<u>Chapter-3:</u> Moment Distribution Method

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VTU – EDUSAT PROGRAMME – 7

Class: B.E. V Sem (Civil Engineering) Sub: Structural Analysis – II (CV51)

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MOMENT DISTRIBUTION METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES:

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I. METHODS OF ANALYSIS:

Following requirements are to be satisfied when analyzing any indeterminate structures.

(i) Equilibrium
(ii) Load-displacement
(iii) Compatibility

Equilibrium requirements are satisfied when the active and reactive forces hold the structure at rest and compatibility is satisfied, when the deformations (displacements & rotations) maintain the continuity between various segments of the structures, which should fit together without break or overlap. Force-displacement relations depend upon the loading pattern and the way material responds. But in all our further study we assume linear elastic response of the material.

Hence depending on the way to satisfy the above requirements we have got basically two following methods of analysis.

1. Force method
2. Displacement method

Force method: This method was originally developed by James Clerk Maxwell in 1864 and latter refined by Otto and Muller Breslau.

Procedure for analysis:

Following are the steps involved in force method of analysis

(i) Determine the degree of static indeterminacy 'n'. Then identify the 'n' unknown redundant forces or moments that must be removed from the structure in order to make it statically determinate and stable.

(ii) Using principle of superposition show that the given indeterminate structure as equal to a sequence of corresponding statically determinate structures. The primary structure supports the same external loads as the given indeterminate structure and each of other structures added to the primary structure shows the structure loaded with a separate redundant force or moment. Also sketch the elastic cure on each structure and indicate symbolically the displacement or rotation at the point of each redundant force or moment.

(iii) Write a compatibility equation for the displacement or rotation at each point where there is redundant force or moment. These equation will be in terms of redundant and flexibility coefficients, obtained from unit loads or moments that are collinear with redundant force or moments.

Determine all the deflections and flexibility coefficients from load displacement relations. And substitute into the above formulated compatibility equation.

Then solve for the redundant forces or moments.

(iv) Then the remaining unknown reactions can be determined by applying equilibrium equations to free body diagram of the structure.

Ex:

Consider a propped cantilever, which is statically indeterminate to degree one is shown in figure 1 (a). In this case identify R_B as the redundant force. Fig. 1 (b) shows application of principle superposition. Fig. 1 (c) depicts the definition of linear flexibility coefficient f_{BB} for unit load, the compatibility equation for the given case is

$$-\Delta_B + \Delta'_{BB} = 0$$

i.e. $-\Delta_B + R_B \times f_{BB} = 0$

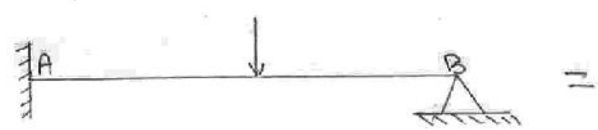


Fig. 1 (a)

Primary Structure

Fig. 1 (b)

Redundant structure. R_B applied $\Delta'_{BB} = R_B \times f_{BB}$

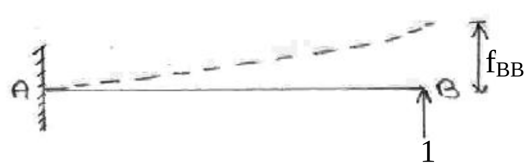


Fig. 1 (c)

Using the load displacement relation, the value of Δ_B and flexibility coefficient f_{BB} can be evaluated. Hence solution for R_B is determined. Once R_B is determined, the reactions at A can be determined from equations of equilibrium.

The choice of redundant is arbitrary. For example the above example can be solved by identifying support moment at A as redundant. As shown in figure 2 b the rotation at A caused by the load is θ_A and the rotation at A caused by redundant M_A is θ'_{AA} .

$\theta'_{AA} = M_A\, \alpha_{AA}$

where α_{AA} is the angular flexibility coefficient, which measures angular displacement per unit couple moment. The compatibility equation for rotation at A therefore will be
$(\theta_A + M_A\, \alpha_{AA} = 0)$

After applying load displacement relation to determine θ_A and α_{AA} the above equation can be solved for M_A.

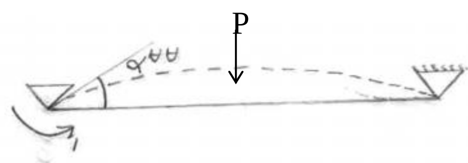
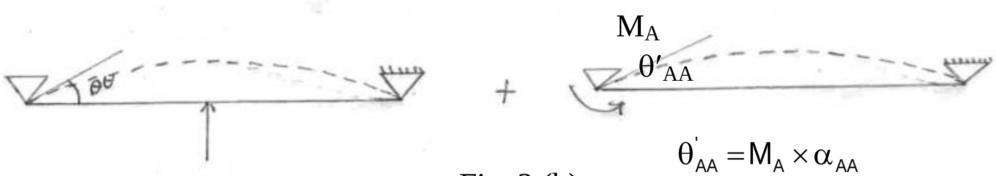


Fig. 2 (a)



$\theta'_{AA} = M_A \times \alpha_{AA}$

Fig. 2 (b)

Fig. 2 (c)

Displacement method of analysis:

This method works the opposite way when compared with force method of

analysis. The following steps are involved in this method.

Steps:

(i) Label all the supports and joints (nodes) in order to identify the spans of the beam or frame between the nodes. Then by drawing the deflected shape of the structure it will be possibly to identity the degrees of freedom hence the unknown displacements. Here each node can possibly have an angular displacement and linear displacement.

(ii) Then these unknown displacements are written in terms of loads by using load displacement relations.

(iii) Then write an equilibrium equation for each unknown degree of freedom for the structure.

(iv) Then these equations are solved for the displacements.

(v) Once the displacements are obtained, the unknown loads are determined from the compatibility equations, using load-displacement relations. Slope deflection method of analysis is one of the examples for displacement method of analysis.

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MOMENT DISTRIBUTION METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES (CONTD.)

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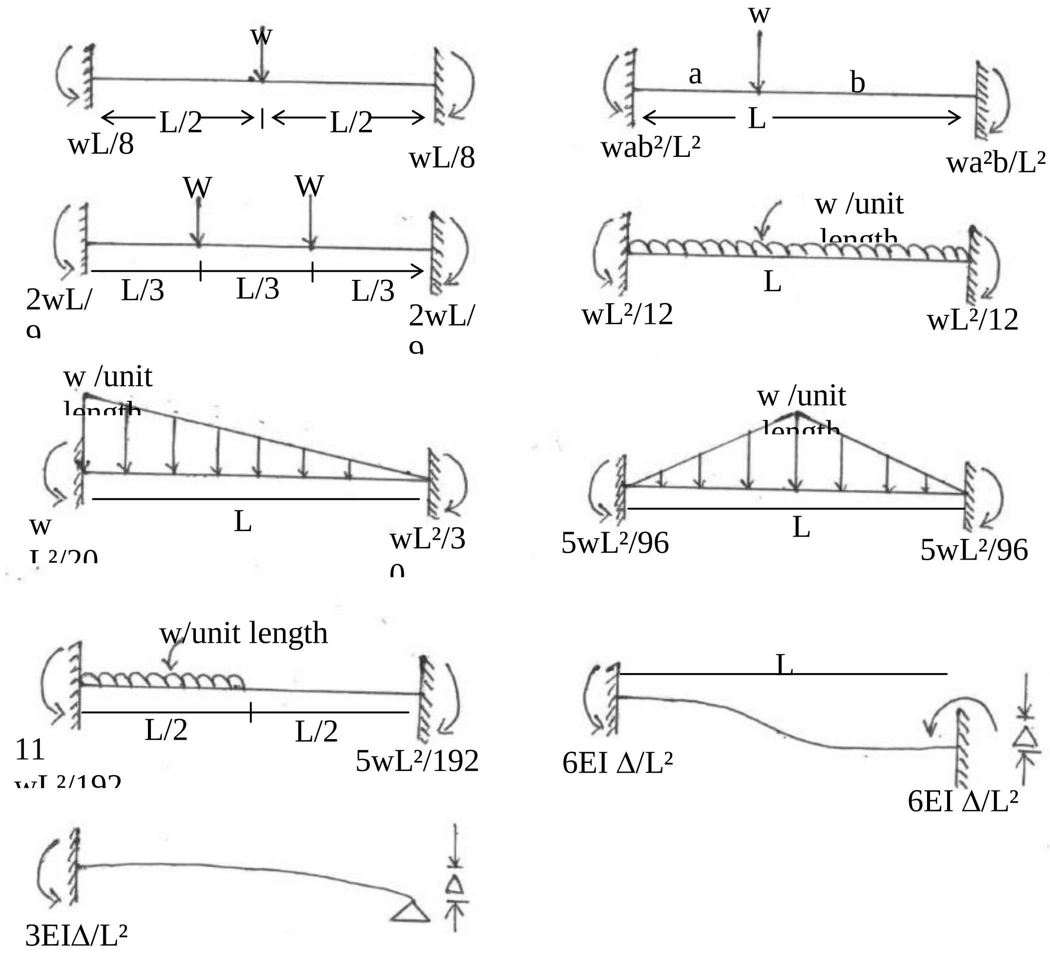
II. MOMENT DISTRIBUTION METHOD:

This method of analyzing beams and frames was developed by Hardy Cross in 1930. Moment distribution method is basically a displacement method of analysis. But this method side steps the calculation of the displacement and instead makes it possible to apply a series of converging corrections that allow direct calculation of the end moments.

This method of consists of solving slope deflection equations by successive approximation that may be carried out to any desired degree of accuracy. Essentially, the method begins by assuming each joint of a structure is fixed. Then by unlocking and locking each joint in succession, the internal moments at the joints are distributed and balanced until the joints have rotated to their final or nearly final positions. This method of analysis is both repetitive and easy to apply. Before explaining the moment distribution method certain definitions and concepts must be understood.

Sign convention: In the moment distribution table clockwise moments will be treated +ve and anti clockwise moments will be treated –ve. But for drawing BMD moments causing concavity upwards (sagging) will be treated +ve and moments causing convexity upwards (hogging) will be treated –ve.

Fixed end moments: The moments at the fixed joints of loaded member are called fixed end moment. FEM for few standards cases are given below:



Member stiffness factor:

a) Consider a beam fixed at one end and hinged at other as shown in figure 3 subjected to a clockwise couple M at end B. The deflected shape is shown by dotted line

BM at any section xx at a distance x from 'B' is given by

$$EI \frac{d^2y}{dx^2} = R_B x - M$$

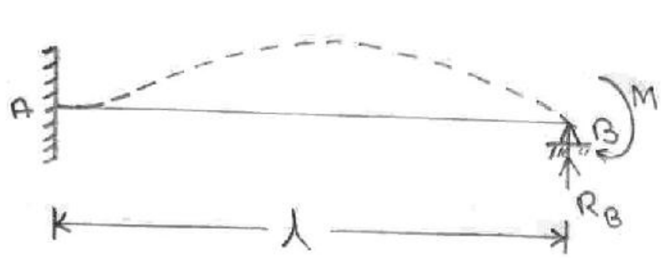


Fig. 3

Integrating EI $\frac{dy}{dx} = \frac{R_B x^2}{2}$ - Mx + C_1

Using condition x = l $\frac{dy}{dx} = 0$

C_1 = Ml - $\frac{R_B l^2}{2}$

$\therefore$ EI $\frac{dy}{dx} = \frac{R_B x^2}{2} - Mx + \left(Ml - \frac{R_B l^2}{2} \right)$……….(1)

Integrating again EI y = $\frac{R_B x^3}{6} - \frac{Mx^2}{2} + \left(Ml - \frac{R_B l^2}{2} \right)$ x + C_2

Using condition at x = 0 y = 0

$C_2 = 0$

$\therefore$ EI y = $\frac{R_B x^3}{6} - \frac{Mx^2}{2} + \left(Ml - \frac{R_B l^2}{2} \right) x$ …………… (2)

Using at x = l y = 0 in the equation (2)

$R_B = \frac{3M}{2l}$

Substituting in equation (1)

EI $\frac{dy}{dx} = \frac{3M}{4l} x^2 - Mx + \frac{Ml}{4}$ ………….. (3)

Substituting x = 0 in the equation (3)

EI $\theta_B = \frac{Ml}{4}$ $\therefore$ M = $\left(\frac{4EI}{l} \right) \theta_B$

The term in parenthesis

$$\left\{ K = \left[\frac{4EI}{L} \right] \right\}$$ For far end fixed …………………. (4)

is referred to as stiffness factor at B and can be defined as moment M required to rotate end B of beam θ_B = 1 radian.

b) Consider freely supported beam as shown in figure 4 subjected to a clockwise couple M at B

By using $\Sigma M_B = 0$

$R_A = \frac{M}{l}$ ($\downarrow$)

And using $\Sigma V = 0$ $R_B = \frac{M}{l}$ ($\uparrow$)

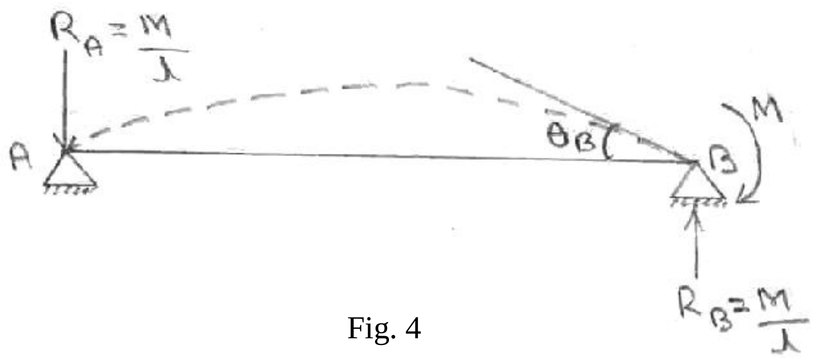


Fig. 4

BM at a section xx at distance x from 'B' is given by EI $\frac{d^2y}{dx^2} = \frac{M}{l}x - M$

Integrating EI $\frac{dy}{dx} = \frac{M}{l}\frac{x^2}{2} - Mx + C_1$

Integrating again EI y = $\frac{M}{l}\frac{x^3}{6} - \frac{Mx^2}{2} + C_1x + C_2$

At x = 0 y = 0 $\therefore$ C_2= 0

At x = l y = 0 $\therefore$ $C_1 = \frac{Ml}{3}$

$\therefore$ EI $\frac{dy}{dx} = \frac{M}{l}\frac{x^2}{2} - Mx + \frac{Ml}{3}$

Substituting x = 0 in above equation

EI $\theta_B = \frac{Ml}{3}$

M = $\left(\frac{3EI}{l}\right)\theta_B$

The term in parenthesis

$\left[K = \frac{3EI}{l}\right]$ For far end hinged ………………(5)

is termed as stiffness factor at B when far end A is hinged.

Joint stiffness factor:

If several members are connected to a joint, then by the principle of superposition the total stiffness factor at the joint is the sum of the member stiffness factors at the joint i.e., $k_T = \Sigma k$

Eg. For joint '0' shown in fig 5

$K_T = K_{0A} + K_{OB} + K_{OC} + K_{OD}$

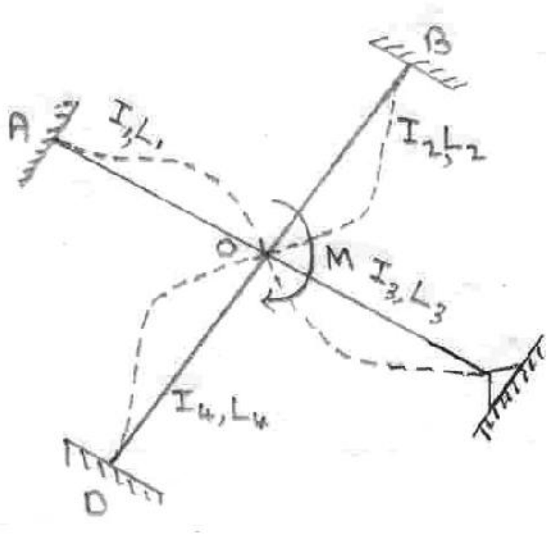


Fig. 5

Distribution factors: If a moment 'M' is applied to a rigid joint 'o', as shown in figure 5, the connecting members will each supply a portion of the resisting moment necessary to satisfy moment equilibrium at the joint. Distribution factor is that fraction which when multiplied with applied moment 'M' gives resisting moment supplied by the members.

To obtain its value imagine the joint is rigid joint connected to different members. If applied moment M cause the joint to rotate an amount 'θ', Then each member rotates by same amount.

From equilibrium requirement

$M = M_1 + M_2 + M_3 + \ldots\ldots\ldots\ldots\ldots\ldots$

$= K_1\theta + K_2\theta + K_3\theta + \ldots\ldots\ldots\ldots$

$= \theta\,\Sigma K$

$\therefore\ DF_1 = \dfrac{M_1}{M} = \dfrac{K_1\theta}{\theta\sum K} = \dfrac{K_1}{\sum K}$

In general $DF = \dfrac{K}{\sum K}$ ………………. (6)

Member relative stiffness factor: In majority of the cases continuous beams and frames will be made from the same material so that their modulus of electricity E will be same for all members.

It will be easier to determine member stiffness factor by removing term 4E & 3E from equation (4) and (5) then will be called as relative stiffness factor.

$K_r = \dfrac{I}{l}$ for far end fixed

$K_r = \frac{3}{4}\frac{I}{l}$ for far end hinged,

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MOMENT DISTRIBUTION METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES (CONTD.)

Carry over factors: Consider the beam shown in figure

We have shown that M = $\frac{4EI}{l}\theta_A$ & $R_B = \frac{3M}{2l}$.

BM at A = $\left(EI\,\frac{d^2y}{dx^2}\right)_{at\ x=l}$ = $\left[\frac{3M}{2l}x - M\right]_{x=1}$

= + $\frac{M}{2}$

+ve BM of $\frac{M}{2}$ at A indicates clockwise moment of $\frac{M}{2}$ at A. In other words the moment 'M' at the pin induces a moment of $\frac{M}{2}$ at the fixed end. The carry over factor represents the fraction of M that is carried over from hinge to fixed end. Hence the carry over factor for the case of far end fixed is + $\frac{1}{2}$. The plus sign indicates both moments are in the same direction.

Moment distribution method for beams:

Procedure for analysis:

(i) Fixed end moments for each loaded span are determined assuming both ends fixed.

(ii) The stiffness factors for each span at the joint should be calculated. Using these values the distribution factors can be determined from equation DF = $\frac{K}{\sum K}$. DF for a fixed end = 0 and DF = 1 for an end pin or roller support.

(iii) Moment distribution process: Assume that all joints at which the moments in the connecting spans must be determined are initially locked.

Then determine the moment that is needed to put each joint in equilibrium.

Release or unlock the joints and distribute the counterbalancing moments into connecting span at each joint using distribution factors.

Carry these moments in each span over to its other end by multiplying each moment by carry over factor.

By repeating this cycle of locking and unlocking the joints, it will be found that the moment corrections will diminish since the beam tends to achieve its final deflected shape. When a small enough value for correction is obtained the process of cycling should be stopped with carry over only to the end supports. Each column of FEMs, distributed moments and carry over moment should then be added to get the final moments at the joints.

Then superimpose support moment diagram over free BMD (BMD of primary structure) final BMD for the beam is obtained.

The above process is illustrated in following examples

Ex: 1 Analyse the beam shown in figure 6 (a) by moment distribution method and draw the BMD. Assume EI is constant

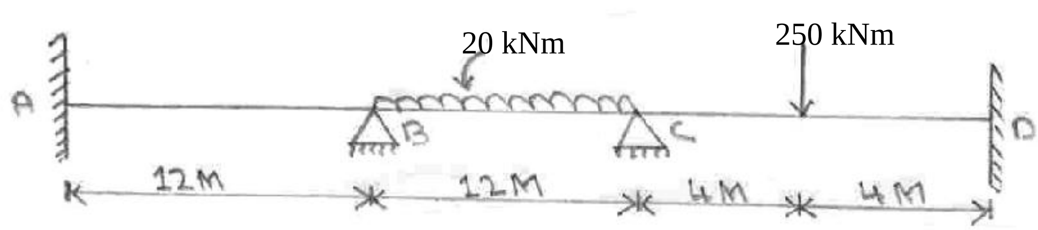

Fig. 6 (a)

Solution:

(i) FEM calculation

$M_{FAB} = M_{FBA} = 0$

$$M_{FBC} = \frac{-20 \times 12^2}{12} = -240\,kNm$$

$M_{FCB} = +240$ kNm

$$M_{FCB} = -\frac{250 \times 8}{8} = -250 \text{ kNm}$$

$F_{FDC} = +250$ kNm

(ii) Calculation of distribution factors:

Jt.	Member	Relative stiffness (K)	pK	$DF = \frac{K}{\sum K}$
B	BA	I/12	I/6	0.5
	BC	I/12		0.5
C	CB	I/12	5I/24	0.4
	CD	I/8		0.6

(iii) The moment distribution is carried out in table below

Jt	A	B		C		D
Member	AB	BA	BC	CB	CD	α
D.F	0	0.5	0.5	0.4	0.6	0
FEM	0	0	-240	+240	-250	+250
Balance		+120	+120	4	6	
C.O	60		2	60		3
Balance		-1	-1	-24	-36	
C.O	-0.5		-12	-0.5		-18
Balance		+6	+6	0.2	0.3	
C.O	3		0.1	3		0.15
Balance		-0.05	-0.05	-1.2	-1.8	
C.O	-0.03		-0.6	-0.03		-0.9
Balance		+0.3	+0.3	0.01	0.02	
C.O	0.15					0.01
Final moments	**62.62**	**125.25**	**-125.25**	**281.48**	**-281.48**	**234.26**

After writing FEMs we can see that there is a unbalancing moment of –240 KNm at B & -10 KNM at Jt.C. Hence in the next step balancing moment of +240 KNM & +10 KNM are applied at B & C Simultaneously and distributed in the connecting members after multiply with D.F. In the next step distributed moments are carried over to the far ends. This process is continued until the resulting moments are diminished an appropriate amount. The final moments are obtained by summing up all the moment values in each column. Drawing of BMD is shown below in figure 6 (b).

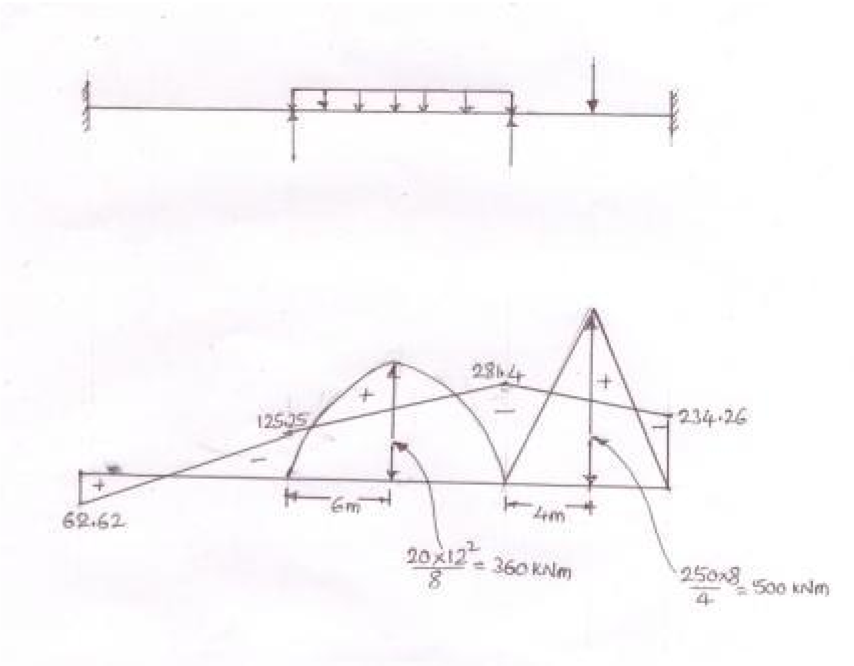

Fig.6(b)

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MOMENT DISTRIBUTION METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES (CONTD.)

Ex 2: Analyse the continuous beam shown in fig 7 (a) by moment distribution method and draw BMD & SFD (VTU July 2006 exam)

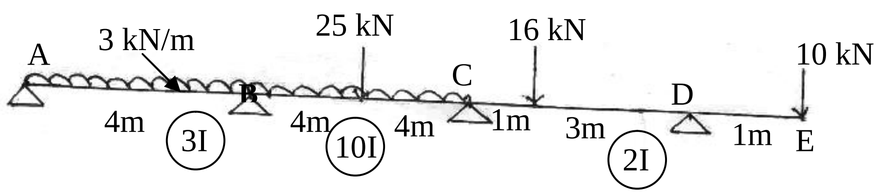


Fig. 7 (a)

Solution:

FEM: $M_{FAB} = -\dfrac{3x4^2}{12} = -4\,kNm;\ M_{FBA} = 4\,kNm$

$M_{FBC} = -\dfrac{3x8^2}{12} - \dfrac{25\,x\,8}{8} = -41\ kNm$ $\quad M_{FAB} = +\dfrac{3x\,8^2}{12} + \dfrac{25\,x\,8}{8} = +41\,kNm$

$M_{FDC} = \dfrac{16\,x\,1^2\ x\,3}{4^2} = +\ 3\ kNm$ $\quad M_{DE} = -10\ x\ 1 = -10kNm$

$M_{FCD} = \dfrac{-16 \times 1 \times 3^2}{4^2} = -9\ kNm$

DF:

Jt.	**Member**	**Relative stiffness (K)**	**pK**	**DF =** $\dfrac{K}{\sum K}$
B	BA	$\dfrac{3}{4}x\dfrac{3I}{4} = 0.56I$	1.81I	0.31
	BC	10I/8 = 1.25I		0.69
C	CB	10I/8 = 1.25I	1.63I	0.77
	CD	$\dfrac{3}{4}x\dfrac{2I}{4} = 0.38I$		0.23

Note: Since support 'A' is simply supported end the relative stiffness value of $\frac{3}{4}\frac{I}{l}$ has been taken and also since 'D' can be considered as simply supported with a definite moment relative stiffness of CD has also been calculated using the formula $\frac{3}{4}\frac{I}{l}$.

Moment distribution table:

Jt	A	B		C		D	
Member	AB	BA	BC	CB	CD	DC	DE
D.F	1	0.31	0.69	0.77	0.23	1	01
FEM	-4	4	-41	+41	-9	3	-10
Release of joint A and adjusting moment at 'D'	+4	2			3.5	+7	
Initial moments	0	6	-41	41	-5.5	+10	-10
Balance		10.9	24.1	-27.3	-8.2		
C.O			-13.7	12.1			
Balance		4.2	9.5	-9.3	-2.8		
C.O			-4.7	4.8			
Balance		1.5	3.2	-3.7	-1.1		
C.O			-1.9	1.6			
Balance		0.6	1.3	-1.2	-0.4		
C.O			-0.6	0.7			
Balance		0.2	0.4	-0.5	-0.2		
C.O			-0.3	0.2			
Balance		0.09	0.21	-0.15	-0.05		
Final moments	**0**	**23.49**	**-23.49**	**18.25**	**-18.25**	**10**	**-10**

FBD of various spans is shown in fig. 7 (b) and 7 (c) and BMD, SFD have been shown in fig. 7 (d)

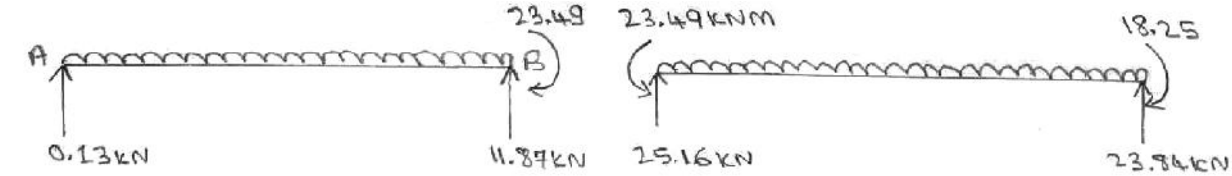


Fig. 7 (b)

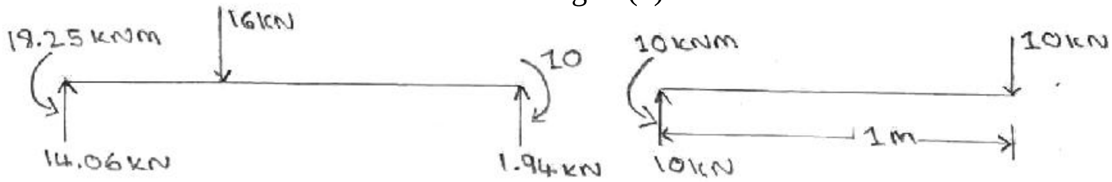


Fig. 7 (c)

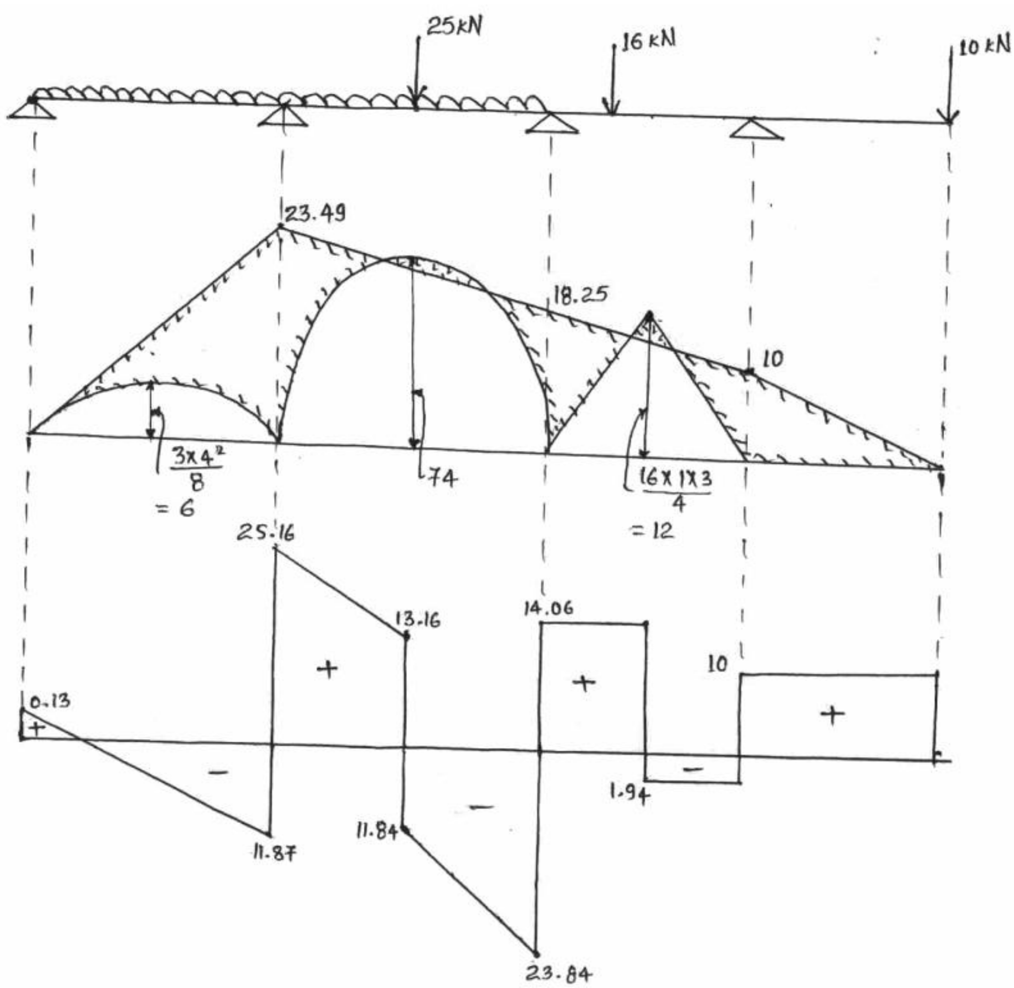


Fig. 7 (d)

Ex 3: Analyse the continuos beam as shown in figure 8 (a) by moment distribution method and draw the B.M. diagrams (VTU January 2005 exam)

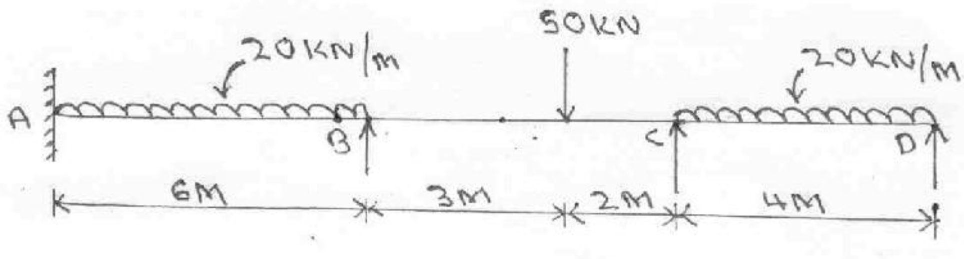


Fig. 8 (a)

Support B sinks by 10mm

$E = 2 \times 10^5$ N/mm², $I = 1.2 \times 10^{-4}$ m^4

Solution: Fixed End Moments:

M_{FAB} = FEM due to load + FEM due to sinking

$$= \frac{-wl^2}{12} + \left[\frac{-6EI\Delta}{l^2}\right]$$

$$= \frac{-20 \times 6^2}{12} - \frac{6 \times 2 \times 10^5 \times 1.2 \times 10^{-4} \times 10^{12} \times 10}{(6000)^2 \times 10^6}$$

$= -60 - 40$

$M_{FAB} = -100$ kNm

M_{FBA} = FEM due to load + FEM due to sinking

$= +60 - 40$

$M_{FBA} = +20$ kNm

M_{FBC} = FEM due to loading + FEM due to sinking

$$= \frac{-Wab^2}{l^2} + \frac{6EI\Delta}{l^2}$$

$$= \frac{-50 \times 3 \times 2^2}{5^2} + \frac{6 \times 2 \times 10^5\, 1.2 \times 10^{-4} \times 10^{12} \times 10}{(5000)^2 \times 10^6}$$

$= -24 + 57.6$

$M_{FBA} = +33.6$ kNm

$$M_{FCB} = +\frac{Wa^2b}{l^2} + \frac{6EI\Delta}{l^2}$$

$$= \frac{50 \times 3^2 \times 2}{5^2} + 57.6$$

$M_{FCB} = 93.6$kNm

M_{FCD} = due to load only ($\because$ C & D are at some level)

$M_{FCD} = \frac{-wl^2}{12} = \frac{-20 \times 4^2}{12} = -26.67 kNm$

$M_{FDC} = +26.67$ kNm

Jt.	**Member**	**Relative stiffness (K)**	p**K**	**DF =** $\frac{K}{\sum K}$
B	BA BC	I/6 I/5	 0.36I	0.46 0.54
C	CB CD	I/5 $\frac{3}{4} x \frac{I}{4} = 0.19I$	 0.39I	0.51 0.49

Jt	A	B		C		D	
Member	AB	BA	BC	CB	CD	DC	
D.F		0.46	0.54	0.51	0.49		
FEM	-100	+20	+33.6	+93.6	-26.67	+26.67	
Release jt. 'D'						-26.67	
CO					-13.34		
Initial moments	-100	+20	+33.6	+93.6	-40.01	0	
Balance		-24.66	-28.94	-27.33	-26.26		
C.O	-12.33		-13.67	--14.47			
Balance		+6.29	+7.38	+7.38	+7.09		
C.O	+3.15		+3.69	+3.69			
Balance		-1.7	-1.99	-1.88	-1.81		
C.O	-0.85		-0.94	-1			
Balance		+0.43	+0.51	+0.51	+0.49		
C.O	+0.22		+0.26	+0.26			
Balance		-0.12	-0.14	-0.13	-0.13		

C.O	-0.06						
Final moments	**-109.87**	**+0.24**	**-0.24**	**+60.63**	**-60.63**		

Bending moment diagram is shown in fig. 8 (b)

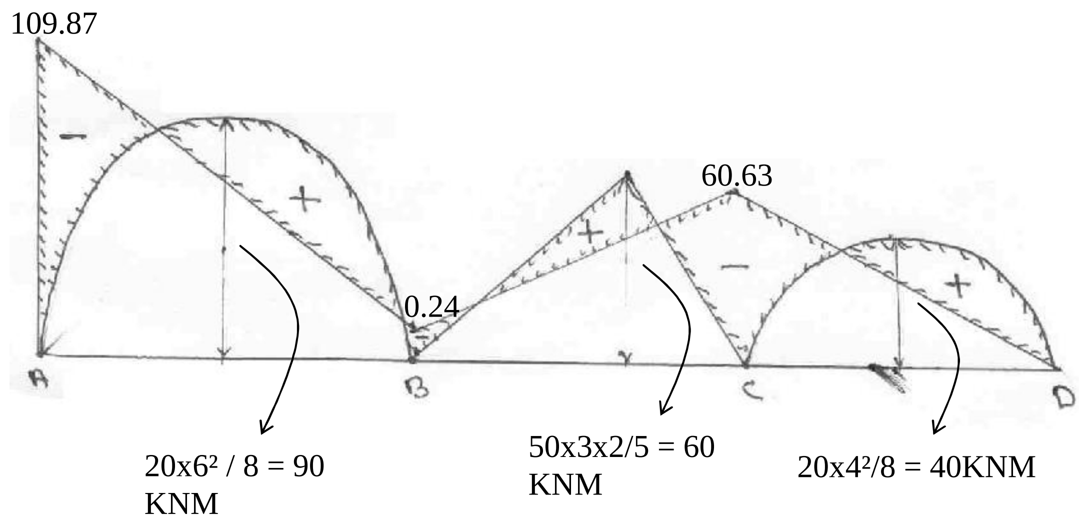


Fig. 8 (b)

VTU – EDUSAT PROGRAMME – 7

Class: B.E. V Sem (Civil Engineering) Sub: Structural Analysis – II (CV51)

Session on 28.09.2007

MOMENT DISTRIBUTION METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES (CONTD.)

I. Moment distribution for frames: (No side sway)

The analysis of such a frame when the loading conditions and the geometry of the frame is such that there is no joint translation or sway, is similar to that given for beams.

Ex 4: Analysis the frame shown in figure 9 (a) by moment distribution method and draw BMD assume EI is constant

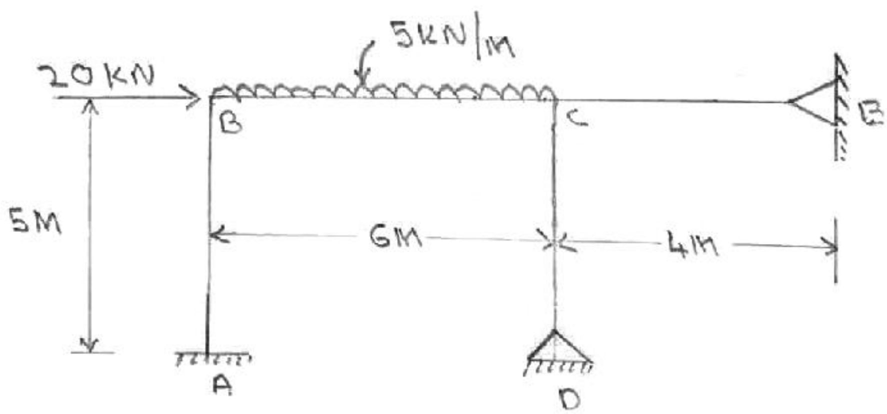


Fig. 9 (a)

Solution:

(i) Calculation of FEM

$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = M_{FCE}$ $M_{FEC} = 0$

$M_{FBC} = -\dfrac{5 \times 6^2}{12} = -15\,\text{kNm}$

$M_{FCB} = +15$ kNm

(ii) Calculation of distribution factors:

Jt.	Member	Relative stiffness (K)	pK	DF = $\dfrac{K}{\sum K}$
B	BA	I/5	$\dfrac{11}{30}I$	0.55
	BC	I/6		0.45
C	CB	I/6 = 0.17 I		0.33
	CD	$\dfrac{3}{4}I/5 = 0.15\,I$	0.51 I	0.3
	CE	$\dfrac{3}{4} \times \dfrac{I}{4} = 0.19\,I$		0.37

(iii) The moment distribution process is shown below:

Jt	A	B		C			D	E
Member	AB	BA	BC	CB	CD	CE	DC	EC
D.F	0	0.55	0.45	0.33	0.3	0.37	1	1
FEM	0	0	-15	+15	0	0	0	0
Balance		8.25	6.75	-4.95	-4.5	-5.55		
C.O	4.13		-2.48	3.38				
Balance		1.36	1.12	-1.12	-1.01	-1.25		
C.O	0.68		0.56	0.56				
Balance		0.31	0.25	-0.18	-0.17	-0.21		
C.O	0.16		-0.09	0.13				
Balance		0.05	0.04	-0.04	-0.04	-0.05		
C.O	0.03							
Final moments	**5**	**9.97**	**-9.97**	**12.78**	**-5.72**	**-7.06**	**0**	**0**

By inspection the hinge at E will prevent the frame from side sway.

The BMD is shown below in fig. 9 (b)

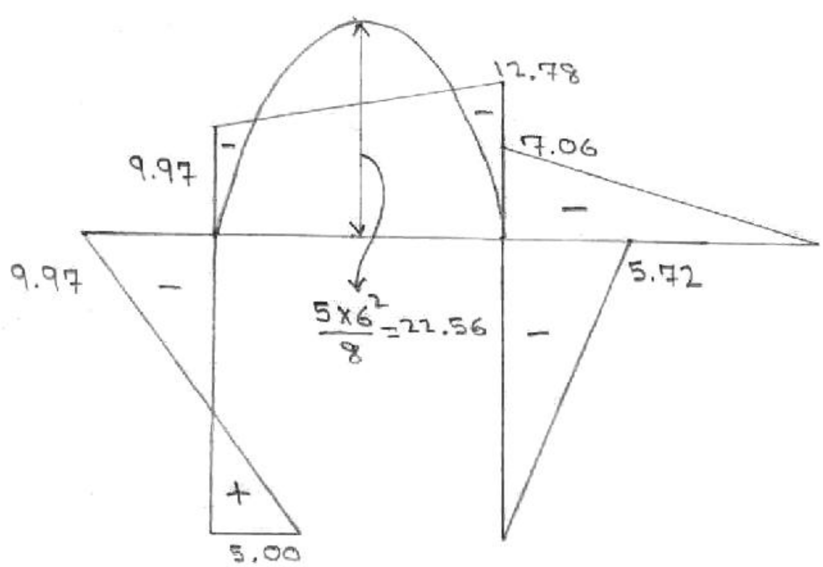


Fig. 9 (b)

Ex 5: Analyse the frame shown in fig. 10 (a) by moment distribution method and draw BMD and SFD (VTU Jan/Feb 2005 exam)

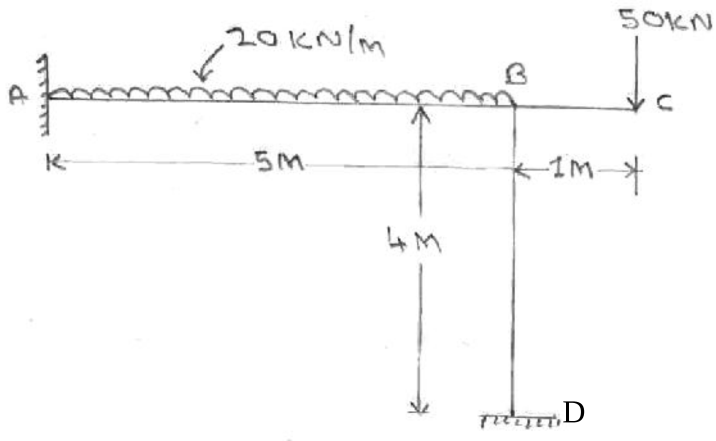


Fig. 10 (a)

Solution:

FEM: $M_{FAB} = -\dfrac{20x5^2}{12} = -41.67$ KNM

$M_{FBA} = +41.67$ KNM

$M_{FBD} = M_{FDB} = 0$

$M_{FBC} = -50 \times 1 = -50$ KNM

DF:

Jt.	**Member**	**K**	p**K**	**DF =** $\dfrac{K}{\sum K}$
B	BA	2I/5 = 0.4I		0.62
	BC	0	0.65I	0
	BD	I/4 = 0.25 I		0.38

Moment Distribution table

Jt	A	B			D	
Member	AB	BA	BC	BD	DB	
D.F	01	0.62	0	0.38	0	
FEM	-41.67	41.67	-50	0	0	
Balance		5.2	0	3.13		

C.O	2.6		1.6	
Final moments	**-39.07** **46.87**	**-50**	**3.13** **1.6**	

FBD of various spans are shown in fig. 10 (b) BMD is shown in fig. 10 (c) & SFD is shown in Fig. 10 (d)

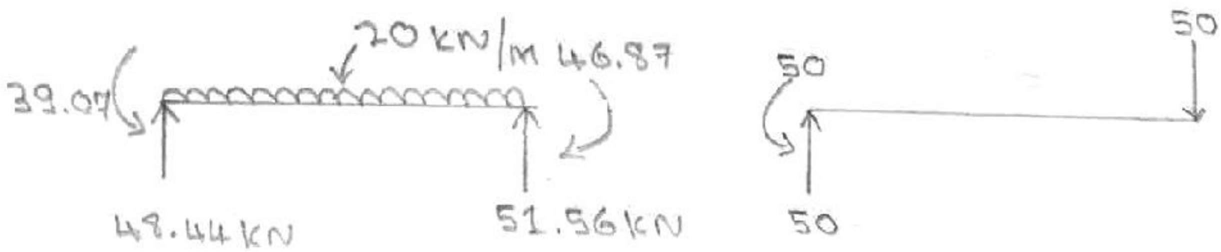


Fig. 10 b

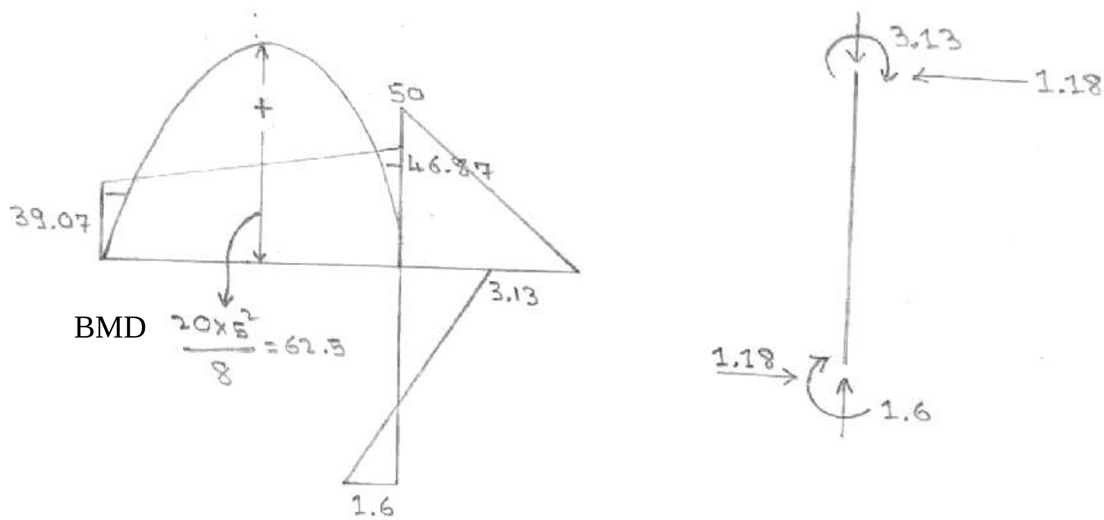
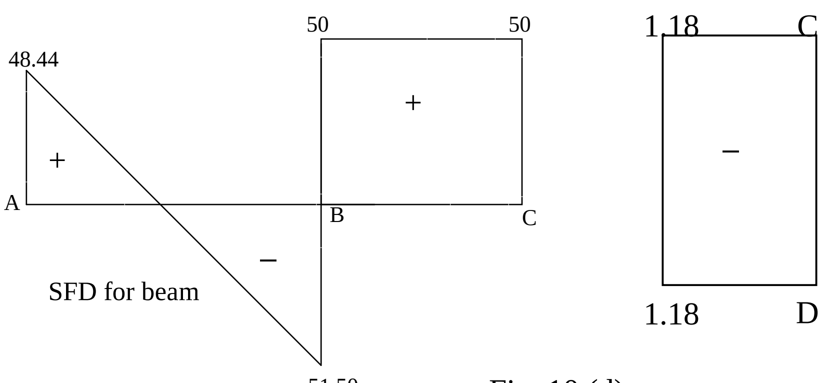


Fig. 10 c



Transvers shear force diagram for

Fig. 10 (d)

VTU – EDUSAT PROGRAMME – 7

Class: B.E. V Sem (Civil Engineering) Sub: Structural Analysis – II (CV51)

Session on 01.10.2007

MOMENT DISTRIBUTION METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES (CONTD.)

By A.B.Harwalkar

P.D.A.College of Engg Gulbarga

Moment distribution method for frames with side sway:

Frames that are non symmetrical with reference to material property or geometry (different lengths and I values of column) or support condition or subjected to non-symmetrical loading have a tendency to side sway. An example has been shown in fig. 11 (a).

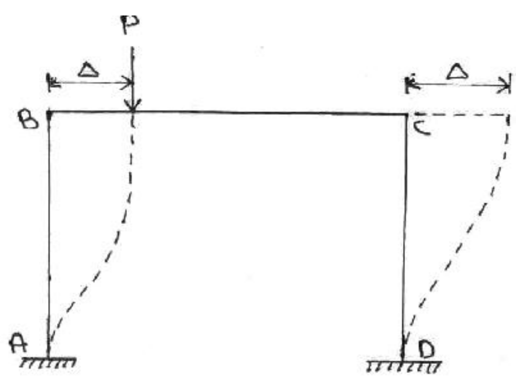


Fig 11 (a)

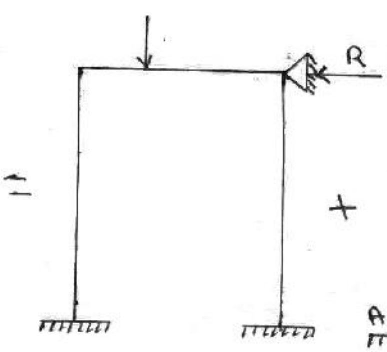


Fig 11 (b)

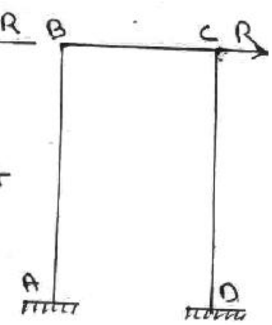


Fig 11 (c)

Analysis of such frames will be carried out in following steps

Step1: First frame is considered as prevented from side sway by applying on artificial support at C as shown in fig11 (b). Then moment distribution is carried out for external loading similar to non-sway frames. Then by using statics restraining force R is determined.

Step 2: The equal but opposite force 'R' is then applied to the frame as shown in figure 11 (c) and moments in the frame are calculated. One method for doing this requires first assuming a numerical value for one of the internal moments, say M'_{BA}. Using moment distribution and statics the deflection Δ' and external force R' corresponding to assumed value of M'_{BA} can then be determined. Since linear elastic determinations occur the force R' develops moments in the frame that are proportional to those developed by R. For example if M'_{BA} & R' are known, the moment at B developed by R will be i.e.; M'_{BA} for R= $M'_{BA}\left(\frac{R}{R'}\right)$ similarly moments due to R at the different joints can be calculated.

Addition of moments for both cases fig 11 (b) & fig 11(c) will yield the actual moments in the frame. Application of this technique is shown in following examples.

Ex 6: Analyze the frame shown in figure 12 (a) by moment distribution method. Assume EI is constant.

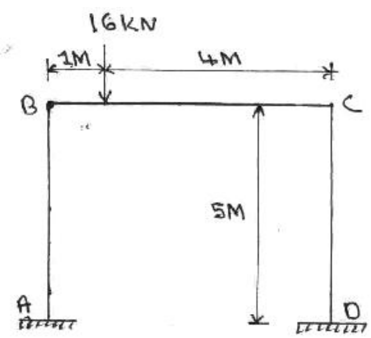


Fig 12 (a)

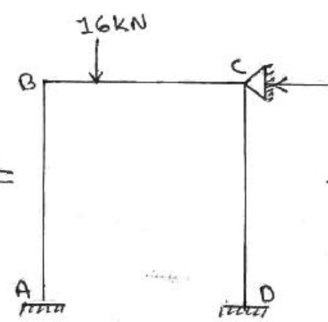


Fig 12 (b)

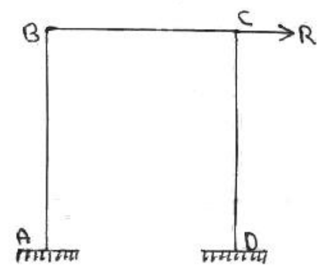


Fig 12 (c)

Solution: i) Non Sway Analysis:

First consider the frame held from side sway as shown in fig 12 (b)

FEM: $M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$

$$M_{FBC} = -\frac{16\times1\times4^2}{5^2} = -10.24 \text{ kNm}$$

$$M_{FCB} = \frac{16\times1^2\times4}{5^2} = 2.56 \text{ kNm}$$

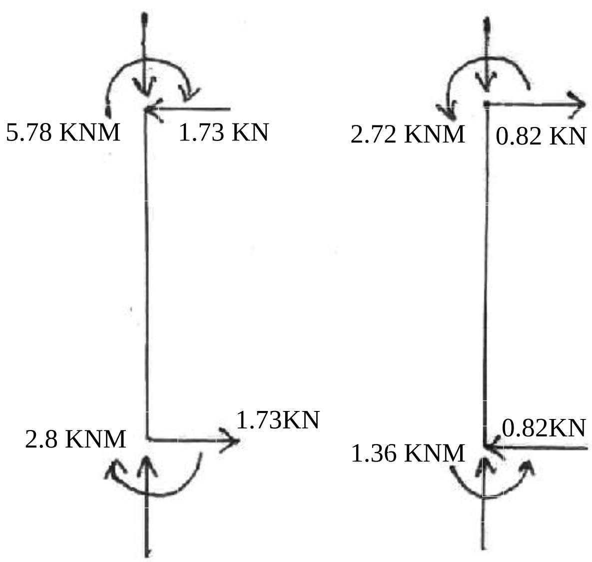
Calculation of distribution factors:

Jt.	**Member**	**Relative stiffness K**	pK	**DF =** $\frac{K}{\sum K}$
B	BA BC	I/5 = 0.2 I I/5 = 0.2 I	0.4 I	0.5 0.5
C	CB CD	I/5 = 0.2I I/5 = 0.2I	0.4I	0.5 0.5

Moment distribution table for non sway analysis :

Joint	A	B		C		D	
Member	AB	BA	BC	CB	CD	DC	
D.F	0	0.5	0.5	0.5	0.5	0	
FEM	0		-10.24	2.56	0	0	
Balance		5.12	5.12	-1.28	-1.28		
CO	2.56		-0.64	2.56		-0.64	
Balance		0.32	0.32	-0.08	-0.08		
CO	0.16		-0.64	0.16		-0.64	
Balance		0.32	0.32	-0.08	-0.08		
C.O	0.16		-0.04	0.16		-0.04	
Balance		0.02	0.02	-0.08	-0.08		
C.O	0.01					-0.04	
Final moments	**2.89**	**5.78**	**-5.78**	**2.72**	**-2.72**	**-1.36**	

FBD of columns:



By seeing of the FBD of columns R = 1.73 – 0.82
(Using $\sum F_x = 0$ for entire frame) = 0.91 KN (←)

Now apply R = 0.91 kN acting opposite as shown in figure 2 (c) for the sway analysis.

ii) **Sway analysis:** For this we will assume a force R′ is applied at C causing the frame to deflect Δ' as shown in figure 12 (d).

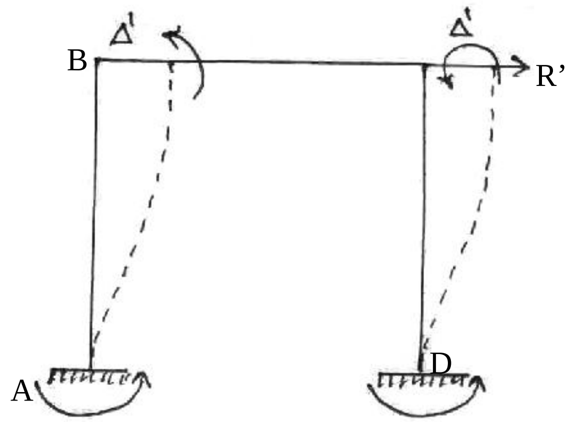


Figure 12 (d)

Since both ends are fixed, columns are of same length & I and assuming joints B & C are temporarily restrained from rotating and resulting fixed end moment are

$$M'_{BA} = M'_{AB} = M'_{CD} = M'_{DC} = \frac{6EI\Delta'}{l^2}$$

Assume $M^1_{BA} = -100\,kNm$

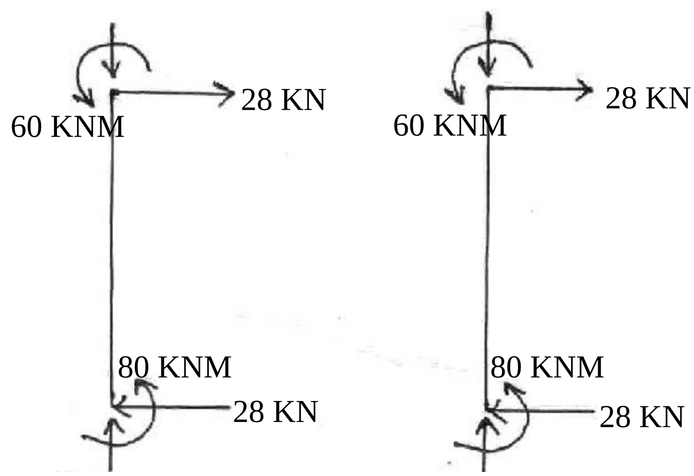
$\therefore\ M'_{AB} = M'_{CD} = M'_{DC} = -100$ kNm

Moment distribution table for sway analysis:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	01	0.5	0.5	0.5	0.5	0
FEM	−100	−100	0	0	−100	−100
Balance		50	50	50	50	
CO	25		25	25		25
Balance		−12.5	−12.5	−12.5	−12.5	
CO	-6.25		−6.25	-6.25		−6.25

Balance		3.125	3.125	3.125	3.125		
C.O	1.56		1.56	1.56		1.56	
Balance		–0.78	–0.78	–0.78	–0.78		
C.O	-0.39		–0.39	–0.39		0.39	
Balance		0.195	0.195	0.195	0.195		
C.O	0.1					0.1	
Final moments	**- 80**	**- 60**	**60**	**60**	J **60**	J **80**	

FBD of columns:



Using $\sum F_x = 0$ for the entire frame

$R' = 28 + 28 = 56$ KN ($\rightarrow$)

Hence $R' = 56$KN creates the sway moments shown in above moment distribution table. Corresponding moments caused by $R = 0.91$KN can be determined by proportion.

Thus final moments are calculated by adding non sway moments and sway moments calculated for $R = 0.91$KN, as shown below

$$M_{AB} = 2.89 + \frac{0.91}{56}(-80) = 1.59 \text{ kNm}$$

$$M_{BA} = 5.78 + \frac{0.91}{56}(-60) = 4.81 \text{ kNm}$$

$$M_{BC} = -5.78 + \frac{0.91}{56}(60) = -4.81 \text{ kNm}$$

$$M_{CB} = 2.72 + \frac{0.91}{56}(60) = 3.7 \text{ kNm}$$

$$M_{CD} = -2.72 + \frac{0.91}{56}(-60) = -3.7 \text{ kNm}$$

$$M_{DC} = -1.36 + \frac{0.91}{56}(-80) = -2.66 \text{ kNm}$$

BMD for the frame is shown in figure 12 (e)

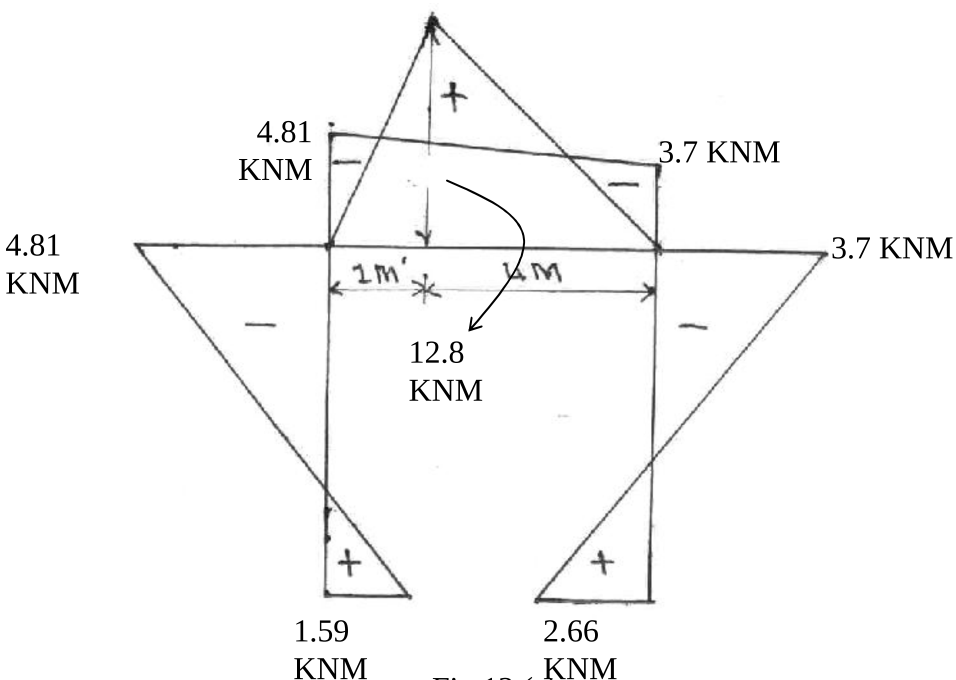


Fig 12 (e)

VTU – EDUSAT PROGRAMME – 7

Class: B.E. V Sem (Civil Engineering) Sub: Structural Analysis – II (CV51)

Session on 05.10.2007

MOMENT DISTRIBUTION METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES (CONTD.)

By A.B.Harwalkar

P.D.A.College of Engg Gulbarga

Example on moment distribution method for frames with side sway:

Ex 7: Analysis the rigid frame shown in figure 13 (a) by moment distribution method and draw BMD (VTU July 2006 exam).

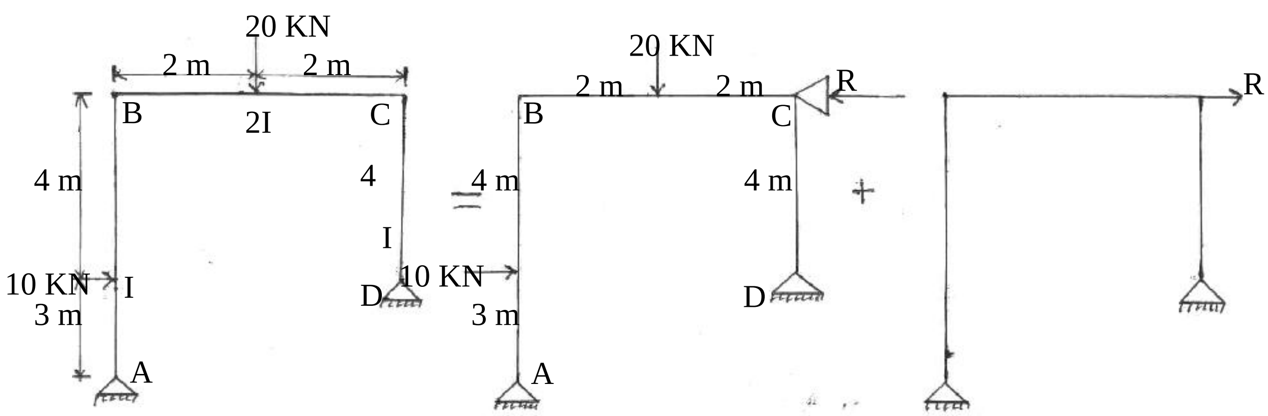


Fig 13 (a) Fig 13 (b) Fig 13 (c)

i) Non Sway Analysis:

First consider the frame held from side sway as shown in figure 13 (b).

FEM calculation:

$$M_{FAB} = -\frac{10\times3\times4^2}{7^2} = -9.8\,KNM$$

$$M_{FBA} = +\frac{10\times3^2\times4}{7^2} = 7.3\,KNM$$

$$M_{FBC} = -\frac{20\times4}{8} = -10\ KNM$$

M_{FCB} = + 10KNM

$M_{FCD} = M_{FDC} = 0$

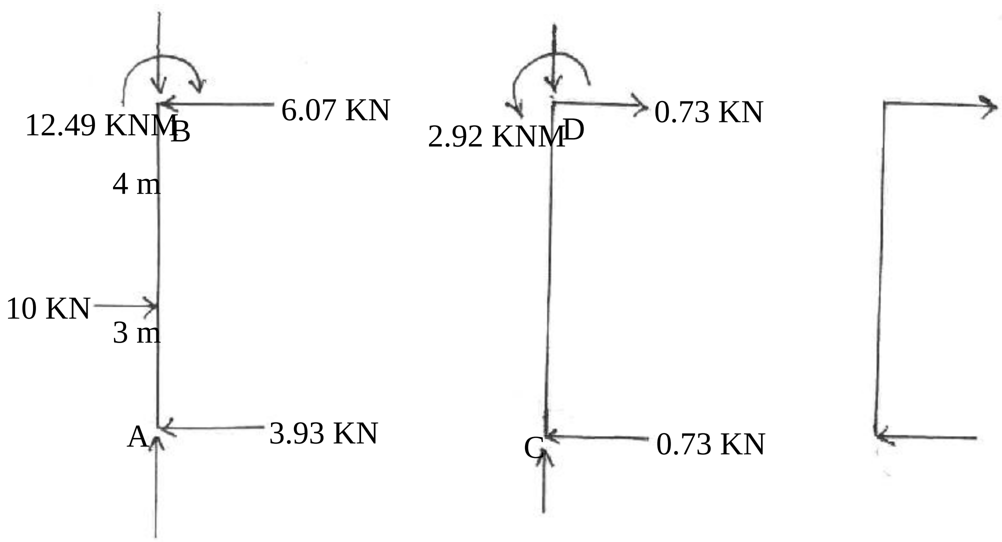
Calculation of distribution factors:

Joint	Member	Relative stiffness k	pk	DF = $\frac{K}{\sum K}$
B	BA	$\frac{3}{4} \times \frac{I}{7} = 0.11I$	0.61 I	0.18
	BC	2I/4 = 0.5I		0.82
C	CB	2I/4 = 0.5I		0.72
	CD	$\frac{3}{4} \times \frac{I}{4} = 0.19\ I$	0.69 I	0.28

Moment distribution table for non sway analysis:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	1	0.18	0.82	0.72	0.28	1
FEM	-9.8	7.3	-10	10	0	0
Release jt. 'D'	+9.8					
CO		4.9				
Initial moments	0	12.2	-10	10	0	0
Balance		-0.4	-1.8	-7.2	-2.8	
CO			-3.6	-0.9		
Balance		0.65	2.95	0.65	0.25	
C.O			0.33	1.48		
Balance		-0.06	-0.27	-1.07	-0.41	
C.O			-0.54	-0.14		
Balance		0.1	0.44	0.1	0.04	
Final moments	**0**	**12.49**	**-12.49**	**2.92**	**-2.92**	**0**

FBD of columns: FBD of columns AB & CD for non-sway analysis is shown in fig. 13 (d) and fig. 13 (e)



Applying $\Sigma F_x = 0$ for frame as a
Whole, $R = 10 - 3.93 - 0.73$
$= 5.34$ kN (←)

Now apply R = 5.34KN acting opposite as shown in figure 13 (c) for sway analysis

ii) **<u>Sway analysis:</u>** For this we will assume a force R' is applied at C causing the frame to deflect Δ' as shown m figure 13 (f).

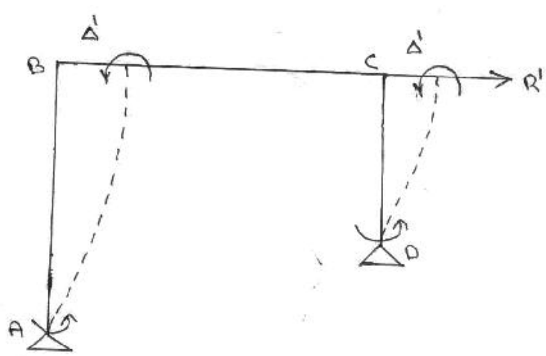


Fig. 13 (f)

Since ends A & D are hinged and columns AB & CD are of different lengths

$M'_{BA} = -3EI\Delta'/L^2_1$

$M'_{CD} = -3EI\Delta'/L^2_2$

$$\therefore \frac{M^1_{BA}}{M^1_{CD}} = \frac{3EI\ \Delta'/l_1^2}{3EI\ \Delta^1/l_2^2} = \frac{l_2^2}{l_1^2} = \frac{4^2}{7^2} = \frac{16}{49}$$

Assume $M^1_{BA} = -16\,\text{kNm}\ \&\ M'_{AB} = 0$

$\&\ M^1_{CD} = -49\,\text{kNm}\ \&\ M'_{DC} = 0$

Moment distribution table for sway analysis:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	1	0.18	0.82	0.72	0.28	1
FEM	0	-16	0	0	-49	0
Balance		2.88	13.12	35.28	13.72	
CO			17.64	6.56		
Balance		-3.18	-14.46	-4.72	-1.84	
CO			-2.36	-7.23		
Balance		0.42	1.94	5.21	2.02	
C.O			2.61	0.97		
Balance		-0.47	-2.14	-0.7	-0.27	
C.O			0.35	-1.07		
Balance		0.06	0.29	0.77	0.3	
C.O			0.39	0.15		
Balance		-0.07	-0.32	-0.11	-0.04	
Final moments	**0**	**-16.36**	**16.36**	**35.11**	**-35.11**	**0**

FBD of columns AB & CD for sway analysis moments is shown in fig. 13 (g)

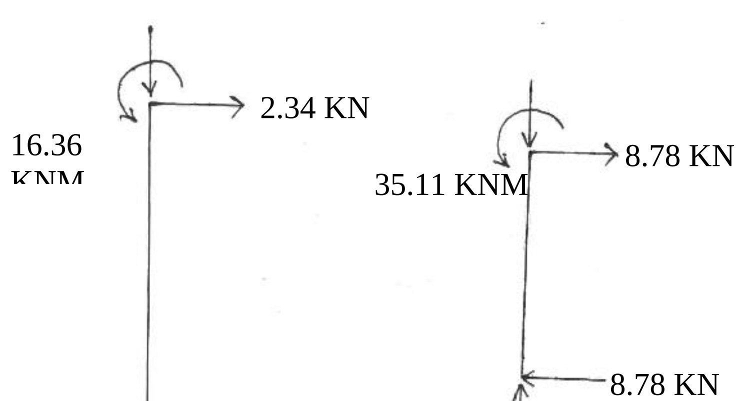


Fig. 13 (g)

Using $\Sigma f_x = 0$ for the entire frame
$R' = 11.12$ kN ($\rightarrow$)

Hence $R' = 11.12$ kN creates the sway moments shown in the above moment distribution table. Corresponding moments caused by R = 5.34 kN can be determined by proportion.

Thus final moments are calculated by adding non-sway moments and sway moments determined for R = 5.34 KN as shown below.

$M_{AB} = 0$

$M_{BA} = 12.49 + \frac{5.34}{11.12}(-16.36) = 4.63$ kNm

$M_{BC} = -12.49 + \frac{5.34}{11.12}(16.36) = -4.63$ KNM

$M_{CB} = 2.92 + \frac{5.34}{11.12}(35.11) = 19.78$ kNm

$M_{CD} = -2.92 + \frac{5.34}{11.12}(-35.11) = -19.78$ KNM

$M_{DC} = 0$

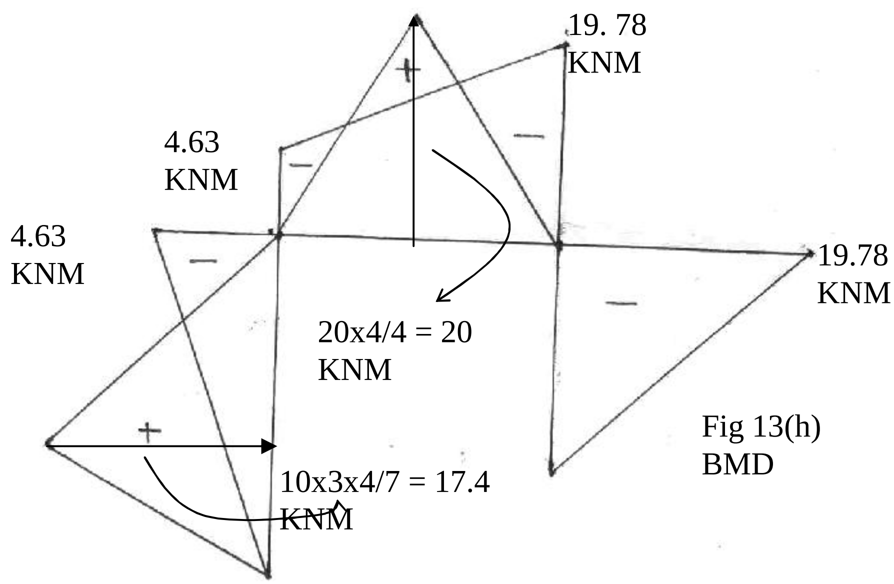


Fig 13(h)
BMD