Chapter-4: Kani's Method

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VTU – EDUSAT Programme – 7

Class: B.E. V Sem (Civil Engineering) Sub: Structural Analysis – II (CV51)

Session on 08.10.2007

KANI'S METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES:

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This method was developed by Dr. Gasper Kani of Germany in 1947. This method offers an iterative scheme for applying slope deflection method. We shall now see the application of Kani's method for different cases.

I. Beams with no translation of joints:

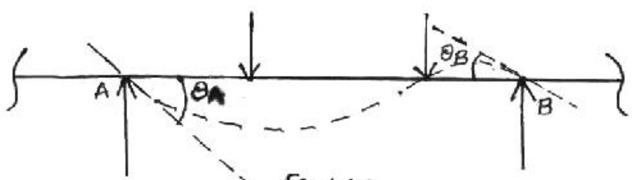

Fig. 1 (a)

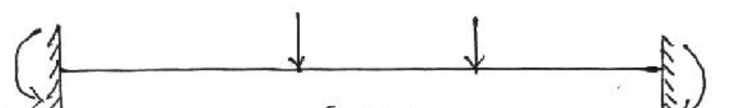

Fig. 1 (b)

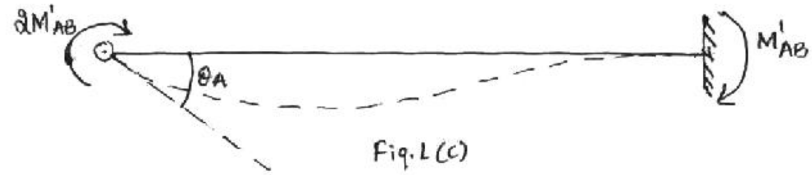

Fig. 1 (c)

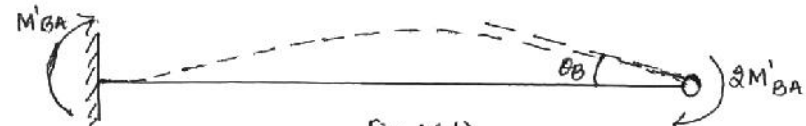

Fig. 1 (d)

Let AB represent a beam in a frame, or a continuous structure under transverse loading, as show in fig. 1 (a) let the M_{AB} & M_{BA} be the end moment at ends A & B respectively.

Sign convention used will be: clockwise moment +ve and anticlockwise moment –ve.

The end moments in member AB may be thought of as moments developed due to a superposition of the following three components of deformation.

1. The member 'AB' is regarded as completely fixed. (Fig. 1 b). The fixed end moments for this condition are written as M_{FAB} & M_{FBA}, at ends A & B respectively.
2. The end A only is rotated through an angle θ_A by a moment $2M'_{AB}$ inducing a moment M'_{AB} at fixed end B.
3. Next rotating the end B only through an angle θ_B by moment $2M'_{BA}$ while keeping end 'A' as fixed. This induces a moment M'_{BA} at end A.

Thus the final moment M_{AB} & M_{BA} can be expressed as super position of three moments

$$\left.\begin{aligned} M_{AB} &= M_{FAB} + 2M'_{AB} + M'_{BA} \\ M_{BA} &= M_{FBA} + 2M'_{BA} + M'_{AB} \end{aligned}\right\} \qquad \ldots\ldots\ldots(1)$$

For member AB we refer end 'A' as near end and end 'B' as far end. Similarly when we refer to moment M_{BA}, B is referred as near end and end A as far end.

Hence above equations can be stated as follows. The moment at the near end of a member is the algebraic sum of (a) fixed end moment at near end. (b) Twice the rotation moment of the near end (c) rotation moment of the far end.

Rotation factors:

Fig. 2 shows a multistoried frame.

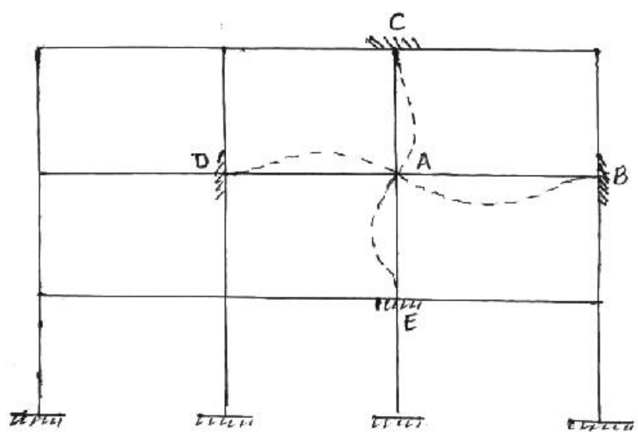

Fig. 2

Consider various members meeting at joint A. If no translations of joints occur, applying equation (1), for the end moments at A for the various members meeting at A are given by:

$$M_{AB} = M_{FAB} + 2M'_{AB} + M'_{BA}$$
$$M_{AC} = M_{FAC} + 2M'_{AC} + M'_{CA}$$
$$M_{AD} = M_{FAD} + 2M'_{AD} + M'_{DA}$$
$$M_{AE} = M_{FAE} + 2M'_{AE} + M'_{EA}$$

For equilibrium of joint A, $\Sigma M_A = 0$

$\therefore \Sigma M_{FAB} + 2\Sigma M'_{AB} + \Sigma M'_{BA} = 0$ ………………….(2)

where ,

ΣM_{FAB} =Algebraic sum of fixed end moments at A of all members meeting at A.

$\Sigma M'_{AB}$ = Algebraic sum of rotation moments at A of all member meeting at A.

$\Sigma M'_{BA}$ = Algebraic sum of rotation moments of far ends of the members meeting at A.

from equation (2)

$\Sigma M'_{AB} = -\frac{1}{2}\left[\sum M_{FAB} + \sum M'_{BA}\right]$ ……………… (3)

We know that $2M'_{AB} = \frac{4EI_{AB}}{L_{AB}}\theta_A = 4EK_{AB}\,\theta_A$

Where $K_{AB} = \frac{I_{AB}}{L_{AB}}$, relative stiffness of member AB

$M'_{AB} = 2E\,K_{AB}\,\theta_A$ …………….(4)

$\therefore \Sigma M'_{AB} = 2E\theta_A\,\Sigma K_{AB}$ …………….(5) (At rigid joint A all the members undergo same rotation θ_A)

Dividing Equation (4)/(5) gives

$$\frac{M'_{AB}}{\Sigma M'_{AB}} = \frac{K_{AB}}{\Sigma K_{AB}}$$

$\therefore M'_{AB} = \frac{K_{AB}}{\Sigma K_{AB}}\Sigma M'_{AB}$ ……………(5)

Substituting value of $\Sigma M'_{AB}$ from (3) in (5)

$$M'_{AB} = \left(-\frac{1}{2}\right)\frac{K_{AB}}{\Sigma K_{AB}}\left[\Sigma M_{FAB} + \Sigma M'_{BA}\right]$$

$= U_{AB}\left[\Sigma M_{FAB} + \Sigma M'_{BA}\right]$………………(6)

where $U_{AB} = -\frac{1}{2}\frac{K_{AB}}{\Sigma K_{AB}}$ is called as rotation factor for member AB at joint A.

Analysis Method:

In equation (6) ΣM_{FAB} is a known quantity. To start with the far end rotation moments M'_{BA} are not known and hence they may be taken as zero. By a similar approximation the rotation moments at other joints are also determined. With the approximate values of rotation moments computed, it is possible to again determine a more correct value of the rotation moment at A from member AB using equation (6).

The process is carried out for sufficient number of cycles until the desired degree of accuracy is achieved.

The final end moments are calculated using equation (1).

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Class: B.E. V Sem (Civil Engineering) Sub: Structural Analysis – II (CV51)

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KANI'S METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES (CONTD.)

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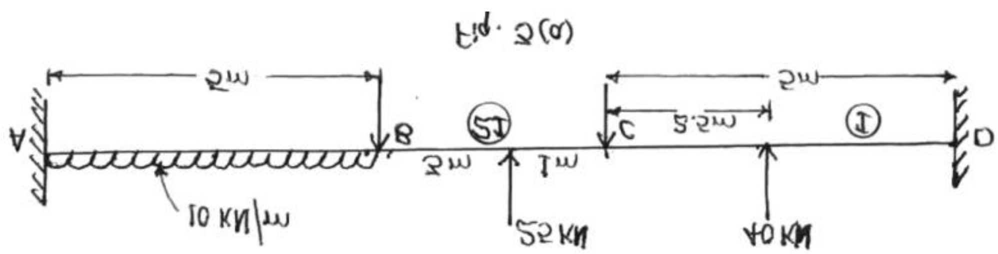
Kani's method for beams without translation of joints, is illustrated in following examples:

Ex: 1 Analyze the beam show in fig 3 (a) by Kani's method and draw bending



Solution:

a) Fixed end moments:

$$M_{FAB} = \frac{-10\times5^2}{12} = -20.83 \text{ kNm}$$

M_{FBA}= + 20.83 kNm

$$M_{FBC} = \frac{-25\times3\times1^2}{4^2} = -4.69 \text{ kNm}$$

$$M_{FCB} = \frac{25\times3^2\times1}{4^2} = 14.06 \text{ kNm}$$

$$M_{FCD} = \frac{-40 \text{ x } 5}{8} = -25 \text{ kNm}$$

M_{FDC} = 25 kNm

b) Rotation Factors:

Jt.	**Member**	**Relative stiffness (K)**	**pK**	**Rotation Factor**

				$= -\frac{1}{2} \mathbf{x} \frac{K}{\sum K}$
B	BA BC	I/5 = 0.2I 2I/4 = 0.5I	0.7I	- 0.14 -0.36
C	CB CD	2I/4 = 0.5I I/5 = 0.2I	0.7I	- 0.36 -0.14

c) Sum fixed end moment at joints:

$\Sigma M_{FB} = 20.83 - 4.69 = 16.14$ kNm

$\Sigma M_{FC} = 14.06 - 25 = -10.94$ kNm

The scheme for proceeding with method of rotation contribution is shown in figure 3 (b). The FEM, rotation factors and sum of fixed end moments are entered in appropriate places as shown in figure 3 (b).

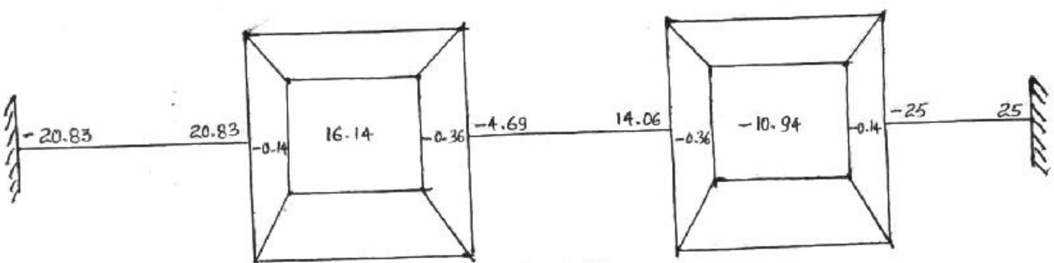


Fig. 3 (b)

d) Iteration Process:

Rotation contribution values at fixed ends A &D are zero. Rotation contributions at joints B & C are initially assumed as zero arbitrarily. These values will be improved in iteration cycles until desired degree of accuracy is achieved.

The calculations for two iteration cycles have been shown in following table. The remaining iteration cycle values for rotation contributions along with these two have been shown directly in figure 3 (c).

Jt	B		C	
Rotation Contribution	M'_{BA}	M'_{BC}	M'_{CB}	M'_{CD}
Iteration 1	-0.14 (16.14 + 0) = -2.26	-0.36 (16.14 + 0) = -5.81	-0.36 (-10.94 – 5.81)	-0.14 (-10.94 – 5.81) = 2.35

Iteration 2	-0.14 (16.14 + 6.03) = -3.1	-0.36 (16.14 + 6.03) = -7.98	= 6.03 -0.36 (-10.94 – 7.98) = 6.81	-0.14 (-10.94 – 7.98) = 2.65

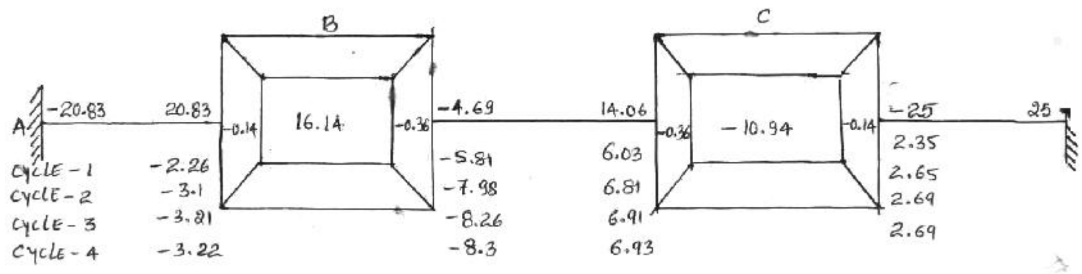


Fig.3(c)

Iterations are done up to four cycles yielding practically the same value of rotation contributions.

e) Final moments:

M_{AB} = - 20.83 + 0 – 3.22 = -24.05 kNm

M_{BA} = 20.83 + 2 × (-3.22) + 0 = 14.39 kNm

M_{BC} = –4.69 + 2 × (-8.3) + 6.93 = – 14.36 kNm

M_{CB} = 14.06 + (2 × 6.93) – 8.3 = 19.62 kNm

M_{CD} = –25 + (2 × 2.69) + 0 = -19.62 kNm

M_{DC} = 25 + 0 + 2.69 = 27.69 kNm

Bending moment diagram is shown in fig.3 (d)

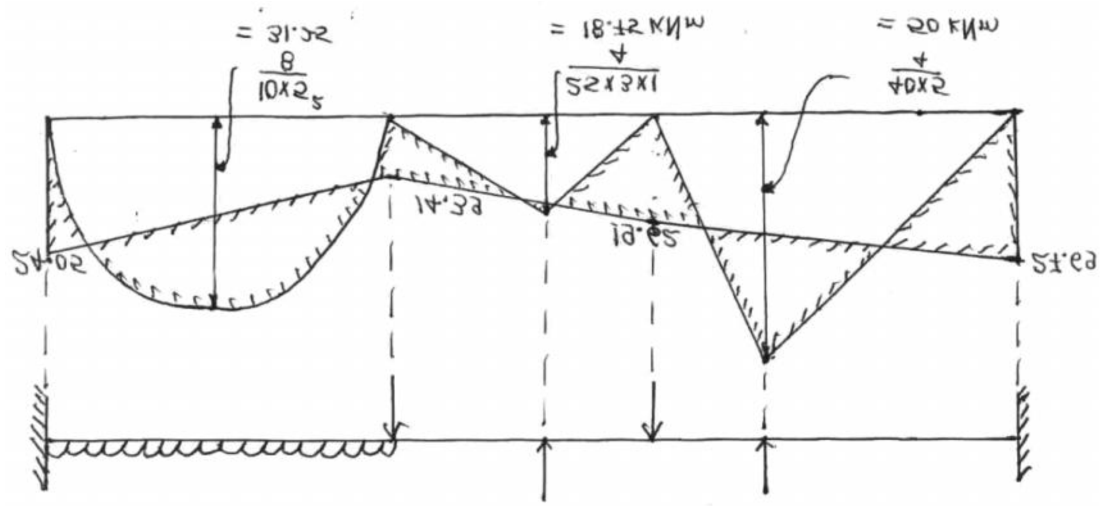


Fig.3 (d)

Ex 2: Analyze the continuous beam shown in fig. 4 (a)

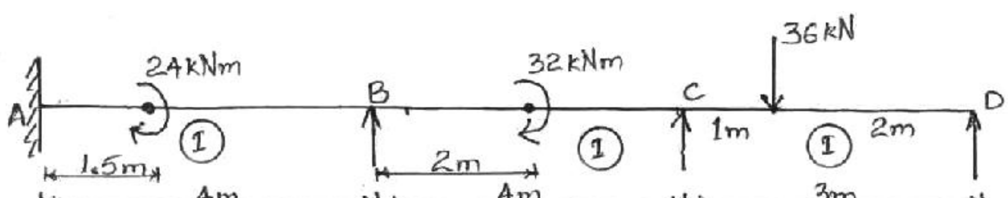


fig. 4(a)

Solution:

a) <u>Fixed end moments:</u>

$$M_{FAB} = \frac{b(3a-l)}{l^2} M_o = \frac{2.5\,(3\times1.5-4)}{4^2}\times 24 = 1.88 \text{ kNm}$$

$$M_{FBA} = \frac{a(3b-l)}{l^2} M_o = \frac{1.5(3\times2.5-4)}{4^2}\times 24 = 7.88 \text{ kNm}$$

$$M_{FBC} = \frac{M_o}{4} = \frac{32}{4} = 8 \text{ kNm}$$

$$M_{FCB} = \frac{M_o}{4} = 8 \text{ kNm}$$

$$M_{FCD} = -\frac{36\times1\times2^2}{3^2} = -16 \text{ kNm}$$

$$M_{FDC} = \frac{36\times1^2\times2}{3^2} = 8 \text{ kNm}$$

b) <u>Modification in fixed end moments:</u>

Actually end 'D' is a simply supported. Hence moment at D should be zero. To make moment at D as zero apply –8 kNm at D and perform the corresponding carry over to CD.

Modified $M_{FDC} = 8 - 8 = 0$

Modified $M_{FCD} = -16 + \frac{1}{2}(-8) = -20$ kNm

Now joint D will not enter the iteration process.

c) <u>Rotation Factors:</u>

Joint	**Member**	**Relative stiffness (K)**	**pK**	**Rotation Factor** $U = -\frac{1}{2} \times \frac{K}{\sum K}$
B	BA	I/4 = 0.25I	0.5 I	– 0.25
	BC	I/4 = 0.25I		–0.25
C	CB	I/4 = 0.25I		–0.25
	CD	$\frac{3}{4}\times\frac{I}{3}$ = 0.25I	0.5I	–0.25

d) <u>Sum of fixed end moments at joints:</u>

$\Sigma M_{FB} = 7.88 + 8 = 15.88$ kNm

$\Sigma M_{FC} = 8 - 20 = -12$kNm

e) <u>Iteration process</u>

Joint	B		C	
Rotation Contribution	M'_{BA}	M'_{BC}	M'_{CB}	M'_{CD}
Rotation factor	–0.25	–0.25	–0.25	–0.25
Iteration 1 started at B assuming M'_{CB} = 0 & taking M'_{AB} = 0′ M'_{DC} = 0.	–0.25 × (15.88 + 0 + 0) = -3.97	–0.25 × (15.88 + 0 + 0) = -3.97	–0.25 × (-12 –3.97 + 0) = 3.97	–0.25 × (-12 –3.97 + 0) = 3.97
Iteration 2	–0.25 × (15.88 + 0 + 3.97) = –4.96	–0.25 × (15.88 + 0 + 3.97) = – 4.96	–0.25 × (-12 –4.96 + 0) = 4.24	–0.25 (–12 –4.96 + 0) = 4.24
Iteration 3	–0.25 (15.88 + 0 + 4.24) = –5.03	–0.25 (15.88 + 0 + 4.24) = –5.03	–0.25 × (-12 –5.03+ 0) = 4.26	–0.25 (–12 –5.03 + 0) = 4.26
Iteration 4	–0.25 (15.88 + 0 + 4.26) = –5.04	–0.25 (15.88 + 0 + 4.26) = –5.04	–0.25 × (-12 –5.03+ 0) = 4.26	–0.25 (–12 –5.03 + 0) = 4.26

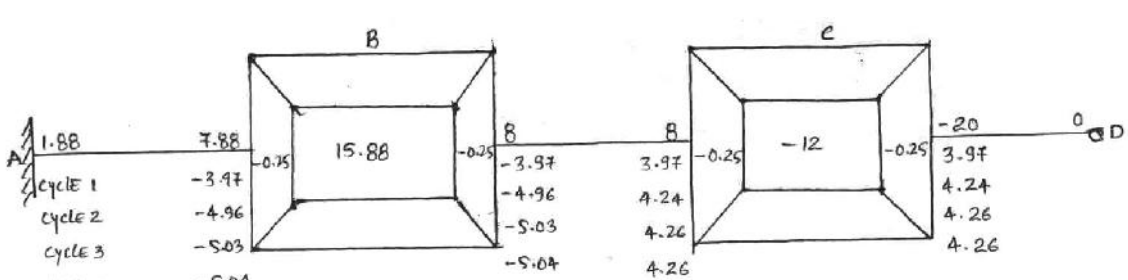


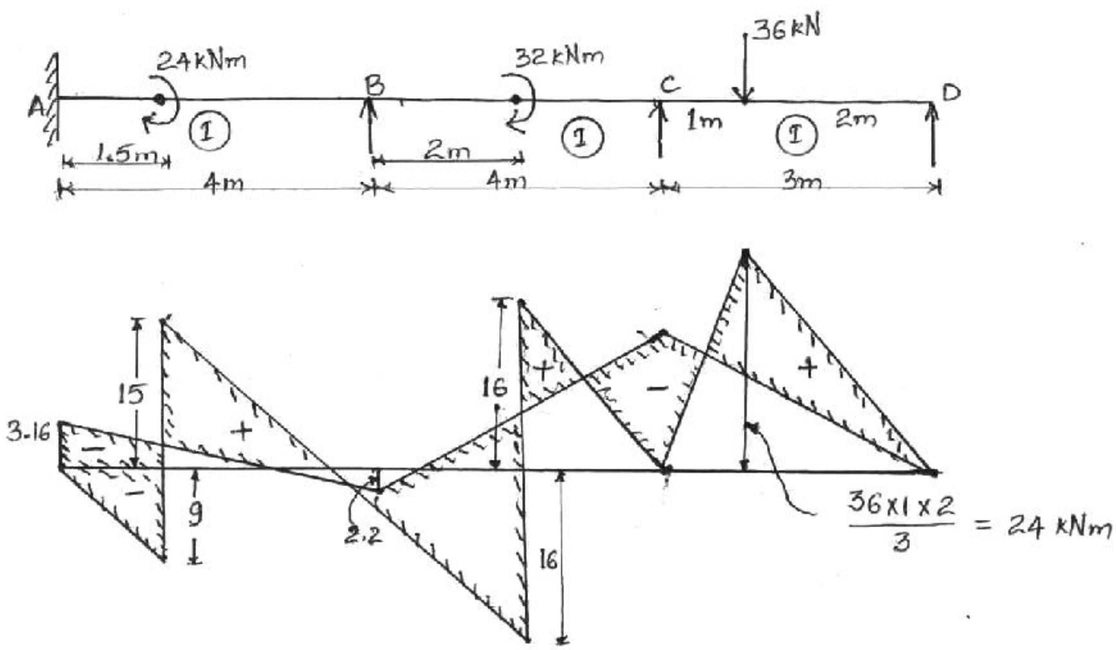
Fig. 4 (b)

Iteration process has been stopped after 4^{th} cycle since rotation contribution values are becoming almost constant. Values of fixed end moments, sum of fixed end moments, rotation factors along with rotation contribution values after end of each cycle in appropriate places has been shown in fig. 4 (b).

f) Final moments

Member (ij)	**FEM M_{Fij} (kNm)**	$2M'_{ij}$ **(kNm)**	M^{1}_{ji} **(kNm)**	**Final moments** $\left(= M_{Fij} + 2M'_{ij} + M'_{ji}\right)$ **(kNm)**
AB	1.88	0	-5.04	-3.16
BA	7.88	2 (-5.04) = 10.08	0	-2.2
BC	8	2 (-504) = −10.08	4.26	+2.2
CB	8	2 × 4.26 = 8.52	-5.04	11.48
CD	-20	2 x 4.26 = 8.52	0	-11.48

BMD is shown below:



VTU – EDUSAT Programme – 7

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KANI'S METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES (CONTD.)

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Ex 3: Analyze the continuous beam shown in fig. 5 (a) and draw BMD & SFD (VTU January 2005 exam)

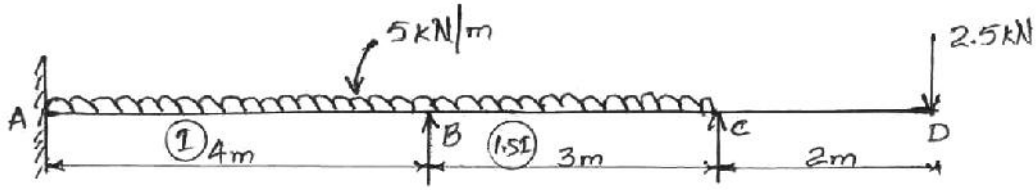


fig. 5 (a)

Solution:

a) Fixed end moments:

$$M_{FAB} = -\frac{5\times4^2}{12} = -6.67 \text{ kNm}$$

$$M_{FBA} = +6.67 \text{ kNm}$$

$$M_{FBC} = \frac{-5\times3^2}{12} = -3.75 \text{ kNm}$$

$$M_{FCB} = +3.75 \text{ kNm}$$

$$M_{CD} = -2.5 \times 2 = -5 \text{ kNm}$$

b) Modification in fixed end moments:

Since $M_{CD} = -5$ kNm; $M_{CB} = +5$kNm, for this add 1.25 kNm to M_{FCB} and do the corresponding carry over to M_{FBC}

$\therefore$ Now $M_{CB} = 5$ kNm

Modified $M_{FBC} = -3.75 + \frac{1}{2}(1.25) = -3.13$ kNm

Now joint C will not enter in the iteration process.

c) Rotation factors:

Jt.	Member	Relative stiffness (K)	pK	Rotation Factor $U = -\frac{1}{2} \times \frac{K}{\sum K}$
B	BA	I/4 = 0.25I	0.625I	– 0.2
	BC	$\frac{3}{4} \times \frac{1.5I}{3} = 0.375I$		–0.3
C	CB	1.5I/3 = 0.5I	0.5I	–0.5
	CD	0		0

d) Sum of fixed end moments at joints:

$\Sigma M_{FB} = 6.67 - 3.13 = 3.54$ kNm

e) Iteration Process

Joint	B	
Rotation Contribution	M'_{BA} (kNm)	M'_{BC} (kNm)
Rotation factor	– 0.2	–0.3
Iteration 1 started at B taking $M'_{AB} = 0$ & $M'_{CB} = 0$	$-0.2 \times (3.54 + 0 + 0) = -0.71$	$-0.3 \times (3.54 + 0 + 0) = -1.06$

Since 'B' is the only joint needing rotation correction, the iteration process will stop after first iteration. Value of FEMs, sum of FEM at joint, rotation factors along with rotation contribution values in appropriate places is shown in fig. 5 (b)

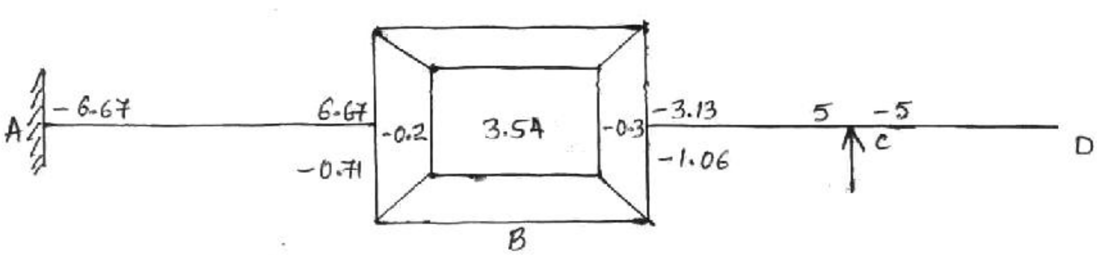
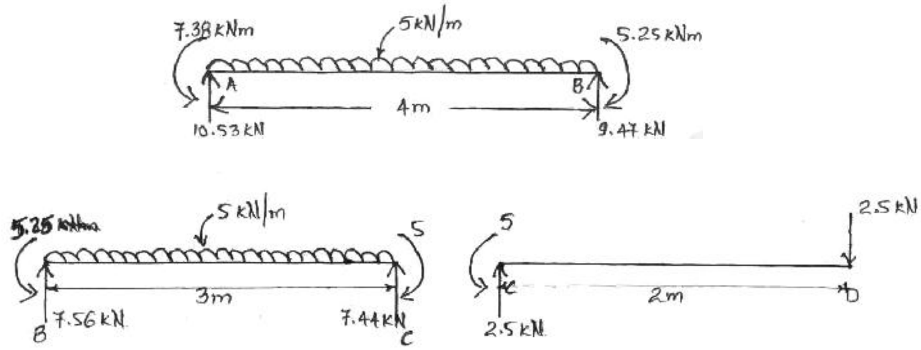


Fig.5(b)

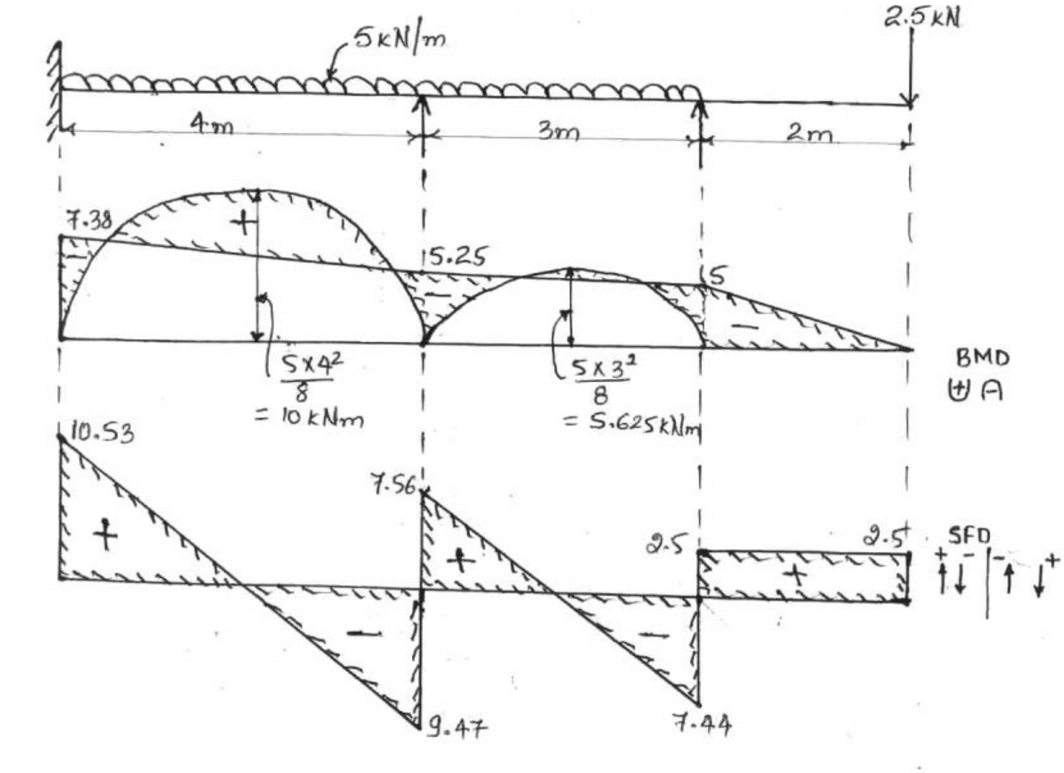
(f) Final moments:

Member (ij)	**FEM M_{Fij} (kNm)**	$2M'_{ij}$ **(kNm)**	M^{1}_{ji} **(kNm)**	**Final moments** $\left(= M_{Fij} + 2M'_{ij} + M'_{ji}\right)$ **(kNm)**
AB	– 6.67	0	–0.71	–7.38
BA	6.67	2 × (-0.71) =	0	5.25
BC	–3.13	2 × (–1.06)	0	–5.25
CB				+ 5
CD	-	-	-	– 5
DC				0

FBD of each span along with reaction values which have been calculated from statics are shown below:



BMD and SFD are shown below



II. Kani's method for members with translatory joints:

Fig. 6 shows a member AB in a frame which has undergone lateral displacement at A & B so that the relative displacement is $\Delta = \Delta_B - \Delta_A$

If ends A & B are restrained from rotation FEM corresponding to this displacement are

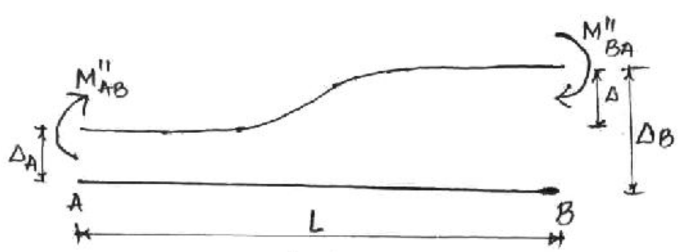


Fig. 6

$$M''_{AB} = M''_{BA} = \frac{6EI\,\Delta}{L^2} \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

When translation of joints occurs along with rotations the true end moments are given by

$M_{AB} = M_{FAB} + 2M'_{AB} + M'_{BA} + M''_{AB}$

$M_{BA} = M_{FBA} + 2M'_{BA} + M'_{AB} + M''_{BA}$

If ‘A’ happens to be a joint where two or more members meet then from equilibrium of joint we have

$\Sigma M_{AB} = 0$

$\Sigma M_{FAB} + 2\Sigma M'_{AB} + \Sigma M'_{BA} + \Sigma M''_{AB} = 0$

$$\therefore \Sigma M'_{AB} = -\frac{1}{2}\left(\Sigma M_{FAB} + \Sigma M'_{BA} + \Sigma M''_{AB}\right) \ldots\ldots\ldots\ldots\ldots(8)$$

we know from equation (5)

$$M'_{AB} = \frac{K_{AB}}{\Sigma K_{AB}}\Sigma M'_{AB}$$

$\therefore$ Using equation (8) $M'_{AB} = -\frac{1}{2}\frac{K_{AB}}{\Sigma K_{AB}}\left(\Sigma M_{FAB} + \Sigma M'_{BA} + \Sigma M''_{AB}\right)$

$$= U_{AB}\left(\Sigma M_{FA} + \Sigma M'_{BA} + \Sigma M''_{AB}\right)$$

Similarly $M'_{BA} = U_{BA}\left(\Sigma M_{FB} + \Sigma M'_{AB} + \Sigma M''_{BA}\right) \ldots\ldots\ldots\ldots\ldots..(9)$

Using the above relationships rotation contributions can be determined by iterative procedure. If lateral displacements are known the displacement moments can be determined from equation (7). If lateral displacements are unknown then additional equations have to be developed for analyzing the member.

Ex 4: In a continuous beam shown in fig. 7 (a). The support ‘B’ sinks by 10mm. Determine the moments by Kani’s method & draw BMD.

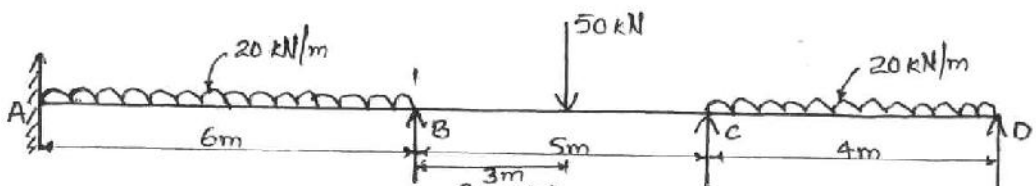


Fig. 7 (a)

Take I = 1.2 x 10^{-4} mm^4 & E = 2 x 10^5 N/mm²

Solution:

(a) Calculation of FEM:

$$M_{FAB} = -\frac{20\times6^2}{12} - \frac{6\times2\times10^5\times1.2\times10^{-4}\times10^{12}\times10}{(6000)^2\times10^6}$$

$= -60 -40$

$= -100$ kNm

M_{FBA} = +60 –40= 20 kNm

$$M_{FBC} = -\frac{50\times3\times2^2}{5^2} + \frac{6\times\ 2\times10^5\ \times1.2\times10^{-4}\times10^{12}\times10}{(5000)^2\times10^6}$$

= –24 + 57.6

= 33.6 kNm

$$M_{FCB} = +\frac{50\times3^2\times2}{5^2} + \frac{6\times2\times10^5\ \times1.2\times10^{-4}\times10^{12}\times10}{(5000)^2\times10^6}$$

= 36 + 57.6

= 93.6 kNm

C & D are at same level

$$\therefore\ M_{FCD} = -\frac{20\times4^2}{12} = -26.67 \text{ kNm}$$

M_{FDC} = + 26.67 kNm

b) Modification in fixed end moments:

Since end 'D' is a simply supported, moment at 'D' is zero. To make moment at D as zero apply a moment of – 26.67 kNm at end D and perform the corresponding carry over to CD.

$\therefore$ Modified M_{FDC} = + 26.67 –26.67 = 0

Modified $M_{FCD} = -26.67 + \frac{1}{2}(-26.67)$

= –40 kNm

Other FEMs will be same as calculated earlier. Now joint 'D' will not enter the iteration process.

c) Rotation factors:

Joint	**Member**	**Relative stiffness (K)**	**pK**	**Rotation Factor** $U = -\frac{1}{2}\times\frac{K}{\sum K}$
B	BA	I/6 = 0.17 I	0.37 I	–0.23
	BC	I/5 = 0.2 I		–0.27
C	CB	I/5 = 0.2I	0.39I	–0.26
	CD	$\frac{3}{4}$ x I/4 = 0.19 I		–0.24

d) Sum of fixed end moments:

$$\Sigma M_{FB} = 20 + 33.6 = 53.6 \text{ kNm}$$
$$\Sigma M_{FC} = 93.6 - 40 = 53.6 \text{ kNm}$$

e) Iteration process:

Joint	B		C	
Rotation Contribution	M'_{BA} (kNm)	M'_{BC} (kNm)	M'_{CB} (kNm)	M'_{CD} (kNm)
Rotation factor	–0.23	–0.27	–0.26	–0.24
Iteration 1 (Started at B by taking $M'_{AB} = 0$ and assuming $M'_{CB} = 0$	–0.23 × (53.6 + 0 + 0) = –12.33	–0.27 x (53.6 + 0+0)= –14.47	–0.26×(53.6 – 14.47+ 0) = –10.17	–0.24 (53.6 – 14.47– 10.96= –9.39
Iteration 2	–0.23 (53.6 – 10.17) = –10.00	–0.27 (53.6 –10.17) = –11.73	–0.26 (53.6 –11.73) = –10.89	–0.24 (53.6 –11.73) =–10.05
Iteration 3	–0.23 (53.6–10.89) = –9.82	–0.27 (53.6 –10.89) =–11.53	–0.26 (53.6 –11.53) = –10.94	–0.24 (53.6 –11.53) =–10.10
Iteration 4	–0.23 (53.6 –10.94) = –9.81	–0.27 (53.6 –10.94) = –11.52	–0.26 (53.6 –11.52) = –10.94	–0.24 (53.6 –11.52) = –10.1

Iteration process has been stopped after fourth cycle since rotation contribution values are becoming almost constant. Values of FEMs, sum of fixed end moments, rotation factors along with rotation contribution values after end of each cycle in appropriate places has been shown in Fig. 7 (b).

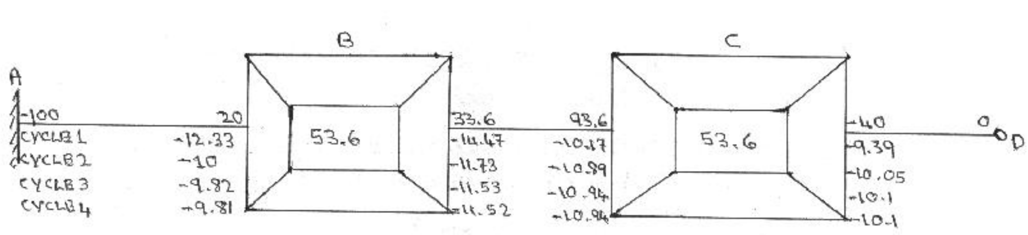
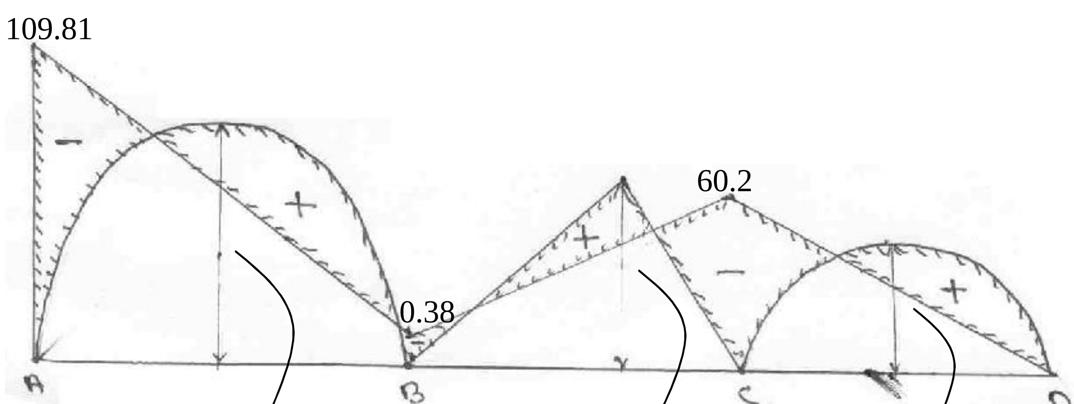


Fig:7(b)

f) Final moments:

Member (ij)	**FEM M_{Fij} (kNm)**	$2M'_{ij}$ **(kNm)**	M^1_{ji} **(kNm)**	**Final moments** $\left(= M_{Fij} + 2M'_{ij} + M'_{ji}\right)$ **(kNm)**
AB	– 100	0	–9.81	–109.81
BA	20	2 × (-9.81)= -19.62	0	+0.38
BC	33.6	2 × (–11.52) = –23.04	–10.94	–0.38
CB	93.6	2 × (–10.94) = –21.88	–11.52	60.2
CD	–40	2 × (–10.1) = –20.2	0	–60.2
DC	0	0	0	0

g) BMD is shown below:



20x6² / 8 = 90 KNM

50x3x2/5 = 60 KNM

20x4²/8 = 40KNM

VTU – EDUSAT Programme – 7

Class: B.E. V SEM (Civil Engineering) SUB: Structural Analysis – II (CV51)

Session on 15.10.2007

KANI'S METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES (CONTD.)

BY A.B.HARWALKAR

P.D.A.College of Engg Gulbarga

III) Analysis of frames with no translation of joints:

The frames, in which lateral translations are prevented, are analyzed in the same way as continuous beams. The lateral sway is prevented either due to symmetry of frame and loading or due to support conditions. The procedure is illustrated in following example.

Example-5. Analyze the frame shown in Figure 8 (a) by Kani's method. Draw BMD. (VTU Jan 2005 Exam).

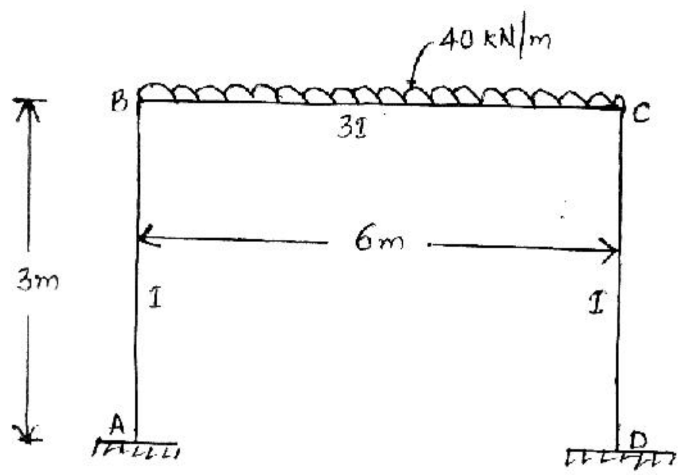


Fig-8(a)

Solution:

(a) Fixed end moments:

$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$

$M_{FBC} = \dfrac{-40x6^2}{12} = -120kNm.$

$M_{FCB} = +120kNm.$

(b) Rotation factors:

Joint	**Member**	**Relative Stiffness (k)**	p**k**	**Rotation factor = -½k/**p**k**
B	BC BA	3I/6 = 0.5I I/3 = 0.33I	0.83I	-0.3 -0.2
C	CB CD	3I/6 = 0.5I I/3 = 0.33I	0.83I	-0.3 -0.2

(c) Sum of FEM:

ΣM_{FB} = -120+0 = -120

ΣM_{FC} = 120+0 = +120

(d) Iteration process:

Joint	**B**		**C**	
Rotation Contribution	**M'_{BA}**	**M'_{BC}**	**M'_{CB}**	**M'_{CD}**
Rotation Factor	**-0.2**	**-0.3**	**-0.3**	**-0.2**
Iteration 1 stated with end B taking M'_{AB}=0 and assuming M'_{CB}=0	-0.2(-120+0) =24	-0.3(-120+0) =36	-0.2(120+36+0) = -46.8	-0.2(120+36+0) = -31.2
Iteration 2	-0.2(-120-46.8) =33.6	-0.3(-120-46.8) =50.04	-0.3(120+50.04) = -51.01	-0.2(120+50.04) = -34.01
Iteration 3	-0.2(-120-51.01) =34.2	-0.3(-120-51.01) =51.3	-0.3(120+51.3) = -51.39	-0.2 (120+51.3) = -34.26
Iteration 4	-0.2(-120-51.39) =34.28	-0.3(-120-51.39) =51.42	-0.3(120+51.42) = -51.43	-0.2 (120+51.42) = -34.28

The fixed end moments, sum of fixed and moments, rotation factors along with rotation contribution values at the end of each cycle in appropriate places is shown in figure 8(b).

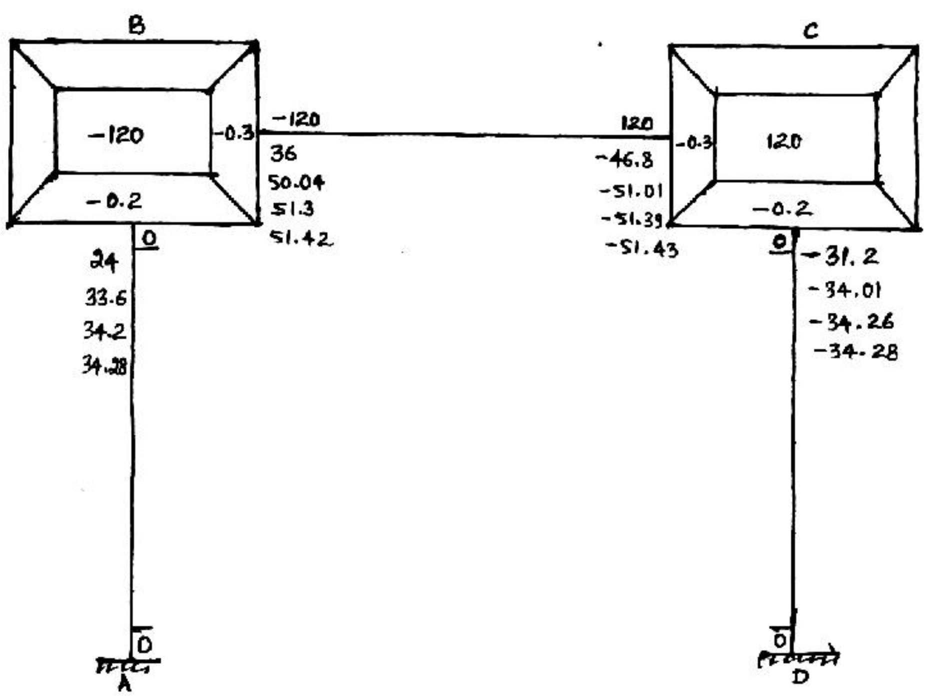


Fig-8(b)

(e) Final moments:

Member (ij)	M_{Fij}	$2M'_{ij}$ (kNm)	M'_{ji} (kNm)	(kNm) Final moment = $M_{Fij} + 2M'_{ij} + M'_{ji}$
AB	0	0	34.28	34.28
BA	0	2 x 34.28	0	68.56
BC	-120	2 x 51.42	-51.43	-68.59
CB	120	2 x (-51.43)	51.42	68.56
CD	0	2 x (-34.28)	0	-68.56
DC	0	0	-34.28	-34.28

BMD is shown below in figure-8 (c)

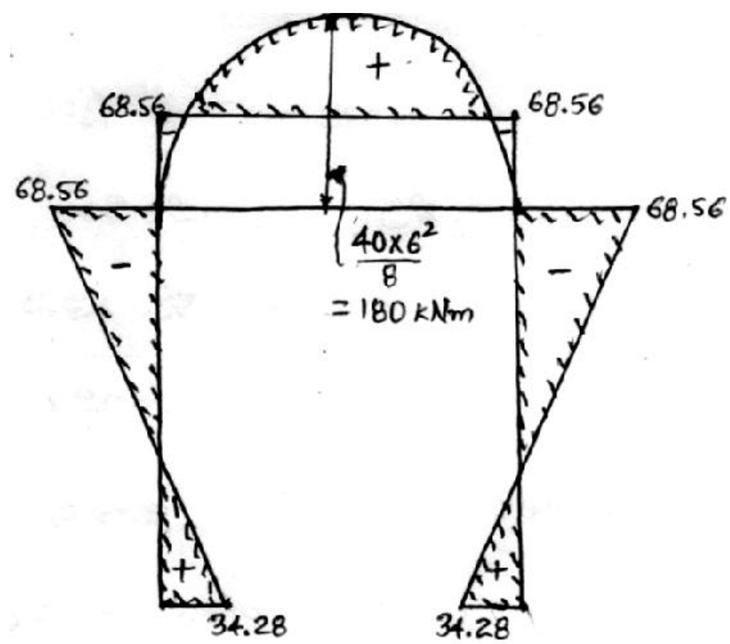


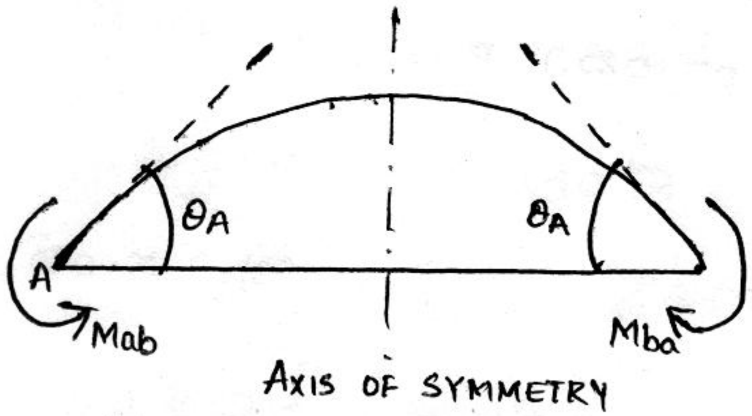
Fig-8 (c)

IV) Analysis of symmetrical frames under symmetrical loading:

Considerable calculation work can be saved if we make use of symmetry of frames and loading especially when analysis is done manually. Two cases of symmetry arise, namely, frames in which the axis of symmetry passes through the centerline of the beams and frames with the axis of symmetry passing through column line.

Case-1: (Axis of symmetry passes through center of beams):

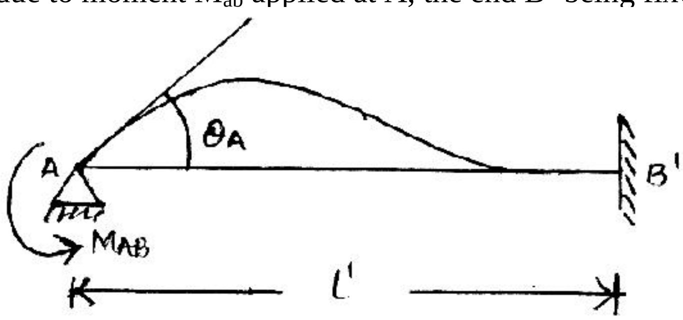
Let AB be a horizontal member of the frame through whose center, axis of symmetry passes. Let M_{ab} and M_{ba} be the end moments. Due to symmetry of deformation M_{ab} and M_{ba} are numerically equal but are opposite in their sense.



θ_A = Slope due to M_{ab} + slope due to M_{ba}

$$= \frac{M_{ab}l}{3EI} + \frac{M_{ba}l}{6EI} = \frac{M_{ab}l}{2EI}$$

Let this member be replaced by member AB' whose end A will undergo the rotation θ_A due to moment M_{ab} applied at A, the end B' being fixed.



$$\therefore \theta_A = \frac{M_{ab}l'}{4EI'}$$

Hence for equality of rotations between original member AB and the substitute member AB'

$$\frac{M_{ab}l}{2EI} = \frac{M_{ab}l'}{4EI'}$$

$$\frac{I}{l} = \frac{2I'}{l'}$$

$K = 2K'$

$\therefore K' = \dfrac{K}{2}$

Thus if K is the relative stiffness of original member AB, this member can be replaced by substitute member AB' having relative stiffness $\dfrac{K}{2}$. With this substitute member, the analysis need to be carried out for only, one half of the frame considering line of symmetry as fixed.

VTU – EDUSAT Programme – 7

Class: B.E. V SEM (Civil Engineering) SUB: Structural Analysis – II (CV51)

Session on 16.10.2007

KANI'S METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES (CONTD.)

BY A.B.HARWALKAR

P.D.A.College of Engg Gulbarga

Example-6: Analyze the frame given in example-5 by using symmetry condition by Kani's method.

Solution:

Since symmetry axis passes through center of beam only one half of frame as shown in figure 9 (a) will be considered

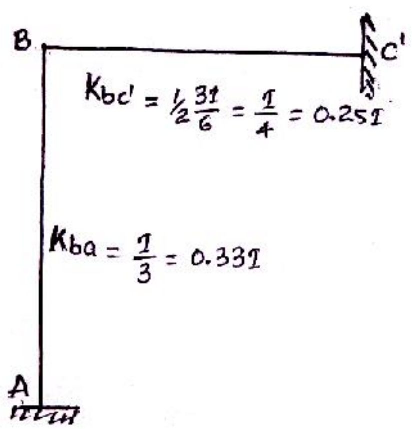


Fig-9(a)

$\therefore$ Rotation factors

$U_{BA} = -\frac{1}{2}$ x (0.33I / 0.33I + 0.25I) = -0.28

$U_{BC} = -\frac{1}{2}$ x (0.25I / 0.33I + 0.25I) = -0.22

The calculation of rotation contribution values is shown directly in figure-9(b)

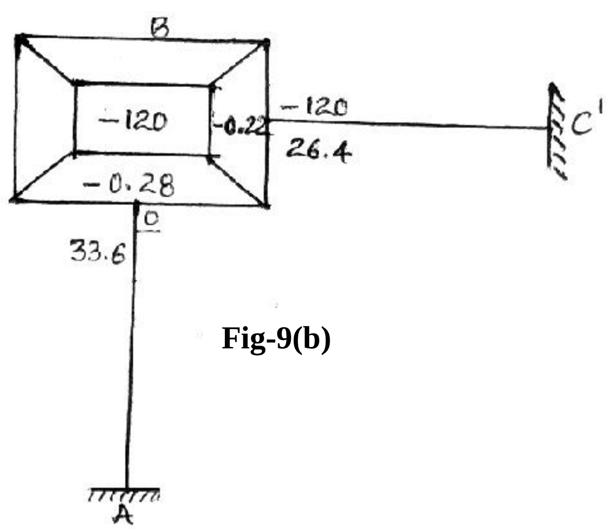


Fig-9(b)

Here we can see that rotation contributions are obtained in the first iteration only. The final moments for half the frame are shown in figure 9(c) and for full frame are shown in figure 9(d).

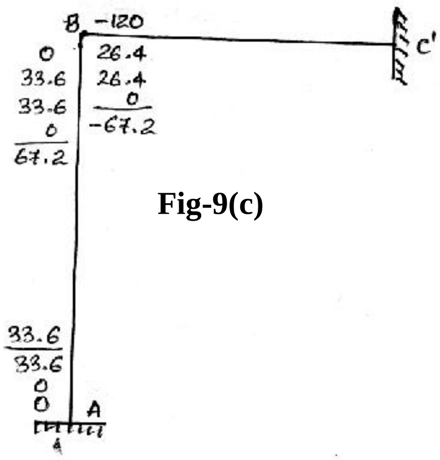


Fig-9(c)

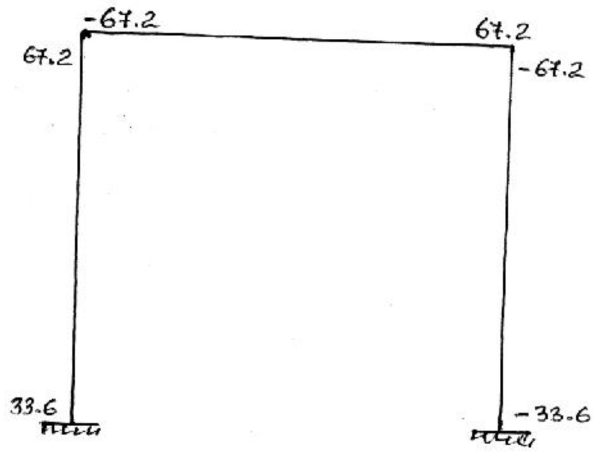


Fig-9(d)

Example-7: Analyze the frame shown in figure 10(a) by Kani's method.

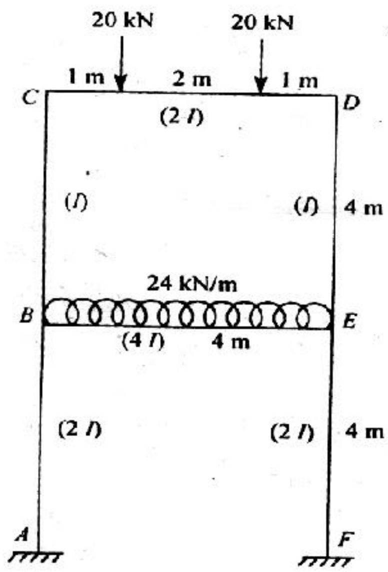


Fig-10(a)

Solution:

Analysis will be carried out taking the advantage of symmetry

(a) Fixed end moments:

$M_{Fcd} = -[(20 \times 1 \times 3^2 / 4^2) + (20 \times 3 \times 1^2 / 4^2)] = -15$kNm

$M_{Fbe} = -24 \times 4^2 / 12 = -32$ kNm.

The substitute frame is shown in figure 10(b)

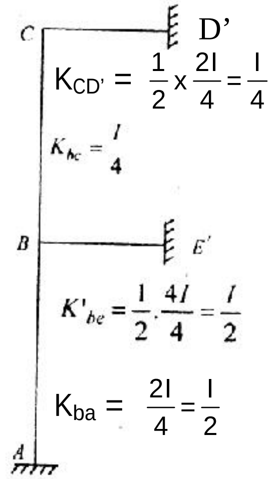


Fig-10(b)

(b) Rotation factors:

Joint	Member	Relative Stiffness K	pk	Rotation factors = $-\frac{1}{2}\frac{K}{\Sigma K}$
B	BA	2I/4	5I/4	-1/5
	BE'	$\frac{1}{2} x \frac{4I}{4} = I/2$		-1/5
	BC	I/4		$-\frac{1}{10}$
C	CB	I/4	2I/4	-1/4
	CD'	$\frac{1}{2} x \frac{2I}{4} = \frac{I}{4}$		-1/4

Rotation contributions calculated by iteration process are directly shown in figure-10(c).

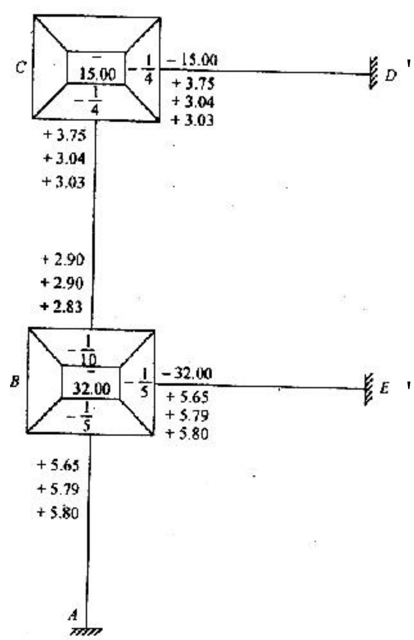


Fig-10(c)

The calculation of final moments for the substitute frame is shown in figure-10(d)

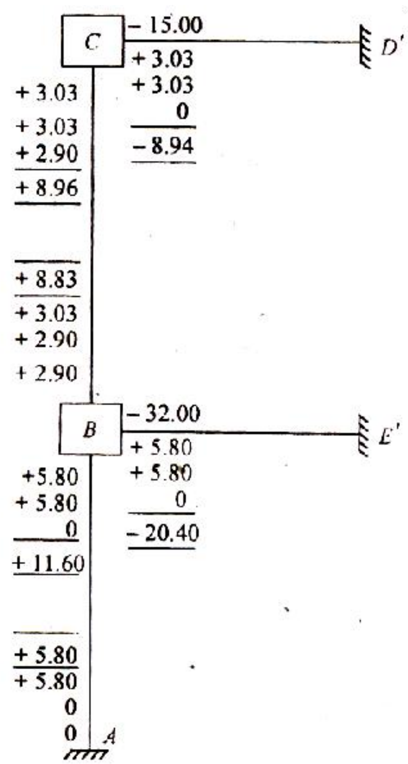


Fig-10(d)

Figure-10(e) shows final end moments for the entire frame.

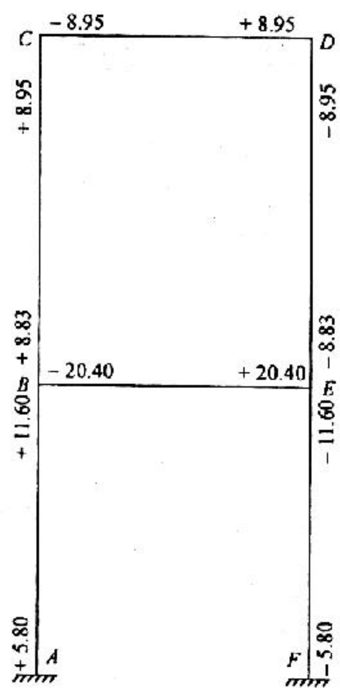


Fig-10(e)

Session on 19.10.2007

KANI'S METHOD FOR ANALYSIS OF INDETERMINATE STRUCTURES (CONTD.)

BY A.B.HARWALKAR
P.D.A.College of Engg Gulbarga

Case 2: When the axis of symmetry passes through the column:

This case occurs when the number of bays is an even number. Due to symmetry of the loading and frame, the joints on the axis of symmetry will not rotate. Hence it is sufficient if half the frame is analyzed. The following example illustrates the procedure.

Example-8: Analyze the frame shown in figure-11(a) by Kani's method, taking advantage of symmetry and loading.

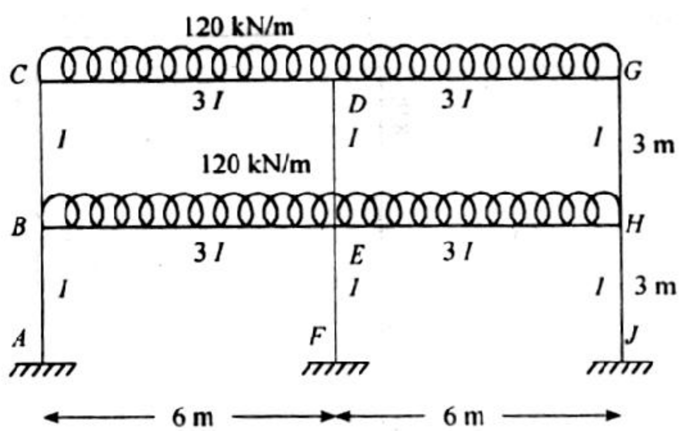


Fig-11(a)

Solution:

Only half frame as shown in figure-11(b) will be considered for the analysis.

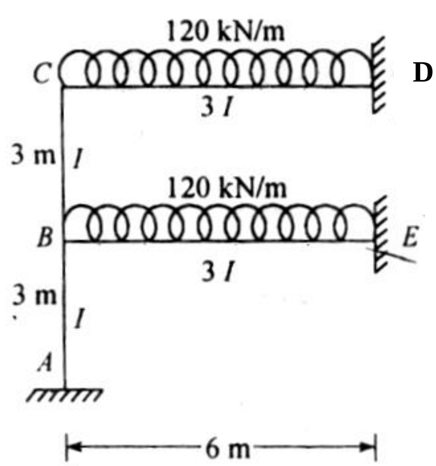


Fig-11(b)

(a) Fixed end moments:

$$M_{FBE} = -\frac{120 \times 6^2}{12} = -360kNm$$

$$M_{FCD} = -\frac{120 \times 6^2}{12} = -360kNm$$

(b) Rotation factors:

Joint	Member	Relative Stiffness k	pk	Rotation factors = $-\frac{1}{2}\frac{K}{\Sigma K}$
B	BA	I/3	7I/6	-1/7
	BE	3I/6 = I/2		-3/14
	BC	I/3		-1/7
C	CB	I/3	5I/6	-1/5
	CD	3I/6 = I/2		-3/10

(c) Iteration process:

The iteration process for calculation of rotation contribution values at C & B was carried up to four cycles and values for each cycle are shown in figure-11(c).

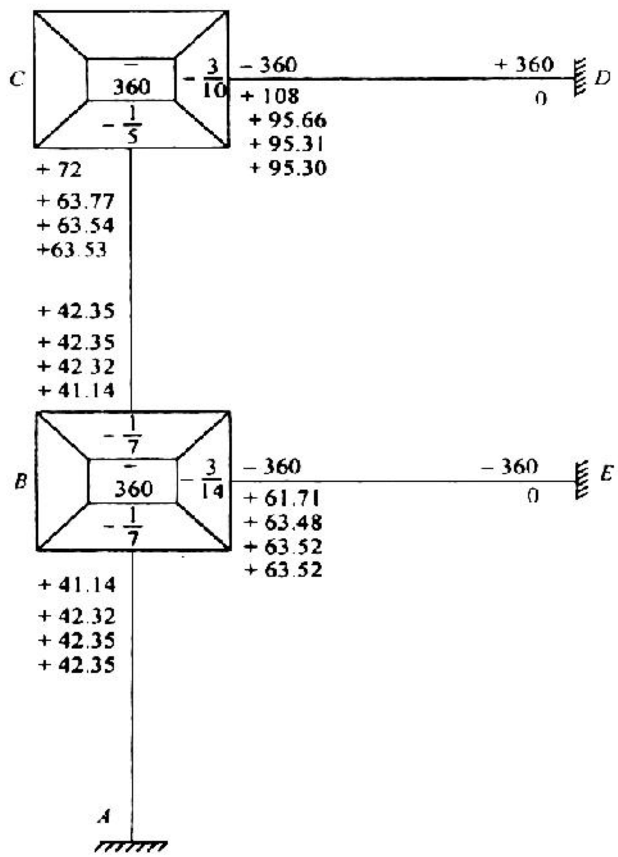


Fig-11(c)

Final moments calculations for half the frame are shown in figure-11(d) and final end moments of all the members of the frame are shown in figure-11(e).

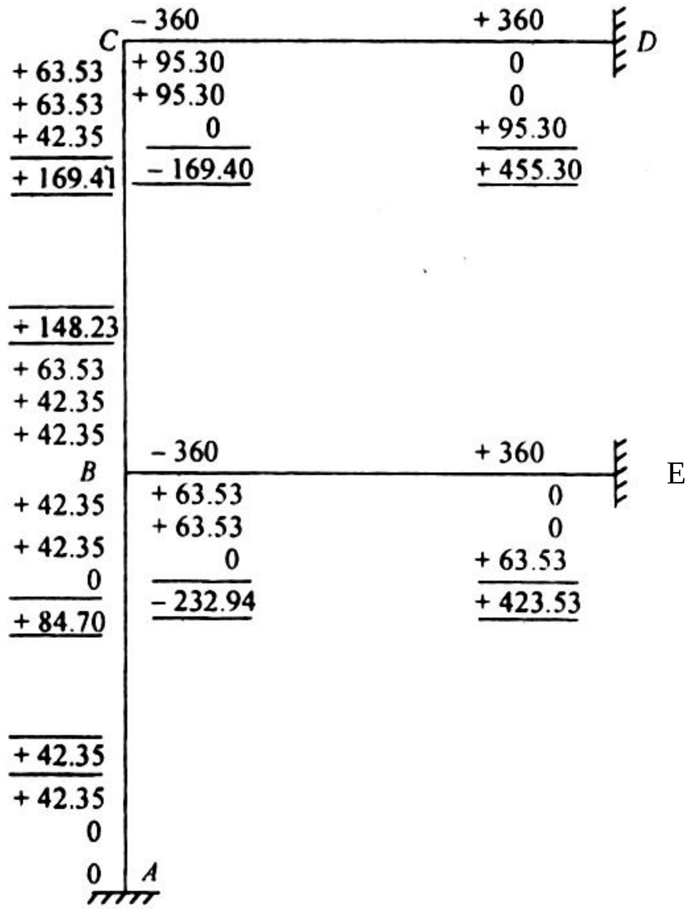


Fig-11(d)

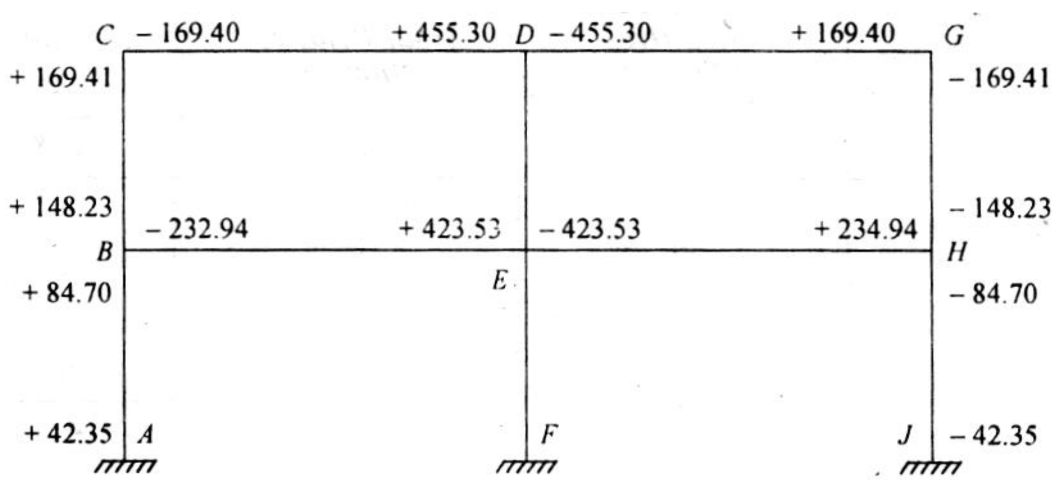


Fig-11(e)