UNIT – 3

Flexure and Serviceability Limit State

Beam

A structural member that support transverse (Perpendicular to the axis of the member) load is called a beam. Beams are subjected to bending moment and shear force. Beams are also known as flexural or bending members. In a beam one of the dimensions is very large compared to the other two dimensions. Beams may be of the following types:

a. Singly or doubly reinforced rectangular beams

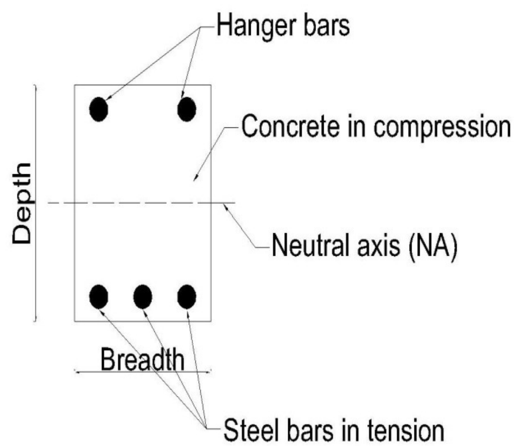

Fig 1: Singly reinforced rectangular beam

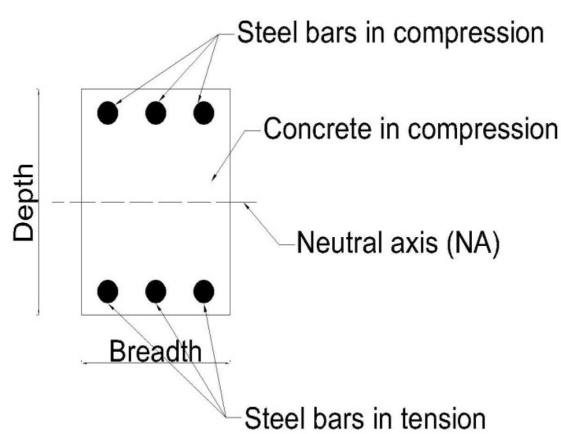

Fig 2: Doubly reinforced rectangular beam

b. Singly or doubly reinforced T-beams

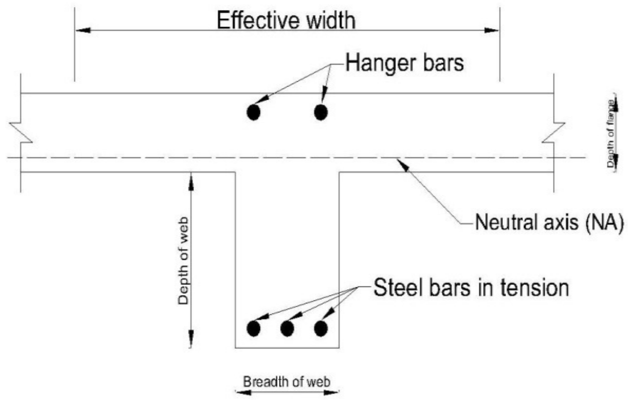

Fig 3: Singly reinforced T beam

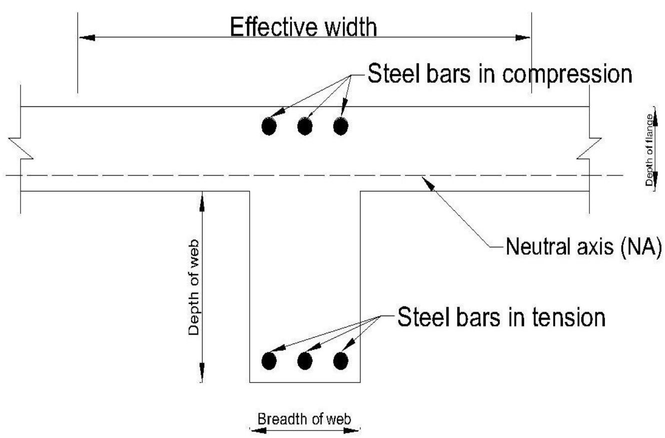

Fig 4: Doubly reinforced T beam

c. Singly or doubly reinforced L-beams

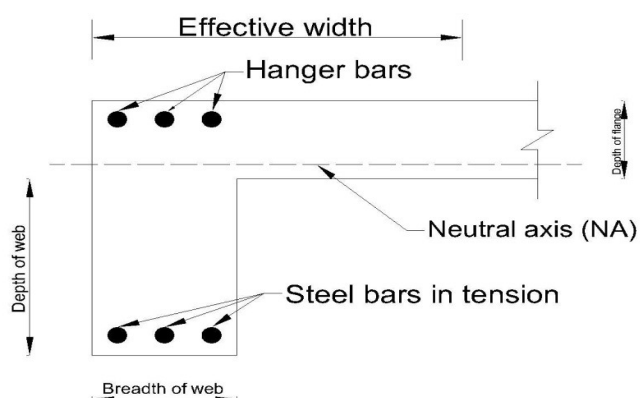

Fig 5: Singly reinforced L beam

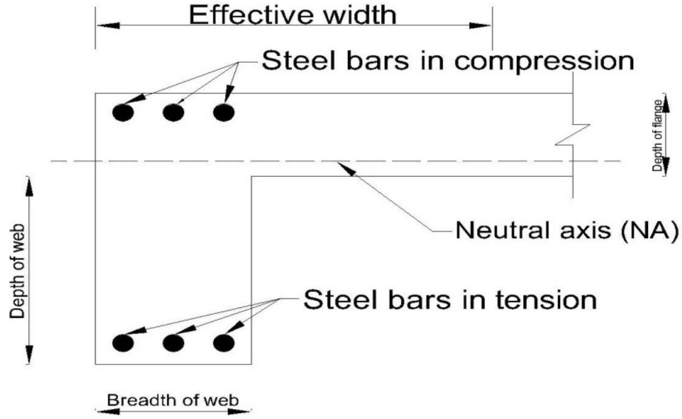

Fig 6: Doubly reinforced L beam

General specification for flexure design of beams

Beams are designed on the basis of limit state of collapse in flexure and checked for other limit states of shear, torsion and serviceability. To ensure safety the resistance to bending, shear, torsion and axial loads at every section should be greater than the appropriate values at that produced by the probable most unfavourable combination of loads on the structure using the appropriate safety factors. The following general specifications and practical requirements are necessary for designing the reinforced cement concrete beams.

a. Selection of grade of concrete

   Apart from strength and deflection, durability shall also be considered to select the grade of concrete to be used. Table 5 of IS 456:2000 shall be referred for the grade of concrete to be used. In this table the grade of concrete to be used is recommended based on the different environmental exposure conditions.

b. Selection of grade of steel

   Normally Fe 250, Fe 415 and Fe 500 are used. In earthquake zones and other places where there are possibilities of vibration, impact, blast etc, Fe 250 (mild steel) is preferred as it is more ductile.

c. Size of the beam

   The size of the beam shall be fixed based on the architectural requirements, placing of reinforcement, economy of the formwork, deflection, design moments and shear. In addition, the depth of the beam depends on the clear height below the beam and the width depends on the thickness of the wall to be constructed below the beam. The width of the beam is usually equal to the width of the wall so that there is no projection or offset at the common surface of contact between the beam and the wall.

   The commonly used widths of the beam are 115 mm, 150 mm, 200 mm, 230 mm, 250 mm, 300 mm.

d. Cover to the reinforcement

Cover is the certain thickness of concrete provided all round the steel bars to give adequate protection to steel against fire, corrosion and other harmful elements present in the atmosphere. It is measured as distance from the outer concrete surface to the nearest surface of steel. The amount of cover to be provided depends on the condition of exposure and shall be as given in the Table 16 of IS 456:2000. The cover shall not be less than the diameter of the bar.

e. Spacing of the bars

The details of spacing of bars to be provided in beams are given in clause 26.3.2 of IS 456. As per this clause the following shall be considered for spacing of bars.

The horizontal distance between two parallel main bars shall usually be not less than the greatest of the following

i. Diameter of the bar if the diameters are equal
ii. The diameter of the larger bar if the diameters are unequal
iii. 5mm more than the nominal maximum size of coarse aggregate

Greater horizontal spacing than the minimum specified above should be provided wherever possible. However when needle vibrators are used, the horizontal distance between bars of a group may be reduced to two thirds the nominal maximum size of the coarse aggregate, provided that sufficient space is left between groups of bars to enable the vibrator to be immersed.

Where there are 2 or more rows of bars, the bars shall be vertically in line and the minimum vertical distance between the bars shall be of the greatest of the following

i. 15 mm
ii. Maximum size of aggregate
iii. Maximum size of bars

Maximum distance between bars in tension in beams:

The maximum distance between parallel reinforcement bars shall not be greater than the values given in table 15 of IS 456:2000.

General Aspects of Serviceability:

The members are designed to withstand safely all loads liable to act on it throughout its life using the limit state of collapse. These members designed should also satisfy the serviceability limit states. To satisfy the serviceability requirements the deflections and cracking in the member should not be excessive and shall be less than the permissible values. Apart from this the other limit states are that of the durability and vibrations. Excessive values beyond this limit state spoil the appearance of the structure and affect the partition walls, flooring etc. This will cause the user discomfort and the structure is said to be unfit for use.

The different load combinations and the corresponding partial safety factors to be used for the limit state of serviceability are given in Table 18 of IS 456:2000.

Load combination	Limit State of Collapse			Limit state of serviceability		
	DL	IL	WL	DL	IL	WL
DL + IL	1.5		1.0	1.0	1.0	-
DL + WL	1.5 or 0.9	-	1.5	1.0	-	1.0
DL + IL + WL	1.2			1.0	0.8	0.8

Limit state of serviceability for flexural members:

Deflection

_The check for deflection is done through the following two methods specified by IS 456:2000 (Refer clause 42.1)

1 Empirical Method

In this method, the deflection criteria of the member is said to be satisfied when the actual value of span to depth ratio of the member is less than the permissible values. The IS code procedure for calculating the permissible values are as given below

a. Choosing the basic values of span to effective depth ratios (l/d) from the following, depending on the type of beam
   1. Cantilever = 8
   2. Simply supported = 20
   3. Continuous = 26
b. Modify the value of basic span to depth ratio to get the allowable span to depth ratio.

   Allowable l/d = Basic l/d x M_t x M_c x M_f

   Where, M_t = Modification factor obtained from fig 4 IS 456:2000. It depends on the area of tension reinforcement provided and the type of steel.

   M_c = Modification factor obtained from fig 5 IS 456:2000. This depends on the area of compression steel used.

   M_f = Reduction factor got from fig 6 of IS 456:2000

Note: The basic values of l/d mentioned above is valid upto spans of 10m. The basic values are multiplied by 10 / span in meters except for cantilever. For cantilevers whose span exceeds 10 m the theoretical method shall be used.

2 Theoretical method of checking deflection

The actual deflections of the members are calculated as per procedure given in annexure 'C' of IS 456:2000. This deflection value shall be limited to the following

i. The final deflection due to all loads including the effects of temperature, creep and shrinkage shall not exceed span / 250.
ii. The deflection including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes shall not exceed span/350 or 20 mm whichever is less.

Cracking in structural members

Cracking of concrete occurs whenever the tensile stress developed is greater than the tensile strength of concrete. This happens due to large values of the following:

1. Flexural tensile stress because of excessive bending under the applied load
2. Diagonal tension due to shear and torsion
3. Direct tensile stress under applied loads (for example hoop tension in a circular tank)
4. Lateral tensile strains accompanying high axis compressive strains due to Poisson's effect (as in a compression test)
5. Settlement of supports

In addition to the above reasons, cracking also occurs because of

1. Restraint against volume changes due to shrinkage, temperature creep and chemical effects.
2. Bond and anchorage failures

Cracking spoils the aesthetics of the structure and also adversely affect the durability of the structure. Presence of wide cracks exposes the reinforcement to the atmosphere due to which the reinforcements get corroded causing the deterioration of concrete. In some cases, such as liquid retaining structures and pressure vessels cracks affects the basic functional requirement itself (such as water tightness in water tank).

Permissible crack width

The permissible crack width in structural concrete members depends on the type of structure and the exposure conditions. The permissible values are prescribed in clause 35.3.2 IS 456:2000 and are shown in table below

Table: Permissible values of crack width as per IS 456:2000

No.	Types of Exposure	Permissible widths of crack at surface (mm)
1	Protected and not exposed to aggressive environmental conditions	0.3
2	Moderate environmental conditions	0.2

Control of cracking

The check for cracking in beams are done through the following 2 methods specified in IS 456:2000 clause 43.1

1. By empirical method:

   In this method, the cracking is said to be in control if proper detailing (i.e. spacing) of reinforcements as specified in clause 26.3.2 of IS 456:2000 is followed. These specifications regarding the spacing have been already discussed under heading general specifications. In addition, the following specifications shall also be considered

   i. In the beams where the depth of the web exceeds 750 mm, side face reinforcement shall be provided along the two faces. The total area of such reinforcement shall not be less than 0.1% of the web area and shall be distributed equally on two faces at a spacing not exceeding 300 mm or web thickness whichever is less. (Refer clause 25.5.1.3 IS456:2000)

   ii. The minimum tension reinforcement in beams to prevent failure in the tension zone by cracking of concrete is given by the following

   $$A_s = 0.85\ f_y / 0.87\ f_y \quad \text{(Refer clause 26.5.1.1 IS 456:2000)}$$

   iii. Provide large number of smaller diameter bars rather than large diameter bars of the same area. This will make the bars well distributed in the tension zone and will reduce the width of the cracks.

2. By crack width computations

   In the case of special structures and in aggressive environmental conditions, it is preferred to compute the width of cracks and compare them with the permissible crack width to ensure the safety of the structure at the limit state of serviceability. The IS 456-2000 has specified an analytical method for the estimation of surface crack width in Annexure-F which is based on the British Code (BS : 8110) specifications where the surface crack width is less than the permissible width, the crack control is said to be satisfied.

Problems:

1. Given the following data of a simply supported T beam, check the deflection criteria by empirical method
   Width of the beam (b) = 230 mm
   Effective depth (d) = 425 mm
   Effective span = 8.0 m
   Area of tension steel required = 977.5 mm^2
   Area of tension steel provided = 1256 mm^2
   Area of compression steel provided = 628 mm^2
   Type of steel = Fe 415
   Width of flange (b_f) = 0.9 m
   Width of web (b_w) = 0.3 m

<u>Solution:</u>

Basic $\frac{l}{d}$ = 20 for simply supported beam from clause 23.2.1

Allowable $\frac{l}{d}$ = Basic $\frac{l}{d}$ x M_t x M_c x M_f ………… (1)

$$P_t = \frac{1265\ x\ 100}{230\ x\ 425} = 1.30\ \%$$

$$f_s = \frac{0.58\ f_y\ \times\ Area\ of\ steel\ required}{Area\ of\ steel\ provided}$$

$$f_s = \frac{0.58\ \times\ 415\ \times\ 977.5}{1256} = 187.3$$

From fig 4, for P_t = 1.3%, f_s = 187.5 N/mm^2

M_t = 1.1 ………. (a)

$$P_c = \frac{628\ \times\ 100}{230\ \times\ 425} = 0.65\%$$

From fig 5, for P_c = 0.65%, M_c = 1.15 ………………(b)

From fig 6, for $\frac{b_w}{b_f} = \frac{0.30}{0.90}$ = 0.33, M_f = 0.80 ………(c)

Substituting a, b and c in equation (1)

We get allowable $\frac{l}{d}$ = 20 x 1.1 x 1.15 x 0.80 = 20.2

Actual $\frac{l}{d} = \frac{8}{0.425} = 18.82 < \text{allowable } \frac{l}{d}$

Hence OK

2. A rectangular beam continuous over several supports has a width of 300 mm and overall depth of 600 mm. The effective length of each of the spans of the beam is 12.0 m. The effective cover is 25 mm. Area of compression steel provided is 942 mm^2 and area of tension steel provided is 1560 mm^2. Adopting Fe 500 steel estimate the safety of the beam for deflection control using the empirical method

Solution:

Allowable $\frac{l}{d}$ = Basic $\frac{l}{d}$ x M_t x M_c x M_f ………… (1)

Basic $\frac{l}{d} = 26$ as the beam is continuous

$$f_s = \frac{0.58\ f_y\ \times\ Area\ of\ steel\ required}{Area\ of\ steel\ provided}$$

$$f_s = \frac{0.58\ \times\ 500\ \times\ 1560}{1560} = 290$$

From fig 4, for $f_s = 290$, $P_t = 0.90$, $M_t = 0.9$……….(a)

From fig 5, for $P_c = 0.54\%$, $M_c = 1.15$ ……………….(b)

From fig 6, for $\frac{b_w}{b_f} = 1.0$, $M_f = 1$ ……………………(c)

The equation (1) shall be multiplied by $\frac{10}{span}$ $i.e$ $\frac{10}{12}$ as the span of the beam is greater than 10.0 m

Allowable $\frac{l}{d} = \frac{10}{12}$ x 26 x 0.9 x 1.15 x 1 = 22.4

Actual $\frac{l}{d} = \frac{12}{0.575} = 20.86 < \text{allowable } \frac{l}{d}$

Hence deflection control is satisfied.

3. Find the effective depth based on the deflection criteria of a cantilever beam of 6m span. Take f_y = 415 N/mm^2, Pt = 1%, Pc = 1%.

Solution:

Allowable $\frac{l}{d}$ = Basic $\frac{l}{d}$ x M_t x M_c x M_f

Basic $\frac{l}{d}$ = 7 for cantilever beam

Assume $\frac{A_{st}\, required}{A_{st}\, provided}$ = 1.0

f_s = 0.58 x 415 x 1= 240.7

From fig 4, for f_s = 240, P_t = 1%, M_t = 1.0

From fig 5, for P_c = 1%, M_c = 1.25

From fig 6, for $\frac{b_w}{b_f} =$ 1.0, M_f = 1

Allowable $\frac{l}{d}$ = 7 x 1.0 x 1.25 x 1.0 = 8.75

$d = \frac{l}{8.75} = \frac{6000}{8.75}$ = 685 mm

4. A simply supported beam of rectangular cross section 250mm wide and 450mm overall depth is used over an effective span of 4.0m. The beam is reinforced with 3 bars of 20mm diameter Fe 415 HYSD bars at an effective depth of 400mm. Two anchor bars of 10mm diameter are provided. The self weight of the beam together with the dead load on the beam is 4 kN/m. Service load acting on the beam is 10 kN/m. Using M20 grade concrete, compute
    a. Short term deflection
    b. Long term deflection

Solution:

Data b = 250 mm, D = 450 mm, d = 400 mm, f_y = 415 N/mm^2

A_{st} = 3 x $\frac{\pi}{4}$ x 20^2 = 942 mm^2, l = 4.0 m, D.L = 4 kN/m, Service load = 10 kN/m,

Total load = 14 kN/m, f_{ck} = 20, A_{sc} = 2 x $\frac{\pi}{4}$ x 10^2 = 158 mm^2

E_s = 2.1 x 10^5 , Ec = 5000 $\sqrt{f_{ck}}$ = 22360 N/mm^2

$m = \frac{280}{3\,\sigma_{cbc}} = \frac{280}{3\ x\ 7} = 13.3$

$f_{cr} = 0.7\sqrt{f_{ck}} = 0.7\sqrt{20} = 3.13$ N/mm^2

a. Short term deflection

To determine the depth of N.A

Equating the moment of compression area to that of the tension area, we get

$b * x * \frac{x}{2} = m * A_{st} * (d-x)$

'm' is used to convert the steel into equivalent concrete area

$250 * \frac{x^2}{2} = 13 * 942 * (400-x)$

Solving, x = 155 mm from the top

Cracked MOI $I_r = \frac{250 \times 155^3}{12} + (250 \times 155)$ x $(155/2)^2$ + 13 x 942 (400 – 155)

= 10.45 x 10^8 mm^4

(2) I_{gr} = Gross MOI = $\frac{250\ x\ 450^3}{12}$ = 18.98 x 10^8 mm^4

(3) M = Maximum BM under service load

$M = \frac{w\ l^2}{8} = \frac{14 \times 4^2}{8}$ = 28 kN = 28 x 10^6 N-mm

(4) Cracked moment of inertia

$M_r = \frac{f_{cr}\, I_{gr}}{y_t} = \frac{3.13 \times 18.98 \times 10^8}{0.5 \times 450}$ = 26 x 10^6 N-mm

Lever arm = z = $\left(d - \frac{x}{3}\right)$

$= \left(400 - \frac{155}{3}\right)$ = 348.34 mm

(5) $I_{eff} = \left[\frac{I_r}{1.2 - \left(\frac{m_r}{m}\right)\left(\frac{z}{d}\right)\left(1 - \frac{x}{d}\right)\left(\frac{b_w}{b}\right)}\right]$

$\left[\frac{10.45 \times 10^8}{1.2 - \left(\frac{26 \times 10^6}{28 \times 10^6}\right)\left(\frac{348.34}{400}\right)\left(1 - \frac{155}{400}\right)(1)}\right]$

$I_{eff} = 14.93 \times 10^8$ mm^4

Further $I_r < I_{eff} < I_{gr}$

(6) Maximum short term deflection

$$a_{i(perm)} = \frac{K_w\, w\, l^4}{E_c\, I_{eff}} = \frac{5}{384}\ \frac{14 \times (4000)^2}{22360 \times 14.93 \times 10^8} = 1.39 \text{ mm}$$

$K_w = \frac{5}{384}$ for SSB with UDL

b. Long term deflection

(1) Shrinkage deflection (a_{cs}):

$a_{cs} = K_3\, \psi_{cs}\, L^2$

K_3 = 0.125 for simply supported beam from Annexure C-3.1

ψ_{cs} = Shrinkage curvature = $K_4 \left(\frac{\epsilon_{cs}}{D}\right)$

ϵ_{cs} = Ultimate shrinkage strain of concrete (refer 6.2.4) = 0.0003

$$P_t = \frac{100 \times 942}{250 \times 400} = 0.942$$

$$P_c = \frac{100 \times 158}{250 \times 400} = 0.158$$

$P_t - P_c = (0.942 - 0.158) = 0.784$

This is greater than 0.25 and less than 1.0 Hence ok.

Therefore $K_4 = 0.72 \times \frac{P_t - P_c}{\sqrt{P_t}} = 0.72 \times \frac{0.942 - 0.158}{\sqrt{0.942}}$

$K_4 = 0.58$

$\psi_{cs} = \frac{0.58 \times 0.0003}{450} = 3.866 \times 10^{-7}$

$a_{cs} = K_3\, \psi_{cs}\, L^2$

$= 0.125 \times 3.866 \times 10^{-7} \times (4000)^2$

$= 0.773$ mm

(2) Creep deflection [$a_{cc(perm)}$]

Creep deflection $a_{cc(perm)} = a_{icc(perm)} - a_{i(perm)}$

Where, $a_{cc(perm)}$ = creep deflection due to permanent loads

$a_{icc(perm)}$ = short term deflection + creep deflection

$a_{i(perm)}$ = short term deflection

$$a_{icc\ (perm)} = K_w \left(\frac{w\, l^3}{E_{ce}\, I_{eff}}\right)$$

$$E_{ce} = \frac{E_c}{(1+\theta)} = \frac{E_c}{(1+1.6)}$$

θ = Creep coefficient = 1.6 for 28 days loading

$a_{icc(perm)}$ = 2.6 x short term deflection

= 2.6 x $a_{i(perm)}$

= 2.6 x 1.39 = 3.614 mm

Creep deflection $a_{cc(perm)}$ = 3.614 – 1.39 = 2.224 mm

Total long term deflection = shrinkage deflection + Creep deflection

= 0.773 + 2.224 = 3.013 mm

Total deflection = Short term deflection + Long term deflection

= 1.39 + 3.013 = 4.402 mm

5. A simply supported beam of rectangular section spanning over 6 m has a width of 300mm and overall depth of 600 mm. The beam is reinforced with 4 bars of 25 mm diameter on the tension side at an effective depth of 550 mm spaced 50 mm centres. The beam is subjected to a working load moment of 160 kN.m at the centre of the span section. Using M-25 grade concrete and Fe-415 HYSD bars, check the beam for serviceability limit state of cracking according to IS:456-2000 method. The beam is protected and not exposed to aggressive environmental conditions.

   a. Data:

b = 300 mm	f_{ck} = 25 N/mm^2
h = 600 mm	f_y = 415 N/mm^2
d = 550 mm	Es = 2 x 10^5 N/mm^2
M = 160 kN.m	spacing between bars s = 5 mm
A_{st} = 1963 mm^2	cover = 50mm

For f_{ck} = 25 N/mm^2, from table 21 IS 456

$$\sigma_{cbc} = 8.5\ N/mm^2$$

$$m = \frac{280}{3\sigma_{cbc}} = 11$$

b. Neutral axis depth

Let x = depth of neutral axis

Then we have 0.5 bx^2 = m A_{st} (d-x)

0.5 x 300x^2 = 11 x 1963 (550-x)

Solving, x = 220 mm

c. Cracked moment of area (I_r):

$I_r = (bx^3/3) + m\ A_{st}\ r^2$

Where r = (d-x) = (550-220) = 330 mm

$$I_r = \left(\frac{300 \times 220^3}{3}\right) + \ (11\ \times 1963\ \times\ 330^2)$$

= 34.1 x 10^8 mm^4

d. Maximum width of cracks

Cover = C_{min} = (50 - 12.5) = 37.5

$a_{cr} = [(0.5\ S)^2 + C^2_{min}]^{1/2}$

$= [(0.5 \times 50)^2 + 37.5^2]^{1/2}$

= 45

Crack width will be maximum at the soffit of the beam

Distance of the centroid of steel from neutral axis

= r = (d-x) = (50-220) = 330 mm

Therefore $\epsilon_1 = \left(\frac{f_s}{E_s}\right)\left[\frac{h-x}{d-x}\right]$

Where, $f_s = m\left[\frac{M\ y}{I_r}\right]\ = 11\left[\frac{160\ \times 10^6\ \times 330}{34.1 \times 10^8}\right]$ = 170 N/mm^2

Therefore, $\epsilon_1\ = \left(\frac{170}{2 \times 10^5}\right)\left[\frac{600-220}{550-220}\right]$ = 9.78 x 10^{-4}

$$\epsilon_m = \epsilon_1 - \left[\frac{b_t\ (h-x)(h-x)}{3\ E_s\ A_s\ (d-x)}\right]$$

$$\epsilon_m = (9.78\ \times\ 10^{-4}) - \left[\frac{300\ (600-220)(600-220)}{3 \times 2 \times 10^5 \times 1963\ (550-220)}\right]$$

= 8.67 x 10^{-4}

Maximum width of crack is expressed as:

$$W_{cr} = \left[\frac{3\ \mathrm{a_{cr}}\ \epsilon_m}{1+2\left\{\frac{a_{cr}-c_{min}}{h-x}\right\}}\right]$$

$$W_{cr} = \left[\frac{3 \times 45 \times 8.67 \times 10^{-4}}{1+2\left\{\frac{45-37.5}{600-220}\right\}}\right]$$

= 0.113 mm $<$ Permissible crack width of 0.3mm from clause 35.3.2 page 67. Hence ok.