}	Q_{i3}	V_{i3}
3	1962.00	1.000	-69.22	**-69.22**
2	1962.00	-2.247	155.54	**86.32**
1	1962.00	0.075	-5.19	**81.13**

<u>SRSS method (Clause 7.8.4.4 – IS1893-2002)</u>:

The contribution of different modes are combined by Square Root of the Sum of the Squares (SRSS) using the following relationship, $V_i = \sqrt{V_{i1}^2 + V_{i2}^2 + V_{i3}^2}$

Then, storey lateral forces are calculated by, $F_i = V_i - V_{i+1}$.

The results obtained are tabulated in the following table.

Storey level	V_{i1}	V_{i2}	V_{i3}	**Combined shear force (SRSS) V_i (kN)**	**Combined lateral force (SRSS) F_i (kN)**
3	**219.04**	**117.25**	**69.22**	**256.32**	**256.32**
2	**385.29**	**22.86**	**-86.32**	**401.13**	**144.81**
1	**458.88**	**-112.79**	**-81.13**	**478.66**	**77.53**

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