EDUSAT Program No. 7

Structural Analysis II

(CV-51)

V Sem B.E (Civil)

Chapter-1: Redundant Trusses

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ANALYSIS OF TRUSSES REDUNDANT TO FIRST DEGREE

The structure shown in figure 1 is internally redundant[Di = m + r – 2j = 6 + 3 - 2 x 4 = 1] first degree. Any member can be taken as redundant member. Member AB is taken as redundant and the frame is made determinant as shown in figure 2.

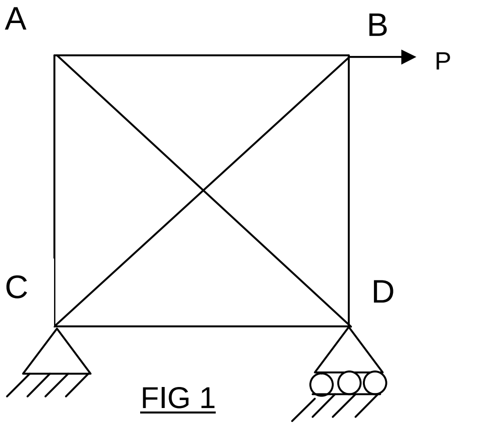


FIG 1

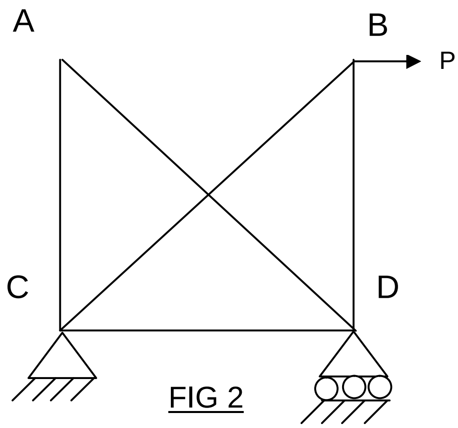


FIG 2

Evaluate the forces in all the members due to external loading. Let the force be F' in any member. Actual force in the redundant member is assumed as tensile force of magnitude 'X' as shown in figure 3. Unit forces are applied at A and B along AB as shown in figure 4.

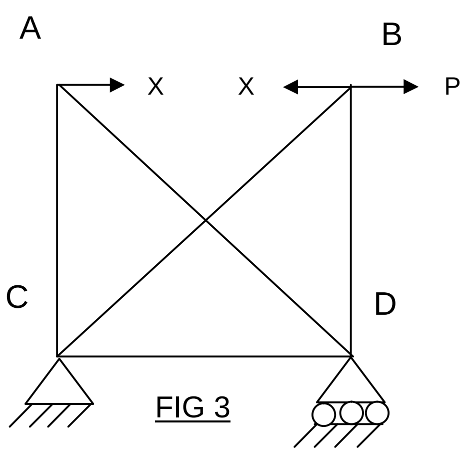


FIG 3

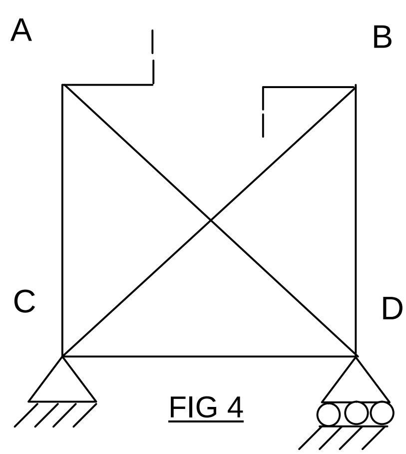


FIG 4

Let the force in any member due to the unit force be u_i

$\therefore$ Due to force X in AB, the force in a member will be $u_i .X$

Then, the total force in a member due to external loading and force in AB = $F_i + u_i .X$

The total strain energy of a structure will be

$$U = \frac{F^2L}{2AE} = \sum \frac{\left(F_i^{'} + u_i X\right)^2 L_i}{2AE} + \frac{X^2 L_R}{2AE_R}$$

By the theorem of least work, W K T,

$$\frac{\partial U}{\partial x} = 0$$

i.e. $$\frac{\partial U}{\partial x} = \sum \frac{\left(F_i^{'} + u_i X\right)^2 L_i}{2AE} + \frac{X^2 L_R}{2AE_R} = 0$$

$$\frac{XL_R}{A_R E} = \frac{L_i u_i^2 X}{AE} + \frac{F_i^{'} u_i L_i}{AE}$$

$$\frac{X}{E}\left(\frac{L_R}{A_R} + \frac{L_i u_i^2}{A}\right) = \frac{F_i^{'} u_i L_i}{AE}$$

$$X = \frac{\sum_{1}^{m\ 1} \frac{F_i^{'} u_i L_i}{AE}}{\sum_{1}^{m\ 1} \frac{u_i^2 L_i}{AE} + \frac{L_R}{A_R E}}$$

<u>IN CASE OF EXTERNAL REDUNDANCY</u>

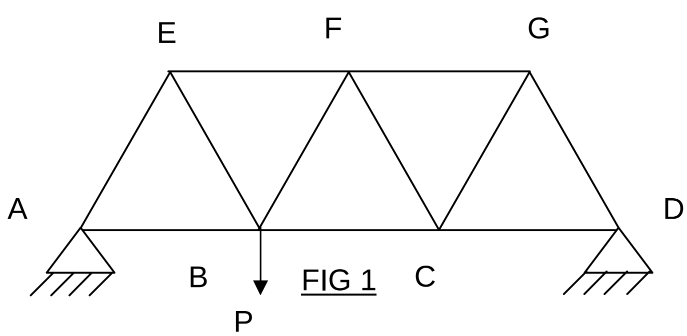


FIG 1

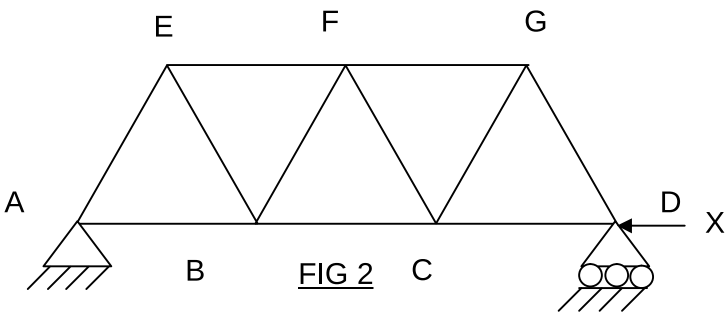


FIG 2

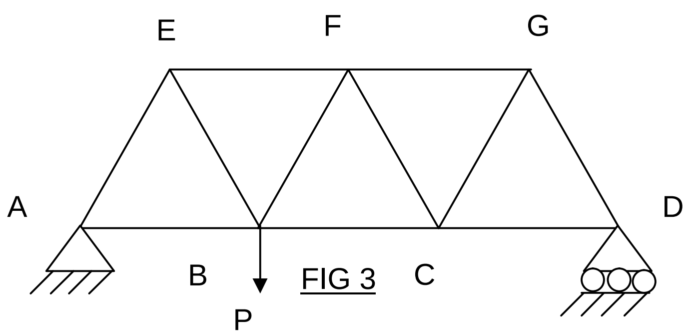


FIG 3

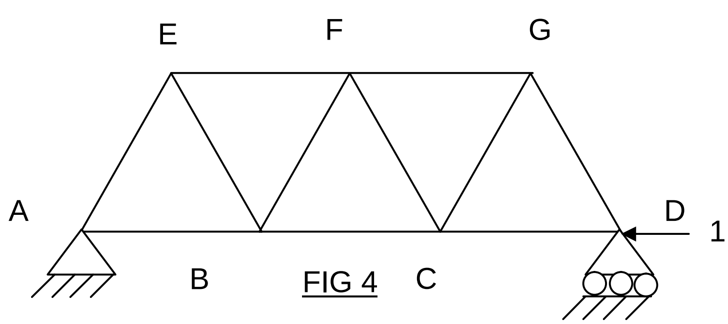


FIG 4

Force in each member, $F = F^{'} + uX$

$$U = \frac{F^2 L}{2AE} = \frac{\left(F^{'} + uX\right)^2}{2AE} L$$

$$\frac{\partial U}{\partial x} = 0 = \sum \frac{\left(F^{'} + uX\right) uL}{AE}$$

$$X = \frac{\sum \frac{F' uL}{AE}}{\sum \frac{u^2 L}{AE}}$$

In case of support ‘D’ yields by an amount ‘δ’ in the direction of X then,

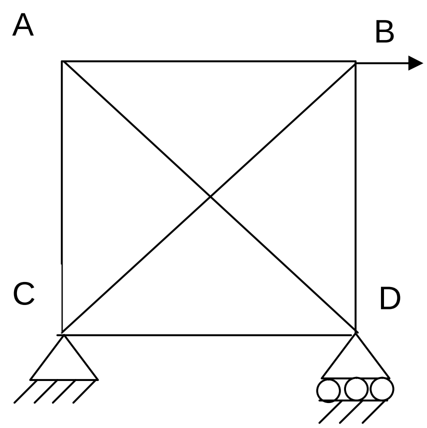
$$\frac{\partial U}{\partial x} = \partial = \sum \frac{(F' + uX)uL}{AE}$$

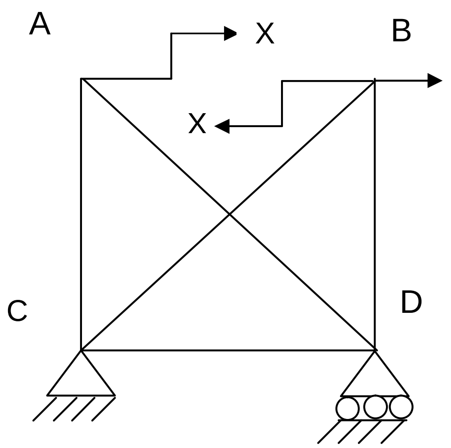
$$\sum \frac{F' uL}{AE} + \sum \frac{u^2 XL}{AE} = \delta$$

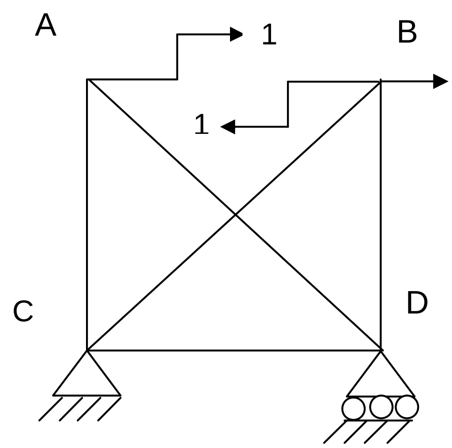
$$\sum \frac{u^2 XL}{AE} = \delta - \sum \frac{F' uL}{AE}$$

$$X = \frac{\delta - \sum \frac{F' uL}{AE}}{\sum \frac{u^2 L}{AE}}$$

Alternative Procedure: Cut the redundant member and let both parts of it remain in the basic structure; the unknown is a pair of forces pulling at the cut ends as shown in figure.







And the compatibility condition is that the total overlap at the cut due to combined action of the applied loads and the redundant forces must be zero.

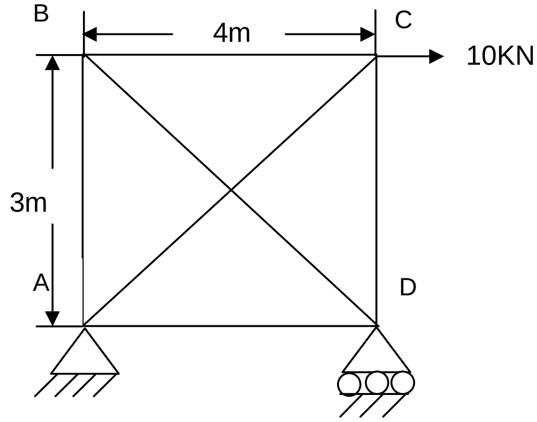
i.e., $$\frac{\partial U}{\partial X} = \delta = 0$$

$$U = \frac{F^2 L}{2AE} = \frac{\left(F' + uX\right)^2}{2AE} L$$

$$X = \frac{-\sum \frac{F' u L}{AE}}{\sum \frac{u^2 L}{AE}}$$

EXAMPLE: 1

Find the forces in all the members for the frame shown in figure. A = 2000mm^2 for all members, E = 2 x 105 N/mm^2.

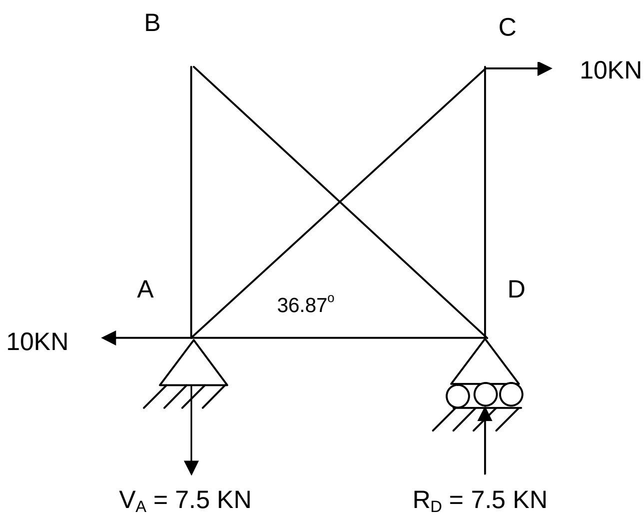


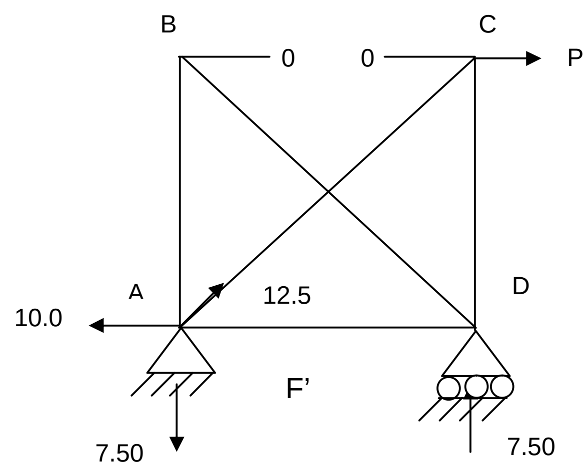
SOLUTION:

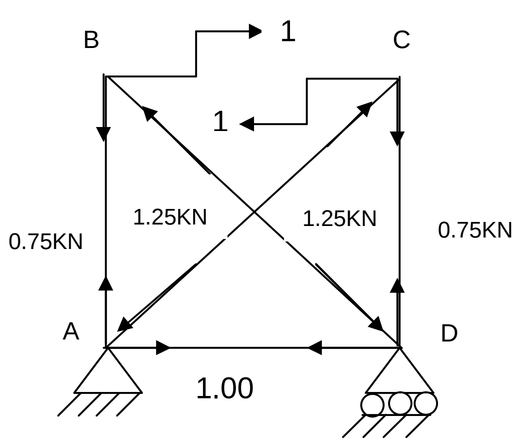
The given structure is statically indeterminate to single degree internally. Let the member BC is considered as redundant. So, introduce a cut in the member BC.

Using method of joints or sections, calculate the forces in all the members.

$R_D = \frac{1}{4} (10 \times 3) = 7.5 KN$







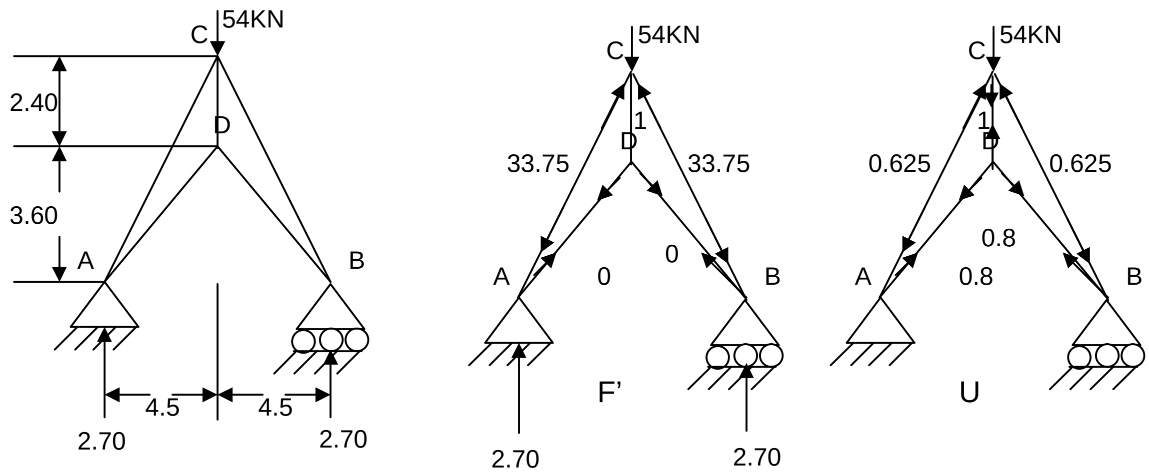
Member	Length	AE in KN	F' in KN	u	$\frac{F'uL}{AE}$	$\frac{u^2L}{AE}$	F = F' + uX
AB	3.0	$4X10^5$	0	0.75	0	$0.4211X10^{-5}$	0.75X = 2.64

BC	4.0	$4X10^5$	0	1.00	0	$1.000X10^{-5}$	1.00 X = 3.52
CD	3.0	$4X10^5$	-7.5	0.75	$-4.211X10^{-5}$	$0.4211X10^{-5}$	-7.5 + 0.75X = -4.86
DA	4.0	$4X10^5$	0	1.00	0	$1.000X10^{-5}$	1X = 3.52
BD	5.0	$4X10^5$	0	-1.25	0	$0.1953X10^{-5}$	-1.25X = - 4.44
AC	4.0	$4X10^5$	12.5	-1.25	$-14.53X10^{-5}$	$0.1953X10^{-5}$	12.5-1.25X = 8.11

$$X = \frac{\sum \frac{F'uL}{AE}}{\sum \frac{u^2L}{AE}} = 3.52$$

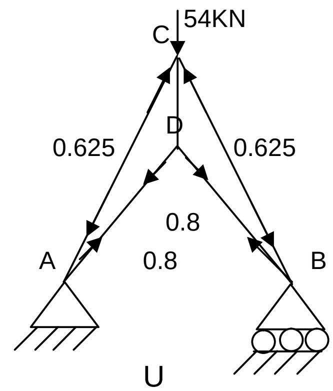
EXAMPLE: 2

Analyse the given truss and find the forces in all the members. Take CD as redundant. Given 4 x 10^{-3} m^2 and E = 200 x 106 KN/m^2



Member	Length	AE in KN	F' in KN	u	$\frac{F'uL}{AE}$	$\frac{u^2L}{AE}$	F = F' + Xu
AC	7.50	$8.00x10^5$	-33.75	-0.625	$1.977x10^{-4}$	$3.6621x10^{-4}$	-20.08
CB	7.50	$8.00x10^5$	-33.75	-0.625	$1.977x10^{-4}$	$3.6621x10^{-4}$	-20.08
AD	5.7628	$1.28x10^6$	0	0.800	0	$2.8814x10^{-4}$	-17.4932
BD	5.7628	$1.28x10^6$	0	0.800	0	$2.8814x10^{-4}$	-17.4932
DC	2.40	$4.80x10^5$	0	1.000	0	$5.0000x10^{-4}$	-21.8665
					$3.955x10^{-4}$	**$18.087x10^{-6}$**	

$$X = \frac{\sum \frac{F'uL}{AE}}{\sum \frac{u^2L}{AE}} = 21.8665$$



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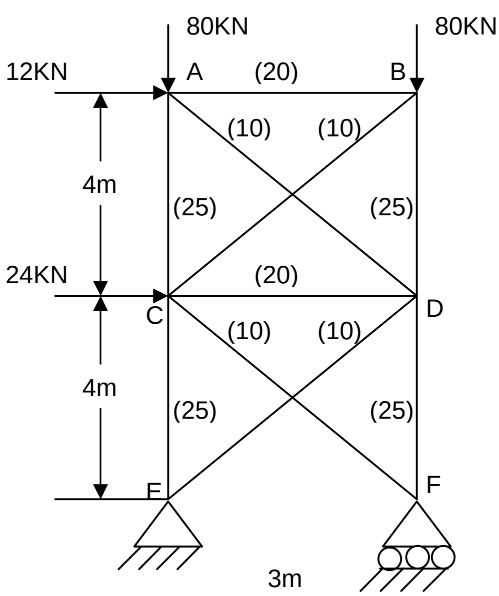
EXAMPLE: 3

Analyse the truss shown in figure. Take CD as redundant. Take E = 200 x 106 KN/m^2



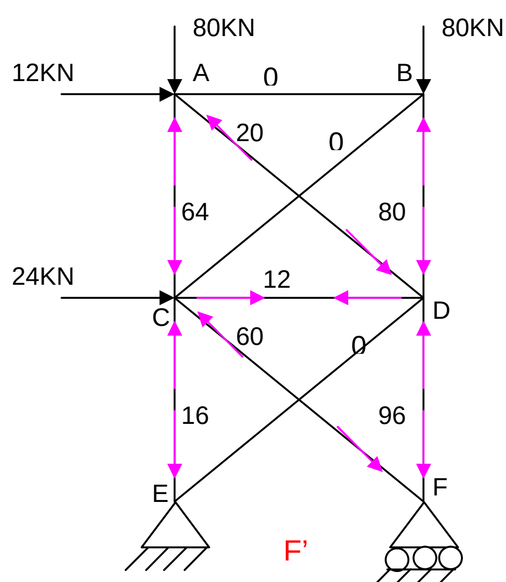


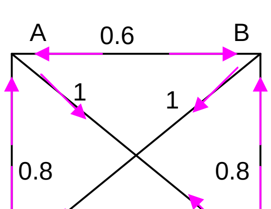
FIG .1

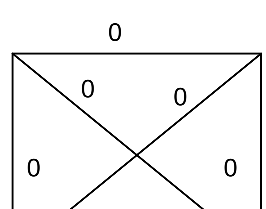
Here, m = 10, j = 6, r = 4

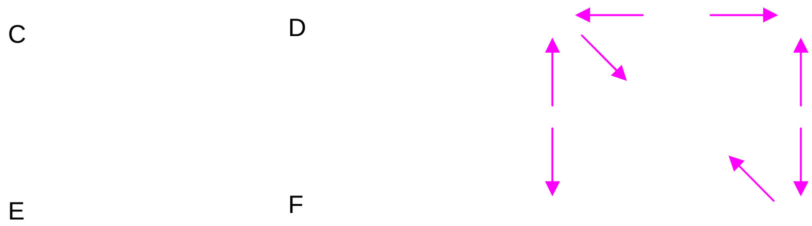
$D_i = 10 + 4 – 2 \times 6 = 2$

$D_{ei} = R – r = 4 – 3 = 1$

Since, the given structure is statically indeterminate to second degree, let the members CB and DE are considered as redundant members. Introduce cut in these two members. The basic determinate structure is as shown in figure.







Memb	L 'm'	AE 10^6	F'	u_i	u_2	$F'u_iL$	$F'u_2L$	u_1^2L	u_2^2L	u_1u_2L	$F=F'+u_1X_1+u_2X_2$
AB	3	40	0	-0.6	0	0	0	$2.7x10^{-8}$	0	0	3.6945
BD	4	50	-80	-0.8	0	$5.12X10^{-6}$	0	$5.12x10^{-6}$	0	0	-75.0736
DF	4	50	-96	0	-0.8	0	$6.144X10^{-6}$	0	$5.12x10^{-8}$	0	-106.848
AC	4	50	-64	-0.8	0	$4.096X10^{-6}$	0	$5.12x10^{-8}$	0	0	-59.0736
CE	4	50	-16	0	-0.8	0	$1.024X10^{-6}$	0	$5.12x10^{-8}$	0	-26.848
AD	5	20	-20	1	0	$-5.00X10^{-6}$	0	$2.5x10^{-7}$	0	0	-26.158
BC	5	20	0	1	0	0	0	$2.5x10^{-7}$	0	0	-6.358
CF	5	20	-60	0	1	0	$-1.5X10^{-5}$	0	$2.5x10^{-7}$	0	-46.440
DE	5	20	0	0	1	0	0	0	$2.5x10^{-7}$	0	13.560
CD	3	40	12	-0.6	-0.6	$-5.40X10^{-7}$	$-5.40X10^{-7}$	$2.7x10^{-8}$	$2.7x10^{-8}$	$2.7x10^{-8}$	7.558

$\Sigma F'.u_1.L + \Sigma u_1^2.L.X_1 + \Sigma u_1.u_2.L..X_2 = 0$

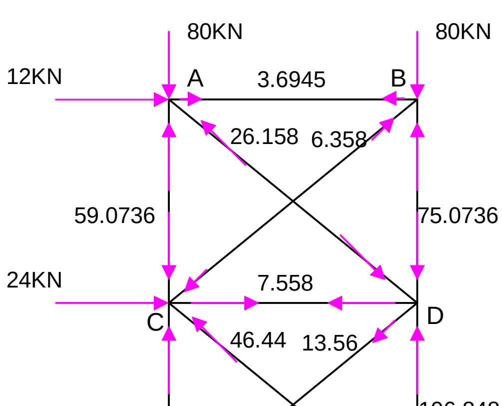
$\Sigma F'.u_2.L + \Sigma u_1.u_2.L..X_1 + \Sigma u_2^2.L.X_2 = 0$

$3.676 + 6.564 X_1 + 0.27 X_2 = 0$

$-8.372 + 0.27 X_1 + 6.294 X_2 = 0$

$X_1 = -6.15$

$X_2 = 13.56$



LACK OF FIT

At the time of construction of structure, If a member is found to be slightly shorter or longer (Lack of Fit), the member is forced in position. This induces forces in that member as well as in all the other members.

Let us consider the structure shown in figure (1) in which the member CF is short by an amount ‘δl’, When this member is forced into position; it is subjected to tensile forces.

Let ‘X’ be the force in the member due to force fitting as shown in figure (2).

Let F_i be the force developed in i^{th} member due to the force ‘X’ in the member CF.

$\therefore F_i = u_i X$

Where U_i is the force in the i^{th} member due to unit load applied to member CF in place of ‘X’.

From Castigliano’s II theorem, the displacement of point ‘F’ relative to ‘C’ in the direction of CF is given by

$$\delta l = \frac{\partial U}{\partial x} = \frac{\partial}{\partial x} \sum \frac{F^2 L}{2AE}$$

$$\delta l = \frac{\partial}{\partial x} \sum \frac{(u_1 X)^2 L}{2AE}$$

$$\delta l = \sum \frac{u_i^2 X L}{AE}$$

$$X = \frac{\delta l}{\sum \frac{u_i^2 L}{AE}}$$ --------For member shorter than the actual member.

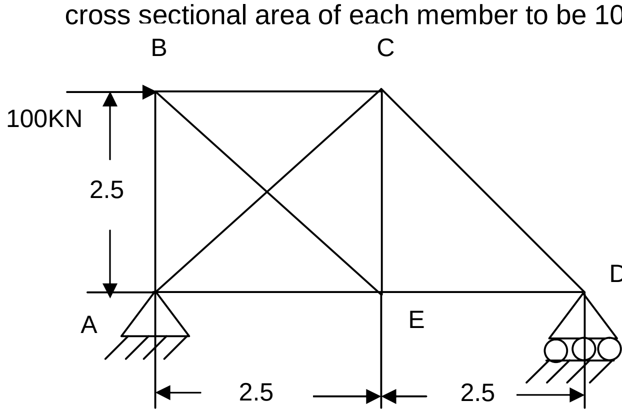
$\therefore$ Final force = $F_i = u_i X$.

NOTE: If the member under consideration is longer than the actual member

$$X = \frac{\delta l}{\sum_1^m \frac{u_i^2 L}{AE}}$$

EXAMPLE: 1

Determine the force in the member BE of the given pin jointed truss shown in figure. Member BE is last to be added to the structure and was initially 0.12cm long. Forces are to be found both before and after the application of external load. Assume the cross sectional area of each member to be $10cm^2$ and E = 200GPa.



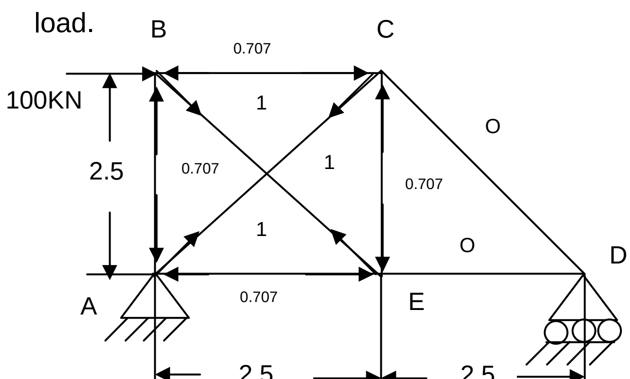
SOLUTION:

M = 8, j = 5, r = 3,

Di = m + r – 2j = 8 + 3 - 2 x 5 = 1

Let BE be redundant.

i) Before External Loading: Let 'X' be the force in member BE. Replace 'X' by unit load.

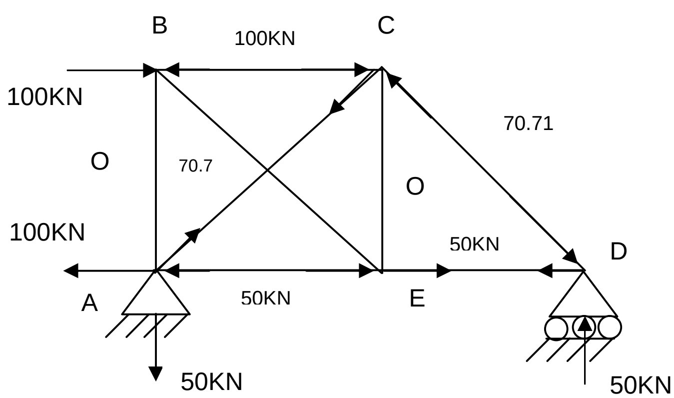


$$\sum \frac{u_i^2 L}{AE} = 6.04 \times 10^{5}$$

$$X = \frac{\delta l}{\sum \frac{u_i^2 L}{AE}} = \frac{1.2 \times 10^{3}}{6.04^{5}} = 19.86$$

Force in the member BE before loading (comp)

ii) After External Loading: The forces in all the members will change. Assuming the member BE as redundant; introduce a cut in the member BE and find the forces in all the members due to external loading. i.e., F'.



Member	Length 'm'	AE KN'	F'	U	$\frac{F'UL}{AE} \times 10^{-4}$	$\frac{U^2 L}{AE} \times 10^{-4}$	F = F' + XU	Force due to lack of fit	Net force
AB	2.50	X 1 0	0	-0.7071	0	6.25	19.82	14.04	33.86

Member	Length 'm'	AE in KN	U_i	$\frac{U_i^2 L}{AE} \times 10^{-6}$	$F_i = U_i X$
AB	2.50	2×10^5	-0.707	6.248	14.04
BC	2.50		-0.707	6.248	14.04
CD	3.54		0	0.00	0.00
DE	2.50		0	0.00	0.00
AE	2.50		-0.707	6.248	14.04
BE	3.54		1.00	17.70	-19.86
AC	3.54		1.00	17.70	-19.86
CE	2.50		-0.707	6.248	14.04

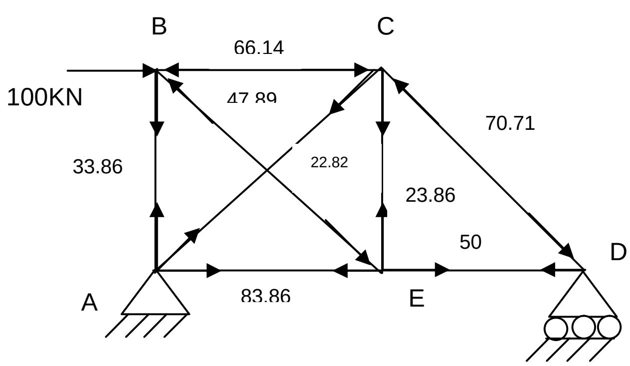
BC	2.50		-100.00	-0.7071	8.84	6.25	-80.18	14.04	-66.14
CD	3.54		-70.71	0	0	0	-70.71	0	-70.71
DE	2.50		50.00	0	0	0	50.00	0	50.00
AE	2.50		50.00	-0.7071	-4.42	6.25	69.82	14.04	83.86
BE	3.54		0	1	0	17.70	-28.03	-19.86	-47.89
AC	3.54		70.71	1	12.51	17.70	42.68	-19.86	22.82
CE	2.50		0	-0.7071	0	6.25	19.82	14.04	33.36
					$1.693X10^{-3}$	**$6.04X10^{-5}$**			

$$X = \frac{\sum \frac{F'uL}{AE}}{\sum \frac{u^2L}{AE}} = \frac{1.693X10^{3}}{6.04X10^{5}} = 28.03$$

Force in the member = Force due to lack of fit + Force due to external loading

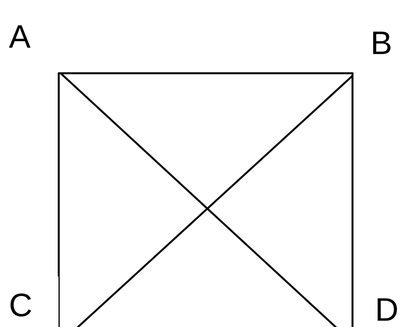
= - 19.86 -28.03

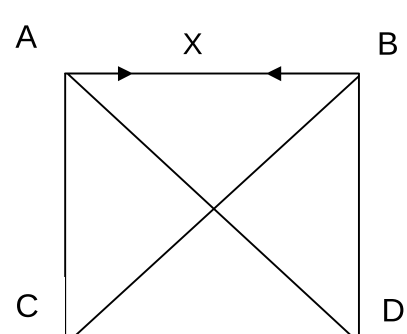
= **<u>- 47.89 KN</u>** (Comp)



<u>TEMPERATURE STRESSES IN REDUNDANT TRUSSES</u>

Change in temperature causes change in length of a member. In redundant trusses, the change in length of any member gives rise to force in all other members. Consider a redundant truss shown in figure below.





Let the temperature of member AB decreases by ‘t’ °C. Then the contraction of the member AB is given by

$\partial l = \alpha.t.l$

But, the free contraction is not possible in the truss. Hence, tensile force of magnitude ‘X’ develops in the member AB. This causes movements of joints A and B in the truss. The compatibility condition demands elongation of member AB and movement of joints A and B. The value of ‘X’ is given by

$$X = \frac{\delta l}{\sum \frac{u_i^2 L}{AE}}$$

i.e., $$X = \frac{\alpha.t.l}{\sum \frac{u_i^2 L}{AE}}$$

Where α = Coefficient of linear expansion or thermal expansion

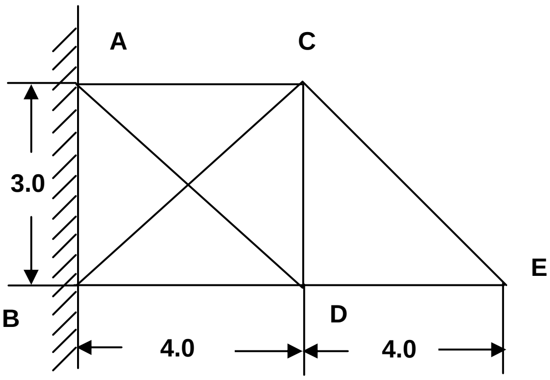
U_i = Forces in various members due to unit load applied for the member under consideration.

NOTE: Use –ve sign for increase in temperature

i.e., $$X = \frac{\alpha.t.l}{\sum \frac{u_i^2 L}{AE}}$$

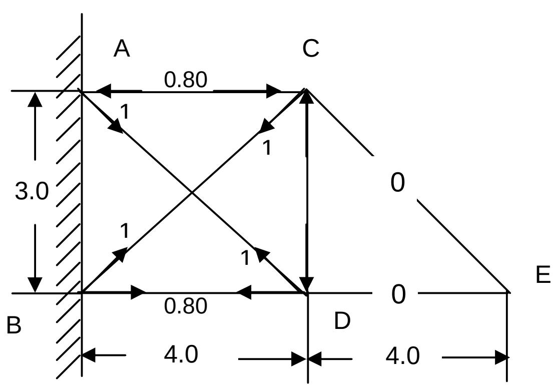
EXAMPLE.1

The members of the frame shown in figure have a cross sectional area of 10 cm^2. If there is a rise in temperature of member AD by 30 °C, determine the forces due to change in temperature. Given, Coefficient of expansion, α = 12 x 10^{-6} / °C. and E = 205 Gpa.



SOLUTION:

Since the free expansion of the member AD is prevented, force develops in the member AD. Let this force be 'X'. Replace 'X' by unit force at A and D and remove the members AD as shown in figure below.

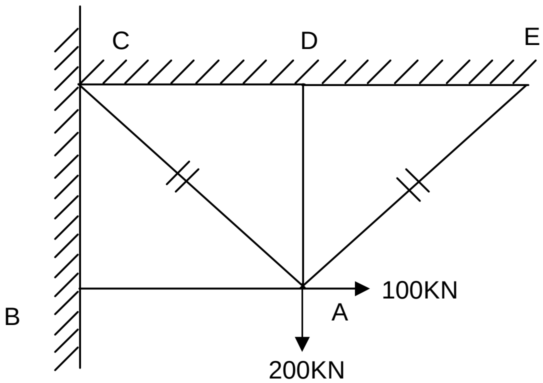


Member	Length 'm'	AE in KN	u_i	$\frac{u_i^2 L}{AE} \times 10^5$	$F_i = u_i X$
AC	4	2.5×10^5	-0.80	1.25	18.205
CE	5		0	0	0
DE	4		0	0	0
BD	4		-0.80	1.25	18.205
DA	5		1	2.44	-22.756
BC	5		1	2.44	-22.756
CD	3		-0.60	0.53	13.653
				7.91×10^{-5}	

$$X = \frac{\alpha . t . l}{\sum \frac{u_i^2 L}{AE}} = \frac{12 \times 10^{6} \times 30 \times 5}{7.91 \times 10^{5}} = 22.756$$

EXAMPLE.2

Determine the internal forces in all the members of the loaded redundant truss shown in figure. Assume same AE for all the members. Release members AC and AE during the analysis.



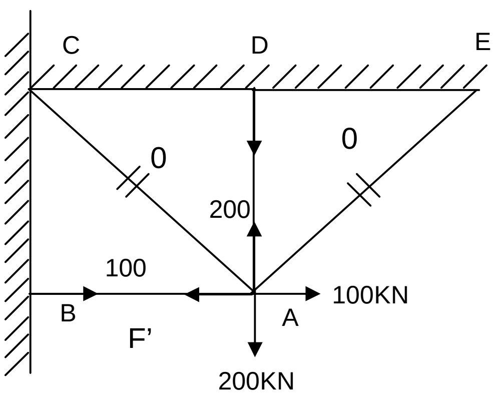
SOLUTION:

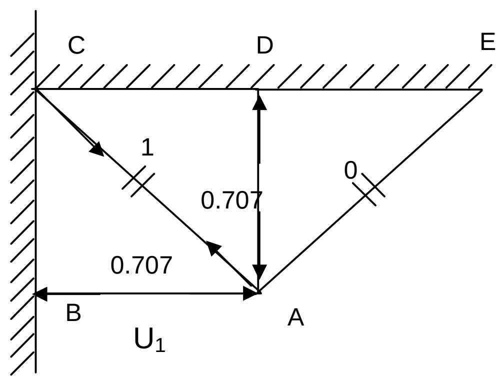
m = 4, j = 5, r = 8

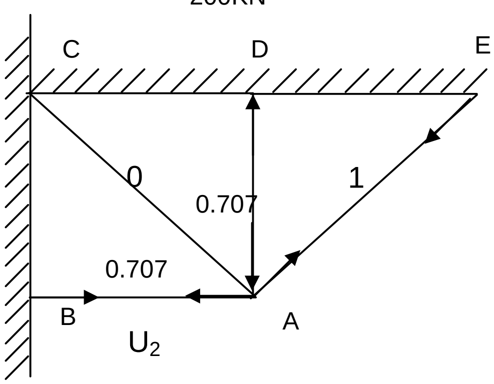
De = m + r -2j

= 4 + 8 – 2 x 5 = 2

Since, the given problem is statically indeterminate to second degree, let the members AC and AE are considered as redundant.







Member	Length 'm'	F'	u_1	u_2	$F'u_iL$	$F'u_2L$	u_1^2L	u_2^2L	u_1u_2L	$F=F'+u_1X_1+u_2X_2$
AB	4.000	100	-0.707	0.707	-282.84	282.84	2	2	-2	58.580
AC	5.657	0	1	0	0	0	5.657	0	0	87.870
AD	4.000	200	-0.707	-0.707	-565.68	-565.68	2	2	2	117.16

AE	5.657	0	0	1	0	0	0	5.657	0	29.29
					-848.52	**-282.84**	**9.657**	**9.657**	**0**	

$\Sigma F'.u_1.L + \Sigma u_1^2.L.X_1 + \Sigma\ u_1.u_2.L..X_2 = 0$

$\Sigma F'.u_2.L + \Sigma u_1.u_2.L..X_1 + \Sigma u_2^2.L.X_2 = 0$

$\therefore - 848.52 + 9.657\ X_1 = 0$

$\therefore$ **X_1 = <u>87.87</u>**

and $-282.84 + 9.657\ X_2 = 0$

$\therefore$ **X_2 = <u>29.29</u>**