Chapter-5: Matrix Method Of Analysis

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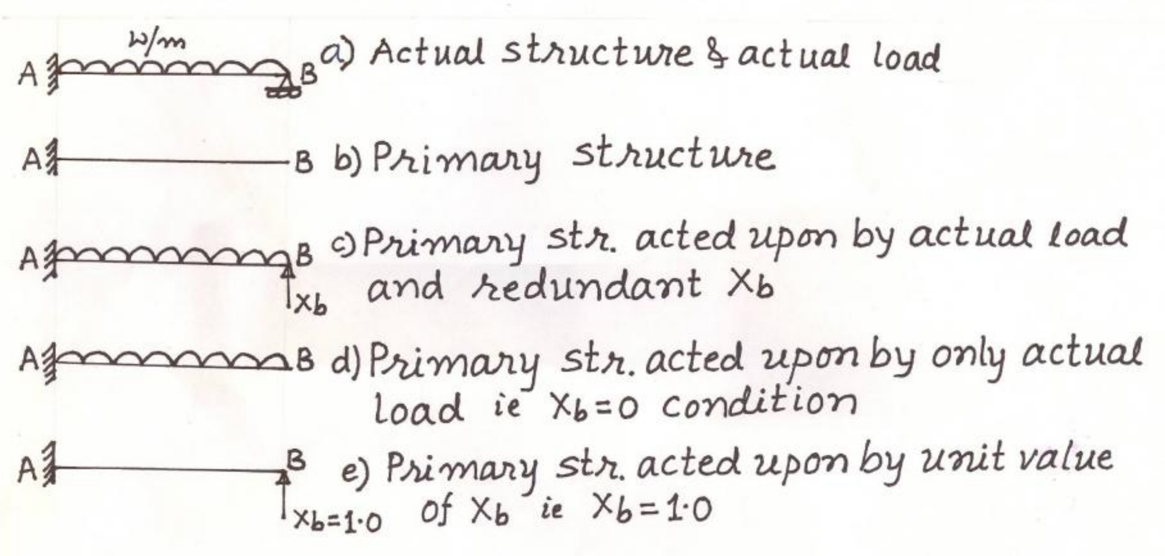
LECTURE No. 1

INTRODUCTION TO MATRIX METHODS

MATRIX METHODS OF ANALYSIS :
Broadly the methods of analysis are categorised in two ways

1. Force Methods : Methods in which forces are made unknowns i.e Method of consistent deformation and strain energy method. In both these methods solution of number of simultaneous equations is involved.
2. Displacement Methods in which displacements are made unknowns i.e slope deflection method, Moment distribution method and Kani's Method (In disguise). In slope deflection method also, the solution of number of simultaneous equations is involved.
   In both of the above methods, for the solution of simultaneous equations matrix approach can be employed & such Method is called **Matrix method of analysis**.

FORCE METHOD :
Method of consistent deformation is the base and forces are made unknown



Δb = Upward Deflection of point B on primary structure due to all causes
Δbo = Upward Deflection of point B on primary structure due to applied load(Redundant removed i.e condition Xb = 0)
Δbb = Upward Deflection of point B on primary structure due to Xb (i.e Redundant)
δbb = Upward Deflection of point B on primary structure due to Xb = 1
Δbb = δbb . Xb

∴ Δb = Δbo + Δbb   Substituting for **Δbb** –
Δb = Δbo + δbb . Xb   Called Super position equation

Using the compatibility condition that the net displacement at B = 0 i.e

$\Delta b = 0$ we get $Xb = -\Delta bo / \delta bb$

To Conclude we can say, **$[\Delta] = [\Delta L] + [\Delta R]$**

DISPLACEMENT METHOD:

This method is based on slope deflection method and displacements are made unkowns
which are computed by matrix approch instead of solving simultaneous equations and finaly unknown forces are calculated using slpoe deflection equations.

$Mab = Mab + 2EI / L\ (2\,\theta a + \theta b + 3\delta / L)$

Mab = Final Moment and may be considered as net force P at the joint

Mab = Fixed end moment i.e Force required for the condition of zero displacements & is called locking force. (i.e. P')

The second term may be considred as the force required to produce the required displacements at the joints. (i.e Pd)Therefore the above equation may be written as **[P] = [P'] + [Pd]**

Thus, there are Two Methods in matrix methods

MATRIX METHODS

FORCE METHOD | **DISPLACEMENT METHOD**

The force method is also called by the names 1) Flexibility Method 2)Static Method 3)Compatibility.

Similarly the displacement method is also called by the names 1)Stiffness Method 2) Kinematic method 3) Equilibrium Method.

In both force method & displacement method there are two different approaches 1) System Approach 2) Element Approach.

To study matrix methods there are some pre-requisites :

i) Matrix Algebra **-** Addition, subtraction ,Multiplication & inversion of matrices (Adjoint Method)
ii) Methods of finding out Displacements i.e. slope & deflection at any point in a structure, such as a)Unit load method or Strain energy method b) Moment area method etc.

According to unit load method the displacement at any point 'j' is given by

$\Delta j = \int_0 Mm_j ds / EI$ Where M – B M due to applied loads & m**j** – B M due to unit load at j

$\delta ij = \int \mathbf{0}$ mi.mj.ds / EI When unit load is applied at i and is called flexibility coefficient.

The values of $\mathbf{\Delta j}$ and $\mathbf{\delta ij}$ can be directly read from the table depending upon the combinations of B M diagrams & these tables are called **Diagram Multipliers.**

iii) Study of Indeterminacies – Static indeterminacy & kinematic indeterminacy

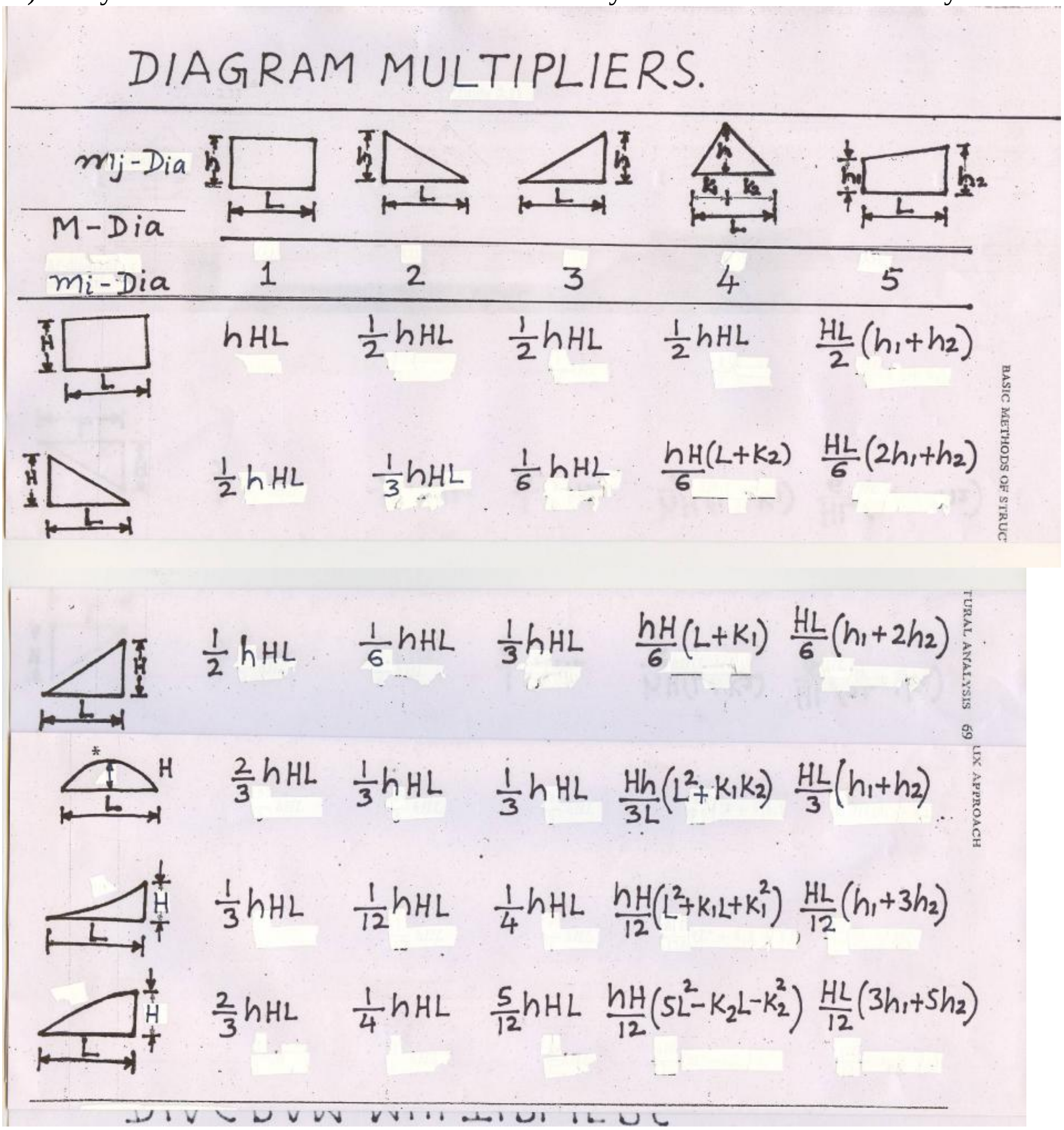


DIAGRAM MULTIPLIERS.

M-Dia (mi-Dia ↓ / mj-Dia →)	1 (rectangle h, L)	2 (triangle h, L)	3 (triangle h, L)	4 (triangle h, k_1, k_2, L)	5 (trapezoid h_1, h_2, L)
Rectangle H, L	hHL	$\frac{1}{2}hHL$	$\frac{1}{2}hHL$	$\frac{1}{2}hHL$	$\frac{HL}{2}(h_1+h_2)$
Triangle H, L	$\frac{1}{2}hHL$	$\frac{1}{3}hHL$	$\frac{1}{6}hHL$	$\frac{hH(L+k_2)}{6}$	$\frac{HL}{6}(2h_1+h_2)$
Triangle H, L	$\frac{1}{2}hHL$	$\frac{1}{6}hHL$	$\frac{1}{3}hHL$	$\frac{hH}{6}(L+k_1)$	$\frac{HL}{6}(h_1+2h_2)$
Parabola* H, L	$\frac{2}{3}hHL$	$\frac{1}{3}hHL$	$\frac{1}{3}hHL$	$\frac{Hh}{3L}(L^2+k_1k_2)$	$\frac{HL}{3}(h_1+h_2)$
Parabola H, L	$\frac{1}{3}hHL$	$\frac{1}{12}hHL$	$\frac{1}{4}hHL$	$\frac{hH}{12}(L^2+k_1L+k_1^2)$	$\frac{HL}{12}(h_1+3h_2)$
Parabola H, L	$\frac{2}{3}hHL$	$\frac{1}{4}hHL$	$\frac{5}{12}hHL$	$\frac{hH}{12}(5L^2-k_2L-k_2^2)$	$\frac{HL}{12}(3h_1+5h_2)$

<u>LECTURE No. 2</u>

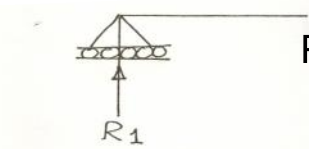
<u>INDETERMINATE STRUCTURES</u>

1. Statically Indeterminate Structure
2. Kinematically Indeterminate Structure

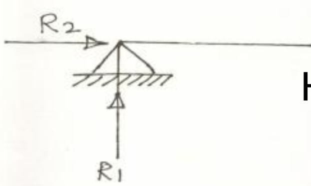
INDETERMINATE STRUCTURES

Statically Indeterminate Structure : Any structure whose reaction components or internal stresses cannot be determined by using equations of static equilibrium alone, (i.e.$\Sigma Fx = 0$, $\Sigma Fy = 0$, $\Sigma Mz = 0$) is a statically Indeterminate Structure.

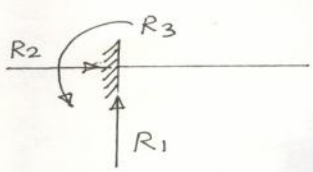
The additional equations to solve statically indeterminate structure come from the conditions of compatibility or consistent displacements.



Roller Support : No. of reactions, r = 1



Hinged Support : No. of reactions, r = 2



Fixed Support : No. of reactions, r = 3

1. Pin Jointed Structures i.e. Trusses

Internal static indeterminacy : (Dsi) No. of members required for stability is given by – 3 joints – 3 members – every additional joint requires two additional members.

∴ $m' = 2(j - 3) + 3$ j = No. of joints

∴ $m' = 2j - 3$ Stable and statically determinate

Dsi = m – m' Where m = No. of members in a structure

Dsi = m – (2j – 3)

External static indeterminacy (Dse)

r = No. of reaction components

Equations of static equilibrium = 3 (i.e.$\Sigma Fx = 0$, $\Sigma Fy = 0$, $\Sigma Mz = 0$)

y **Dse = r – 3**

Total static indeterminacy Ds = Dsi + Dse ∴ Ds = m –(2j – 3)+(r – 3)

y **Ds = (m+r) – 2j**

Rigid Jointed Structures : No. of reaction components over and above the no. of equations of static equilibrium is called a degree of static indeterminacy.

Ds = r-3 Equations of static equilibrium = 3 (i.e.ΣFx = 0, ΣFy = 0, ΣMz = 0)

Example 1

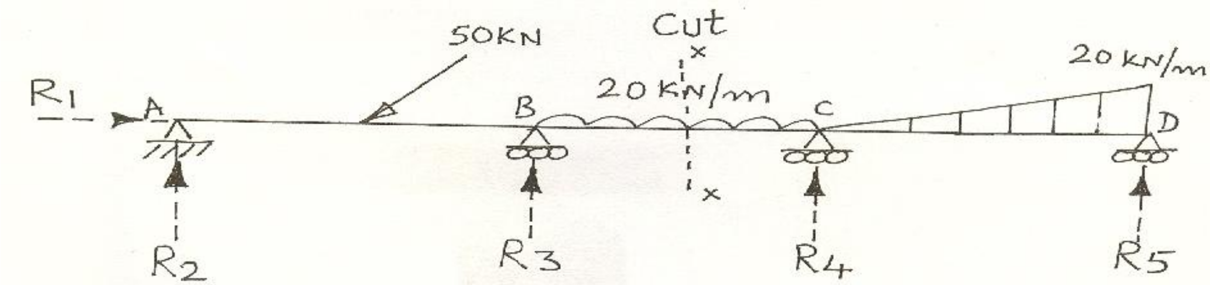


Fig 1

No. of reaction components r = 5 (as shown)

∴ Ds = r – 3 = 5 – 3 = 2 **Ds = 2**

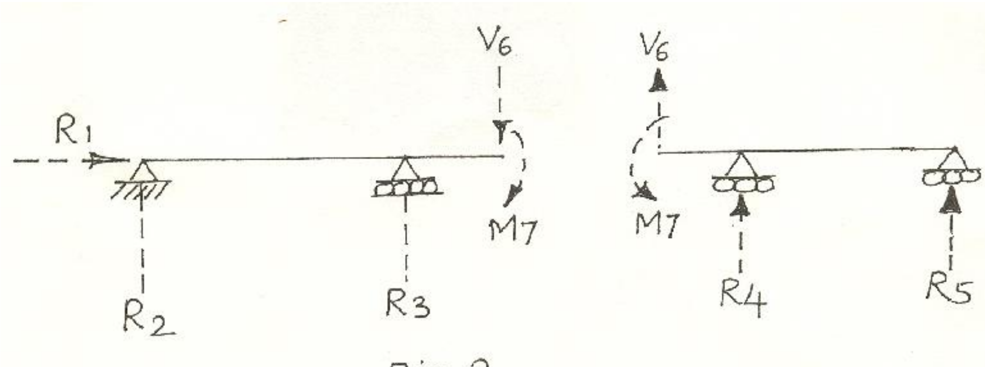


Fig. 2

Introduce cut in the member BC as shown. At the cut the internal stresses are introduced i.e. shear force and bending moment as shown.

Left part : No. of unknowns = 5 Equations of equilibrium = 3

y **Ds = 5 – 3 = 2**

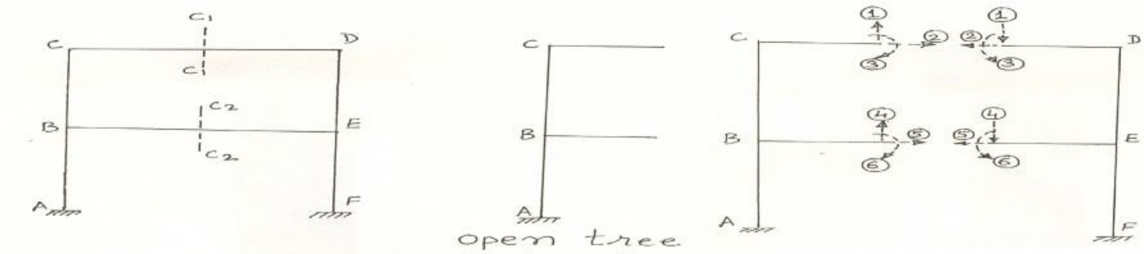
Right Part : No. of unknowns = 4 Equations of equilibrium = 2

y **Ds = 4 – 2 = 2**

∴ Ds = Static Indeterminacy = 2

Example 2

Fig. (A) Fig. (B) Fig. (C)



Open tree
Statically
Determinate

At every cut 3 forces are introduced

∴ Ds = 3 × No. of cuts = 3×C

∴ In the problem, —

Ds = 3 × 2 = 6 ∵ c = 2

Another Approach : For Every member in a rigid jointed structure there will be 3 unknowns i.e. shear force, bending moment, axial force.

Let r be the no. of reaction components and m be the no. of members

Total no. of unknowns = 3m + r

At every joint **three** equations of static equilibrium are available

∴ no. of static equations of equilibrium = 3j (where j is no. of joints)

∴ **Ds = (3m + r) – 3j**

∴ In the example r = 6, m = 6, j = 6

∴ Ds = (3 x 6 + 6) – (3 x 6) = 6

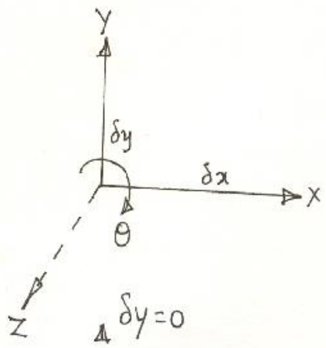
LECTURE No. 3

Kinematic Indeterminacy :

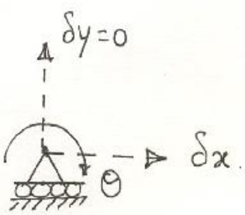
A structure is said to be kinematically indeterminate if the displacement components of its joints cannot be determined by compatibility conditions alone. In order to evaluate displacement components at the joints of these structures, it is necessary to consider the equations of static equilibrium. i.e. no. of unknown joint displacements over and above the compatibility conditions will give the degree of kinematic indeterminacy.

Fixed beam : Kinematically determinate :

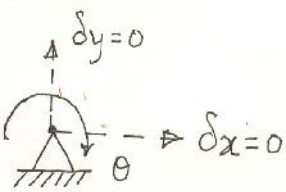
Simply supported beam Kinematically indeterminate



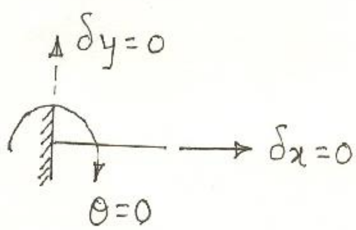
Any joint – Moves in three directions in a plane structure Two displacements δx in x direction, δy in y direction, θ rotation about z axis as shown.



Roller Support :
r = 1, $\delta y = 0$, θ & δx exist – DOF = 2   e = 1



Hinged Support :
r = 2, $\delta x = 0$, $\delta y = 0$, θ exists – DOF = 1   e = 2



Fixed Support :
r = 3, $\delta x = 0$, $\delta y = 0$, $\theta = 0$ DOF = 0   e = 3

i.e. reaction components prevent the displacements $\therefore$ no. of restraints = no. of reaction components.

Degree of kinematic indeterminacy :

Pin jointed structure : Every joint – two displacements components and no rotation

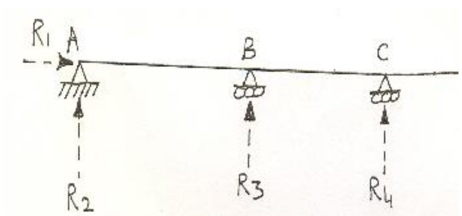
y **Dk = 2j – e** where, e = no. of equations of compatibility
= no. of reaction components

Rigid Jointed Structure : Every joint will have three displacement components, two displacements and one rotation.
Since, axial force is neglected in case of rigid jointed structures, it is assumed that the members are inextensible & the conditions due to inextensibility of members will add to the numbers of restraints. i.e to the '**e'** value.

y **Dk = 3j – e** where, e = no. of equations of compatibility
= no. of reaction components +
constraints due to in extensibility

Example 1 : Find the static and kinematic indeterminacies

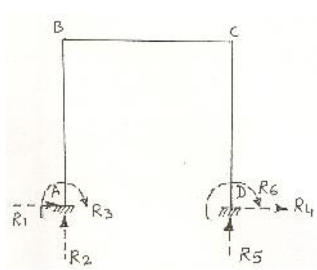
r = 4, m = 2, j = 3



Ds = (3m + r) – 3j
= (3 x 2 + 4) – 3 x 3 = 1
Dk = 3j – e
= 3 x 3 – 6 = 3
i.e. rotations at A,B, & C i.e. θa, θb & θc are the displacements.
(e = reaction components + inextensibility conditions = 4 + 2 = 6)

Example 2 :

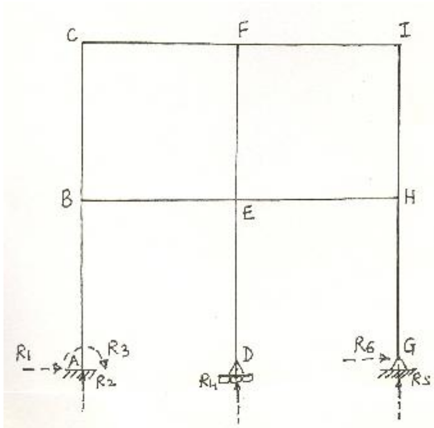
Ds = (3m+r) – 3j
m = 3, r = 6, j = 4
∴ Ds = (3 x 3 + 6) – 3 x 4 = 3



Dk = 3j – e e = no. of reaction components + conditions of inextensibility
= 6+3 = 9
Dk = 3 x 4 – 9 = 3 i.e. rotation θb, θc & sway.

Example 3 :

Ds = (3m + r) – 3j
r = 6, m = 10, j = 9



∴ Ds = (3 x 10 + 6) – 3 x 9 = 9

Conditions of inextensibility :

Joint :	B	C	E	F	H	I	
	1	1	2	2	2	2	Total = 10

Reaction components r = 6

y **e = 10 + 6 = 16**

y Dk = 3j – e
= 3 x 9 – 16 = 11

LECTURE No. 4

FORCE METHOD :

This method is also known as flexibility method
or compatibility method. In this method the degree of static indeterminacy of the structure is determined and the redundants are identified. A coordinate is assigned to each redundant. Thus,P1, P2 - - - - - -Pn are the redundants at the coordinates 1,2, - - - - - n.If all the redundants are removed , the resulting structure known as released structure, is statically determinate. This released structure is also known as basic determinate structure. From the principle of super position the net displacement at any point in statically indeterminate structure is some of the displacements in the basic structure due to the applied loads and the redundants. This is known as the compatibility condition and may be expressed by the equation;

$\Delta_1 = \Delta_1 L + \Delta_1 R$ Where $\Delta 1$ - - - - Δn = Displ. At Co-ord.at 1,2 - -n
$\Delta_2 = \Delta_2 L + \Delta_2 R$ $\Delta_1 L$ ---- $\Delta_n L$ = Displ.At Co-ord.at 1,2 - - - - -n
| | | Due to aplied loads
| | | $\Delta_1 R$ ----$\Delta_n R$ = Displ.At Co-ord.at 1,2 - - - - -n
$\Delta_n = \Delta_n L + \Delta_n R$ Due to Redudants

The above equations may be return as $[\Delta] = [\Delta_L] + [\Delta_R]$ - - - - **(1)**

$\Delta_1 = \Delta_1 L + \delta_{11} P_1 + \delta_{12} P_2 +$ - - - - - $\delta_{1n} P_n$
$\Delta_2 = \Delta_2 L + \delta_{21} P_1 + \delta_{22} P_2 +$ - - - - - $\delta_{2n} P_n$
| | | | |
| | | | | - - - - - **(2)**

$\Delta_n = \Delta_n L + \delta_{n1} P_1 + \delta_{n2} P_2 +$ - - - - - $\delta_{nn} P_n$

$\therefore \Delta = [\Delta L] + [\delta][P]$ - - - - - - (3)

$\therefore [P] = [\delta]^{-1} \{[\Delta] - [\Delta L]\}$ - - - - - - (4)

If the net displacements at the redundants are zero then

$\Delta 1, \Delta 2$ - - - - $\Delta n = 0$,

Then $\therefore [P] = -[\delta]^{-1}[\Delta L]$ - - - - - -(5)

The redundants P1,P2, - - - - - Pn are Thus determined

DISPLACEMENT METHOD :

This method is also known as stiffness or equilibrium. In this method the degree of kinematic indeterminacy (D.O.F) of the structure is determined and the coordinate is assigned to each independent displacement component.

In general, The displacement components at the supports and joints are treated as independent displacement components. Let 1,2, - - - - - n be the coordinates assigned to these independent displacement components $\Delta 1, \Delta 2$ - - - - Δn.

In the first instance **lock all the supports and** the joints to obtain the **restrained structure** in which no displacement is possible at the coordinates. Let P'1, P'2 , - - - - P'n be the forces required at the coordinates 1,2, - - - - - n in the restrained structure in which the displacements $\Delta 1, \Delta 2$ - - - - Δn are zero. Next, Let the supports and joints be unlocked permitting displacements $\Delta 1, \Delta 2$ - - - - Δn at the coordinates. Let these displacements require forces in P1d, P2d, - - - - Pnd at coordinates 1,2, - - - - n respectively.

If P1, P2- - - - - - Pn are the external forces at the coordinates 1,2, - - - - n, then the conditions of equilibrium of the structure may be expressed as:

$$
\begin{aligned}
P1 &= P'1 + P1d \\
P2 &= P'2 + P2d \qquad \text{- - - - - - - - - - - - -(1)} \\
&\vdots \\
Pn &= P'n + Pnd
\end{aligned}
$$

or $[P] = [P'] + [Pd]$ - - - - - - - - -- -(2)

$$
\begin{aligned}
P1 &= P'1 + K11\,\Delta 1 + K12,\ \Delta 2 + K13\Delta 3 + \text{ - - - - -} K1n\,\Delta n \\
P2 &= P'2 + K21\,\Delta 1 + K22\,\Delta 2 + K23\Delta 3 + \text{ - - - - -} K2n\,\Delta n \\
&\vdots \qquad \text{- - - - - - - - - - (3)} \\
Pn &= P'n + Kn1\,\Delta 1 + Kn2,\ \Delta 2 + Kn3\Delta 3 + \text{ - - - - - - - - -} Knn\,\Delta n \quad \therefore
\end{aligned}
$$

i.e $[P] = [P'] + [K][\Delta]$ - - - - - - - - - - (4)

$\therefore [\Delta] = [K]^{-1}\{[P] - [P']\}$ - - - - - - - - - (5)

Where P = External forces

P' = Locking forces

Pd=Forces due to displacements

If the external forces act only at the coordinates the terms P'1, P'2 , - - - - P'n vanish. i. e the Locking forces are zero,then

$[\Delta] = [K]^{-1}[P]$ - - - - - - - - - - (6)

On the other hand if there are no external forces at the coordinates then [P]=0 then

$[\Delta] = -[K]^{-1}[P']$ - - - - - - - - - - - -(7)

Thus the displacements can be found out. Knowing the displacements the forces are computed using slope deflection equations:

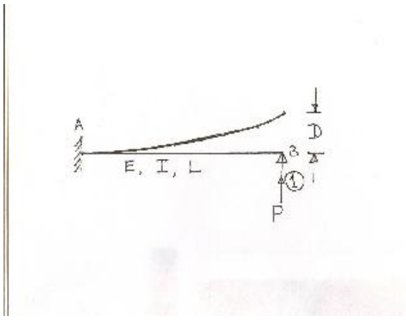
$Mab = Mab + 2EI / L\,(2\theta a + \theta b + 3\delta / L)$

$Mba = Mba + 2EI / L\,(\theta a + 2\theta b + 3\delta / L)$

Where Mab& Mba are the fixed end moments for the member AB due to external loading

LECTURE No. 5

FLEXIBILITY AND STIFFNESS MATRICES : **SINGLE CO-ORD.**



$D = \delta \times P$

$D = PL^3/3EI = \delta \times P$

$\therefore \delta = L^3/3EI$

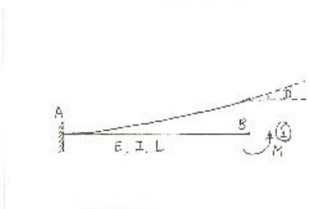
δ=Flexibility Coeff.

$P = K \times D$

$P = K \times PL^3/3EI$

$K = 3EI/L^3$

K=Stiffness Coeff.



$D = ML/EI$

$D = \delta \times M = ML/EI$

$\therefore \delta = L/EI$

δ=Flexibility Coeff.

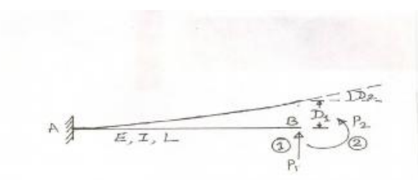
$M = K \times D = K \times ML/EI$

$\therefore K = EI/L$

K=Stiffness Coeff.

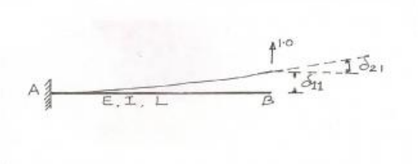
$\delta \times K = 1$

TWO CO-ORDINATE SYSTEM



$D_1 = \delta_{11}P_1 + \delta_{12} \times P_2$ & $D_2 = \delta_{21}P_1 + \delta_{22}P_2$

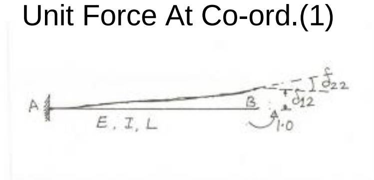
$$\therefore \begin{matrix} D_1 \\ D_2 \end{matrix} \quad \begin{matrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{matrix} \quad \begin{matrix} P_1 \\ P_2 \end{matrix}$$



Unit Force At Co-ord.(1)

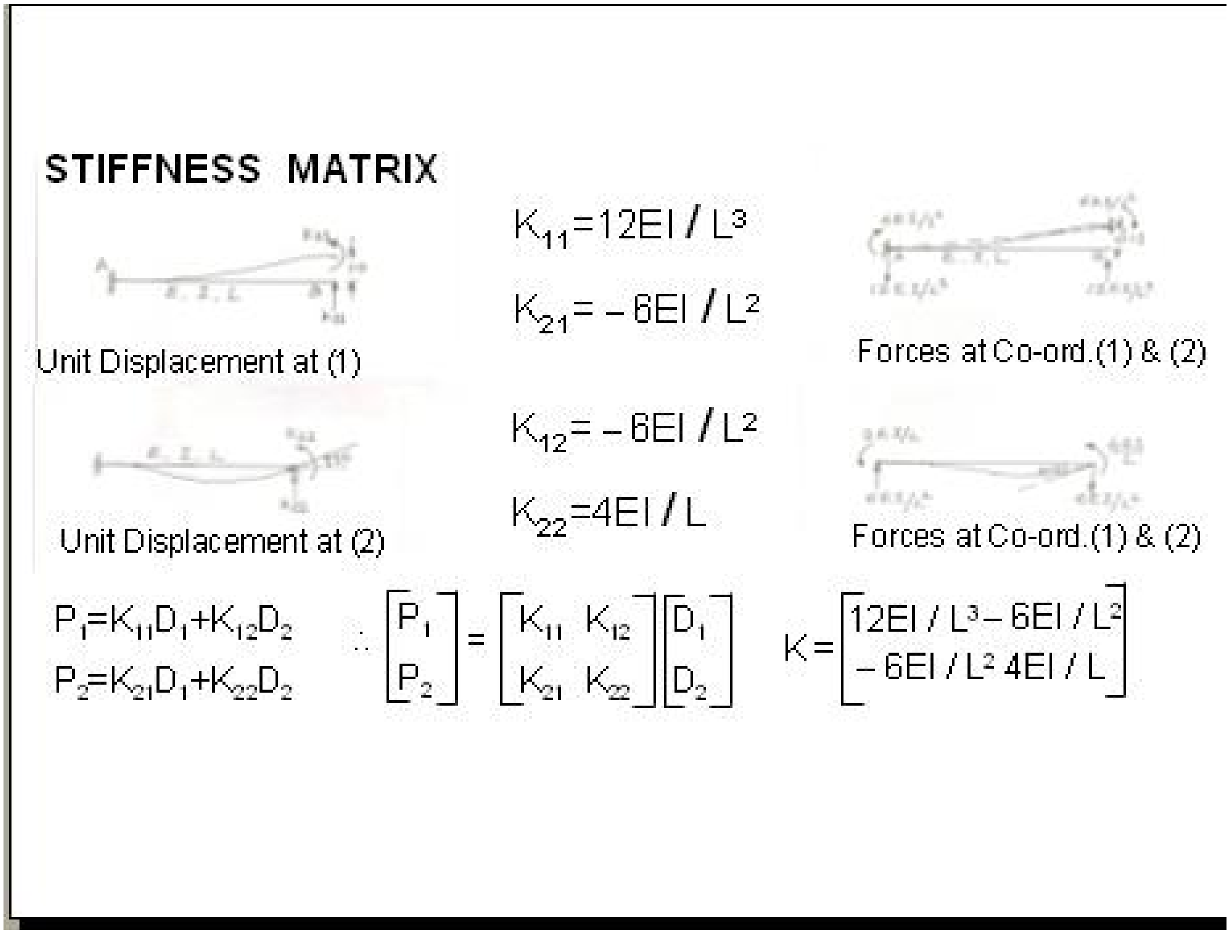
$$[\delta] = \begin{bmatrix} L^3/3EI & L^2/2EI \\ L^2/2EI & L/EI \end{bmatrix}$$

$\delta_{11} = L^3/3EI$ $\qquad \delta_{21} = L^2/2EI$

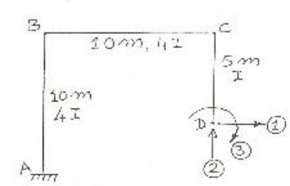


Unit Force At Co-ord.(2)

$\delta_{12} = \delta_{21} = L^2/2EI \quad \delta_{22} = L/EI$

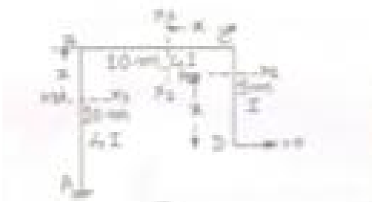


Develop the Flexibility and stiffness matrices for frame ABCD with reference to Coordinates shown

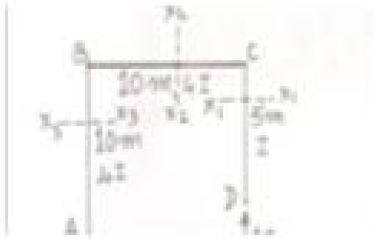


The Flexibility matrix can be developed by applying unit force successively at coordinates (1),(2) &(3) and evaluating the displacements at all the coordinates

$\delta_{ij} = \int mi\ mj / EI \times ds$ δ_{ij} =displacement at I due to unit load at j



Unit Load at (1)



Unit Load at (2)

Unit Load at (3)

Portion	DC	CB	BA
I	I	4I	4I
Origin	D	C	B
Limits	0 - 5	0 - 10	0 - 10
m_1	x	5	5 - x
m_2	0	x	10
m_3	- 1	-1	-1

$\delta_{11} = \int m_1 . m_1 dx / EI = 125 / EI$

$\delta_{21} = \delta_{12} = \int m_1 . m_2 dx / EI = 125 / 2EI$

$\delta_{31} = \delta_{13} = \int m_1 . m_3 dx / EI = -25 / EI$

$\delta_{22} = \int m_2 . m_2 dx / EI = 1000 / 3EI$

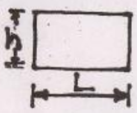
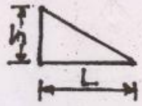
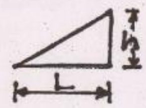
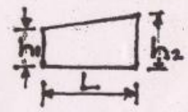
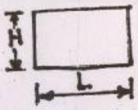
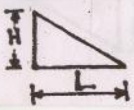
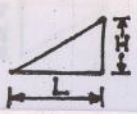
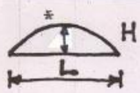
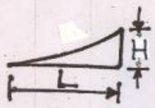
$\delta_{23} = \delta_{32} = \int m_2 . m_3 dx / EI = -75 / 2EI$

$\delta_{33} = \int m_3 . m_3 dx / EI = 10 / EI$

$$\therefore \boldsymbol{\delta} = 1 / 6EI \begin{bmatrix} 750 & 375 & -150 \\ 375 & 2000 & -225 \\ -150 & -225 & 60 \end{bmatrix}$$

INVERSING THE **FLEXIBILITY MATRIX [δ]**
THE **STIFENESS MATRIX [K]** CAN BE OBTAINED

DIAGRAM MULTIPLIERS.

m_j-Dia / M-Dia / m_i-Dia	1 	2 	3 	4 	5 
	hHL	$\frac{1}{2}hHL$	$\frac{1}{2}hHL$	$\frac{1}{2}hHL$	$\frac{HL}{2}(h_1+h_2)$
	$\frac{1}{2}hHL$	$\frac{1}{3}hHL$	$\frac{1}{6}hHL$	$\frac{hH}{6}(L+k_2)$	$\frac{HL}{6}(2h_1+h_2)$
	$\frac{1}{2}hHL$	$\frac{1}{6}hHL$	$\frac{1}{3}hHL$	$\frac{hH}{6}(L+k_1)$	$\frac{HL}{6}(h_1+2h_2)$
	$\frac{2}{3}hHL$	$\frac{1}{3}hHL$	$\frac{1}{3}hHL$	$\frac{Hh}{3L}(L^2+k_1k_2)$	$\frac{HL}{3}(h_1+h_2)$
	$\frac{1}{3}hHL$	$\frac{1}{12}hHL$	$\frac{1}{4}hHL$	$\frac{hH}{12}(L^2+k_1L+k_1^2)$	$\frac{HL}{12}(h_1+3h_2)$
	$\frac{2}{3}hHL$	$\frac{1}{4}hHL$	$\frac{5}{12}hHL$	$\frac{hH}{12}(5L^2-k_2L-k_2^2)$	$\frac{HL}{12}(3h_1+5h_2)$

LECTURE No. 6

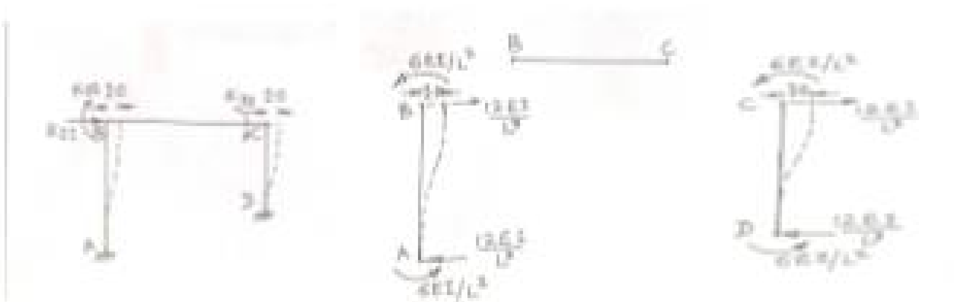
STIFFNESS MATRIX

Given Co-ordinates

$$K_{11}=(12EI / L^3)_{AB} + (12EI / L^3)_{CD} = 0.144EI$$

$$K_{21}= -(6EI / L^2)_{AB} = -0.24EI$$

$$K_{31}= -(6EI / L^2)_{CD} = -0.24EI$$



Unit Displacement at Co-ordinate (1)

Unit Dspl. at Co-ord.(2)

$$K_{12}=K_{21}= -(6EI / L^2)_{AB} = -0.24EI$$

$$K_{22}=(4EI / L)_{AB} +(4EI / L)_{BC}= 3.2EI$$

$$K_{32}=(2EI / L)_{BC}= 0.82EI$$

Unit Dspl. at Co-ord.(3)

$$K_{13}=K_{31}= -(6EI / L^2)_{CD} = -0.24EI$$

$$K_{23}= K_{32}=(2EI / L)_{BC}= 0.82EI$$

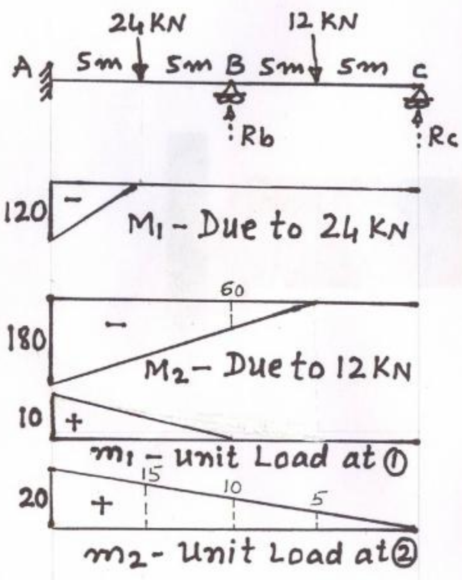
$$K_{33}= (4EI / L)_{BC}+(4EI/L)_{CD}=2.4EI$$

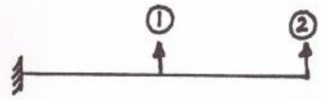
$$K=EI\begin{bmatrix} 0.144 & -0.24 & -0.24 \\ -0.24 & 3.2 & 0.8 \\ -0.24 & 0.8 & 2.4 \end{bmatrix}$$

Force Method (Flexibility or compatibility method)	**Displacement method** (Stiffness or equilibrium method)
1. Determine the degree of static indeterminacy (degree of redundancy), n	1. Determine the degree of kinematic indeterminacy , (degree of freedom), n
2. Choose the redundants.	2. Identify the independent displacement components
3. Assign coordinates 1, 2, … ,n to the redundants	3. Assign coordinates 1, 2, … , n to the independent displacement components.
4. Remove all the redundants to obtain to obtain the released structure.	4.Prevent all the independent displacement components to obtain the restrained structure.
5.Determine $[\Delta_L]$, the displacements at the coordinates due to applied loads acting on the released structure	5. Determine $[P^1]$ the forces at the coordinates in the restrained structure due to the loads other than those acting at the coordinates
6. Determine $[\Delta_R]$, the displacements at the coordinates due to the redundants acting on the released structure	6. Determine $[P_\Delta]$ the forces required at the coordinates in the unrestrained structure to cause the independent displacement components$[\Delta]$
7. Compute the net displacement at the coordinates $[\Delta] = [\Delta_L] + [\Delta_R]$	7. Compute the forces at the coordinates $[P] = [P^1] + [P_\Delta]$
8. Use the conditions of compatibility of displacement s to compute the redundants $[P] = [\delta]^{-1} \{[\Delta] - [\Delta_L]\}$	8. Use the conditions of equilibrium of forces to compute the displacements $[\Delta] = [k]^{-1} \{[P] - [P^1]\}$
9. Knowing the redundants, compute the internal member forces by using equations of statics.	9. Knowing the displacements, compute the internal member forces by using slope deflection equation.

LECTURE No .7

Example : ①





Ds = 2 , Rb, Rc Redundants

$\frac{HL}{6}(2h_1+h_2)$

$$\Delta_{1L} = \frac{1}{EI}\left[-\frac{120\times5}{6}(2\times10+5) + \frac{10\times10}{6}\{2\times(-180)+(-60)\}\right]$$
$$= -9500/EI$$

$$\Delta_{2L} = \frac{1}{EI}\left[-\frac{120\times5}{6}(2\times20+15) + \frac{-180\times15}{6}(2\times20+5)\right]$$
$$= -25750/EI$$

$\frac{hHL}{3}$

$$\delta_{11} = \frac{1}{EI}\left[\frac{1}{3}\times10\times10\times10\right] = \frac{1000}{3EI}$$
$$\delta_{12} = \delta_{21} = \frac{1}{EI}\left[\frac{1}{6}\times10\times10(2\times20+10)\right] = \frac{2500}{EI}$$
$$\delta_{22} = \frac{1}{EI}\left[\frac{1}{3}\times20\times20\times20\right] = \frac{8000}{3EI}$$

$$\therefore [\delta] = \frac{1}{3EI}\begin{bmatrix}1000 & 2500\\ 2500 & 8000\end{bmatrix}$$

Using, $[P] = [\delta]^{-1}\{[\Delta]-[\Delta_L]\}$

$[\Delta] = 0$ Net displacements zero

$$\therefore \begin{bmatrix}P_1\\ P_2\end{bmatrix} = -3EI\begin{bmatrix}1000 & 2500\\ 2500 & 8000\end{bmatrix}^{-1}\begin{bmatrix}-9500/EI\\ -25750/EI\end{bmatrix} = \begin{bmatrix}19.93\\ 3.43\end{bmatrix}$$

ie $P_1 = R_b = 19.93$ KN & $P_2 = R_c = 3.43$ KN.

Draw B.M. & S.F. Diagram.

② If supports B & C sinks down by units $\frac{200}{EI}$ & $\frac{100}{EI}$ units analyse the beam.

In this, net displacements, at ① $\Delta_1 = -\frac{200}{EI}$ & at ② $\Delta_2 = -\frac{100}{EI}$

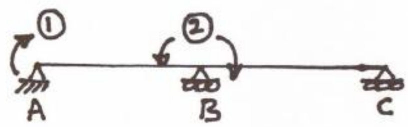
Compatibility Eqn is modified as –

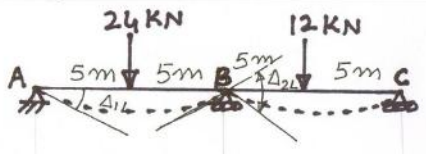
$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \frac{1}{3EI}\begin{bmatrix} 1000 & 2500 \\ 2500 & 8000 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} -\frac{200}{EI} \\ -\frac{100}{EI} \end{bmatrix} - \begin{bmatrix} -\frac{9500}{EI} \\ -\frac{25750}{EI} \end{bmatrix} \right\}$$

$$= 3EI \begin{bmatrix} 1000 & 2500 \\ 2500 & 8000 \end{bmatrix}^{-1} \begin{bmatrix} \frac{9300}{EI} \\ \frac{25650}{EI} \end{bmatrix} = \begin{bmatrix} 17.61 \\ 4.11 \end{bmatrix}$$

$\therefore\ P_1 = R_b = 17.61$ KN & $P_2 = R_C = 4.11$ KN.

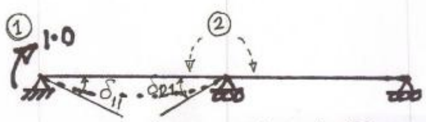
③ Choose the support moments Ma, Mb as the redundants. ie basic determinate structure will be two simply supported beams:





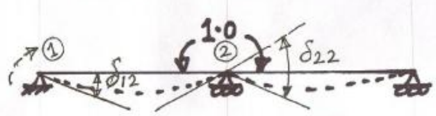
$$\Delta_{1L} = \frac{WL^2}{16EI} = \frac{24 \times 10^2}{16EI} = \frac{150}{EI}$$

$$\Delta_{2L} = \frac{24 \times 10^2}{16EI} + \frac{12 \times 10^2}{16EI} = \frac{225}{EI}$$



unit Load at ①

$$\delta_{11} = \frac{L}{3EI} = \frac{10}{3EI} \qquad \delta_{21} = \frac{L}{6EI} = \frac{10}{6EI} = \frac{5}{3EI}$$



Unit load at ②

$$\delta_{12} = \frac{L}{6EI} = \frac{5}{3EI} = \delta_{21}$$

$$\delta_{22} = \left(\frac{L}{3EI}\right)_{AB} + \left(\frac{L}{3EI}\right)_{BC}$$

$$= \frac{10}{3EI} + \frac{10}{3EI}$$

$$= \frac{20}{3EI}$$

$$\therefore [\delta] = \begin{bmatrix} \frac{10}{3EI} & \frac{5}{3EI} \\ \frac{5}{3EI} & \frac{20}{3EI} \end{bmatrix}$$

Using — $[P] = [\delta]^{-1} \{[\Delta] - [\Delta_L]\}$ $\quad [\Delta] = 0$

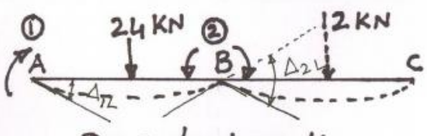
$$\therefore \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = - \begin{bmatrix} \frac{10}{3EI} & \frac{5}{3EI} \\ \frac{5}{3EI} & \frac{20}{3EI} \end{bmatrix}^{-1} \begin{bmatrix} \frac{150}{EI} \\ \frac{225}{EI} \end{bmatrix} = \begin{bmatrix} -32.10 \\ -25.70 \end{bmatrix}$$

$\therefore$ $P_1 = M_a = -32.10$ KNm $\quad \therefore M_a = 32.10$ KNm (hogging)

$P_2 = M_b = -25.70$ " $\quad \therefore M_b = 25.70$ " (")

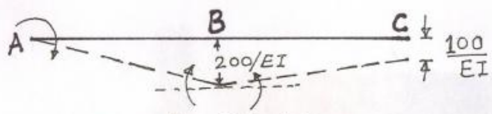
(Both Ma & Mb were assumed sagging)

④ If sinking of supports B & C, which are $\frac{200}{EI}$ & $\frac{100}{EI}$, are considered:



Due to Loading

$$\Delta_{1L} = \frac{24 \times 10^2}{16EI} + \frac{200}{EI} \times \frac{1}{10} = \frac{170}{EI}$$



Due to Sinking

$$\Delta_{2L} = \frac{24 \times 10^2}{16EI} + \frac{12 \times 10^2}{16EI} - \frac{200}{EI} \times \frac{1}{10} - \left(\frac{200}{EI} - \frac{100}{EI}\right) \times \frac{1}{10} = \frac{195}{EI}$$

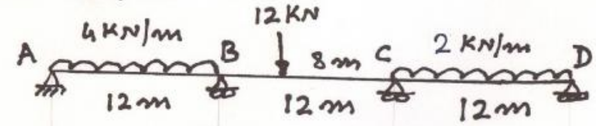
Considering net displ. $[\Delta] = 0$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = - \begin{bmatrix} \frac{10}{3EI} & \frac{5}{3EI} \\ \frac{5}{3EI} & \frac{20}{3EI} \end{bmatrix}^{-1} \begin{bmatrix} \frac{170}{EI} \\ \frac{195}{EI} \end{bmatrix} = \begin{bmatrix} -41.57 \\ -18.86 \end{bmatrix}$$

ie $P_1 = M_a = -41.57$ (hogging), $P_2 = M_b = -18.86$ (hogging)

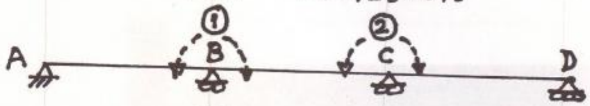
LECTURE No .8

Example:

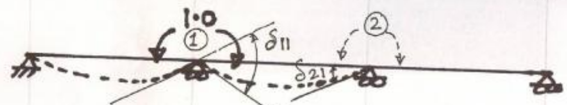


$D_s = 2$

EI – constant



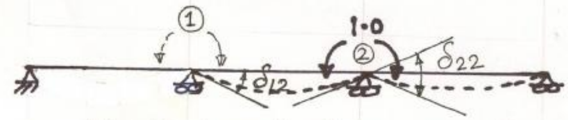
M_b & M_c redundants.



unit Load at co-ord ①

$$\delta_{11} = \left(\frac{L}{3EI}\right)_{AB} + \left(\frac{L}{3EI}\right)_{BC} = \frac{12}{3EI} + \frac{12}{3EI} = \frac{8}{EI}$$

$$\delta_{21} = \left(\frac{L}{6EI}\right)_{BC} = \frac{12}{6EI} = \frac{2}{EI}$$

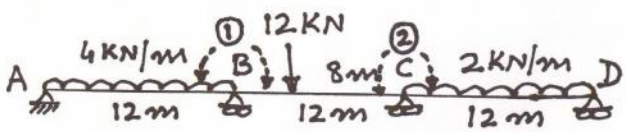


Unit Load at co-ord ②

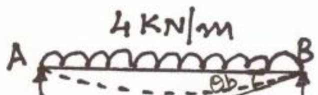
$$\delta_{12} = \left(\frac{L}{6EI}\right)_{BC} = \delta_{21} = \frac{2}{EI}$$

$$\delta_{22} = \left(\frac{L}{3EI}\right)_{BC} + \left(\frac{L}{3EI}\right)_{CD} = \frac{12}{3EI} + \frac{12}{3EI} = \frac{8}{EI}$$

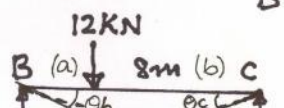
$$\therefore [\delta] = \begin{bmatrix} \frac{8}{EI} & \frac{2}{EI} \\ \frac{2}{EI} & \frac{8}{EI} \end{bmatrix}$$



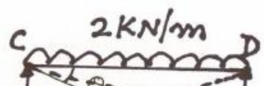
Since Mb & Mc are redundants, Basic determinate str.– AB, BC, & CD – Simply supported beams



$\theta_a = \theta_b = \frac{wl^3}{24EI}$



$\theta_b = \frac{Pab(2l-a)}{6EIL}$ $\quad \theta_c = \frac{Pa(L^2-a^2)}{6EIL}$



$\theta_c = \theta_d = \frac{wl^3}{24EI}$

Δ_{1L} = Displ. at co-ord. ① due to applied loads for determinate str

$$= (\theta_b)_{AB} + (\theta_b)_{BC} = \frac{4\times12^3}{24EI} + \frac{12\times4\times8\,(2\times12-4)}{6EI\times12} = \frac{1184}{3EI}$$

Δ_{2L} = Displ. at co-ord. ② due to applied loads for determinate str

$$= (\theta_c)_{BC} + (\theta_c)_{CD} = \frac{12\times4\,(12^2-4^2)}{6EI\times12} + \frac{2\times12^3}{24EI} = \frac{688}{3EI}$$

Using compatibility condition,

$$[P] = [\delta]^{-1}\{[\Delta] - [\Delta_L]\} \quad \text{Net displ. } [\Delta]=0$$

$$\therefore \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = -\begin{bmatrix} \frac{8}{EI} & \frac{2}{EI} \\ \frac{2}{EI} & \frac{8}{EI} \end{bmatrix}^{-1} \begin{bmatrix} \frac{1184}{3EI} \\ \frac{688}{3EI} \end{bmatrix} = \begin{bmatrix} -45.0 \\ -17.4 \end{bmatrix}$$

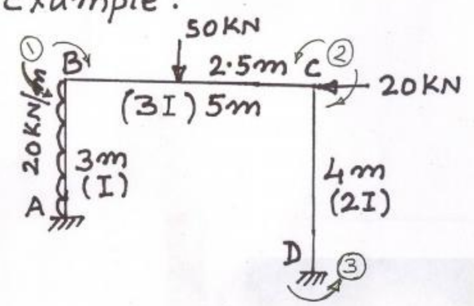
ie $P_1 = M_b = -45.0$ KNm (hogging)

$P_2 = M_c = -17.4$ KNm (hogging)

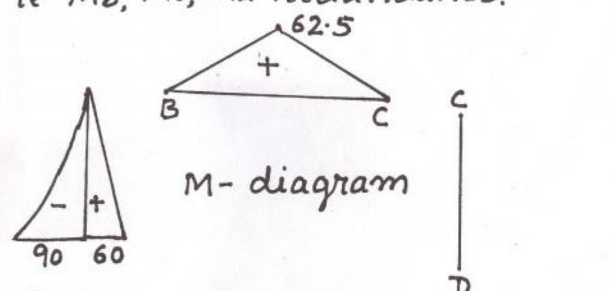
Draw B.M. & S.F. Diagrams.

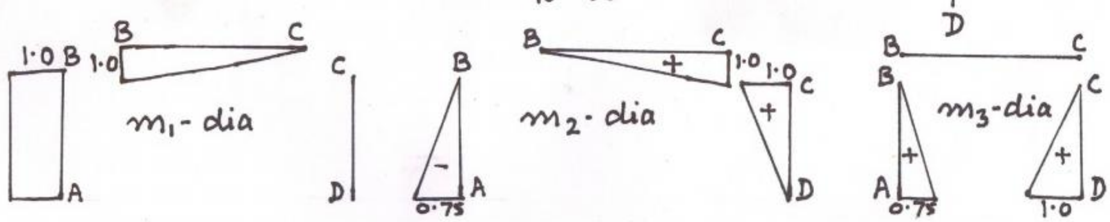
The problem may also be solved by taking Rb & Rc redundant.

Example :



Ds = 3 Introduce hinges at B, C, & D ie Mb, Mc, Md redundants.





FLEXIBILTY MATRIX:

$$\delta_{11} = \underset{(AB)}{\frac{1\times1\times3}{EI}} + \underset{(BC)}{\frac{1\times1\times5}{3\times3EI}} + 0 = \frac{3.56}{EI}$$

$$\delta_{22} = \underset{(AB)}{\frac{-0.75\times(-0.75)\times3}{3EI}} + \underset{(BC)}{\frac{1\times1\times5}{3\times3EI}} + \underset{(CD)}{\frac{1\times1\times4}{3\times2EI}} = \frac{1.785}{EI}$$

$$\delta_{33} = \frac{0.75\times0.75\times3}{3EI} + 0 + \frac{1\times1\times4}{3\times2EI} = \frac{1.223}{EI}$$

$$\delta_{12} = \delta_{21} = \frac{-0.75\times1\times3}{2\times EI} + \frac{1\times1\times5}{6\times3EI} + 0 = -\frac{0.847}{EI}$$

$$\delta_{23} = \delta_{32} = -\frac{0.75\times0.75\times3}{3\times EI} + 0 + \frac{1\times1\times4}{6\times2EI} = -\frac{0.23}{EI}$$

$$\delta_{31} = \delta_{13} = \frac{1\times0.75\times3}{2\times EI} + 0 + 0 = \frac{1.125}{EI}$$

$$\therefore [\delta] = \frac{1}{EI}\begin{bmatrix} 3.56 & -0.847 & 1.125 \\ -0.847 & 1.785 & -0.23 \\ 1.125 & -0.23 & 1.22 \end{bmatrix}$$

$$\Delta_{1L} = \frac{1\times(-90)\times3}{3\times EI} + \frac{1\times60\times3}{2\times EI} + \frac{1\times62.5(5+2.5)}{6\times3EI} + 0$$

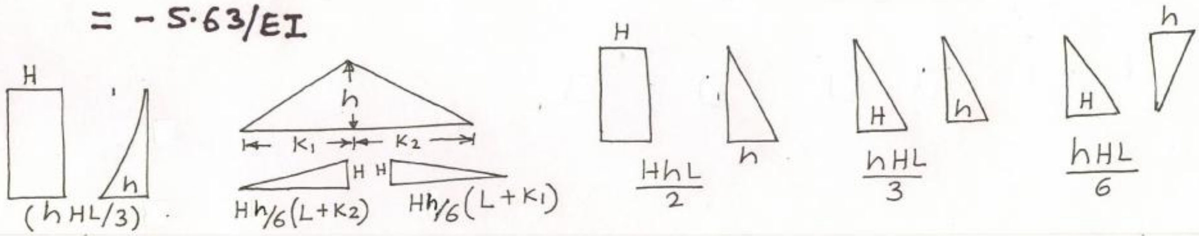
$$= 26.04/EI$$

$$\Delta_{2L} = \frac{-0.75(-90)\times3}{4\times EI} + \frac{-0.75\times60\times3}{3\times EI} + \frac{1\times62.5(5+2.5)}{6\times3EI} + 0$$

$$= 31.67/EI$$

$$\Delta_{3L} = \frac{0.75(-90)\times3}{4\times EI} + \frac{0.75\times60\times3}{3\times EI} + 0 + 0$$

$$= -5.63/EI$$



Using Compatibility condition,

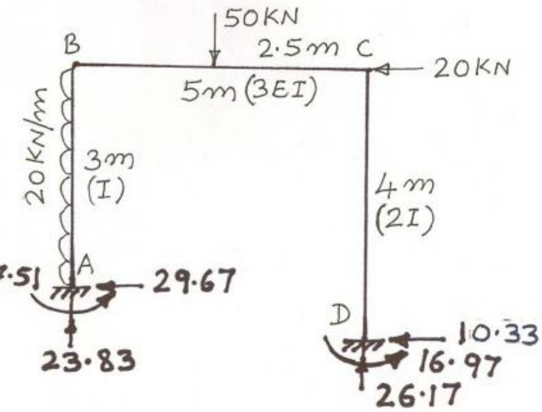
$$[P] = [\delta]^{-1}\{[\Delta] - [\Delta_L]\} \qquad [\Delta] = 0$$

$$\therefore \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = -EI \begin{bmatrix} 3.56 & -0.847 & 1.125 \\ -0.847 & 1.785 & -0.23 \\ 1.125 & -0.23 & 1.22 \end{bmatrix}^{-1} \frac{1}{EI} \begin{bmatrix} 26.04 \\ 31.67 \\ -5.63 \end{bmatrix} = \begin{bmatrix} -18.50 \\ -24.34 \\ +16.97 \end{bmatrix}$$

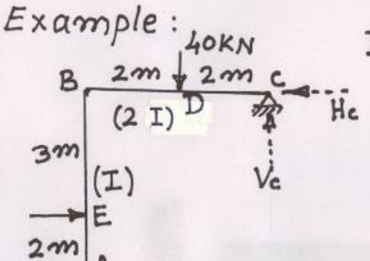
$\therefore$ $P_1 = M_b = 18.50$ (ten. out) $-ve$

$P_2 = M_c = 24.34$ (- do -)

$P_3 = M_d = 16.97$ (ten. in) $+ve$

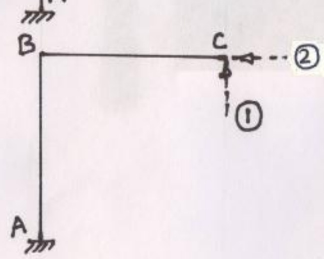


Example :



Ds = 2 Remove hinge at C.

ie V_c & H_c redundants



S. No.	Seg·ment	Ori·gin	Limits	M	m_1	m_2	I
1	CD	C	0-2	0	x	0	2I
2	DB	C	2-4	$-40(x-2)$	x	0	2I
3	BE	B	0-3	-80	4	x	I
4	EA	B	3-5	$-80-50(x-3)$	4	x	I

$$\Delta_{1L} = \int_0^l \frac{M m_1}{EI}\,dx = -\frac{2133.33}{EI}$$

$$\Delta_{2L} = \int_0^l \frac{M m_2}{EI}\,dx = -\frac{1433.33}{EI}$$

$$\delta_{11} = \int_0^l \frac{m_1 \cdot m_1}{EI} dx = \frac{90.67}{EI} \qquad \delta_{12} = \delta_{21} = \int_0^l \frac{m_1 m_2}{EI} dx$$

$$\delta_{22} = \int_0^l \frac{m_2 \cdot m_2}{EI} dx = \frac{41.67}{EI} \qquad = \frac{50}{EI}$$

Using compatibility condition, $[P] = [\delta]^{-1}\{[\Delta] - [\Delta_L]\}$

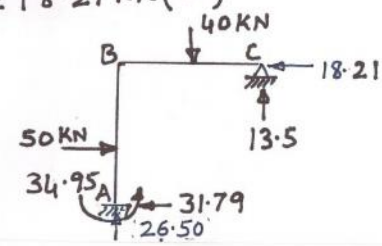
$$[\Delta] = 0 \quad \therefore \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = -\begin{bmatrix} 90.67/EI & 50/EI \\ 50/EI & 41.67/EI \end{bmatrix}^{-1} \begin{bmatrix} -2133.33/EI \\ -1433.33/EI \end{bmatrix} = \begin{bmatrix} 13.50 \\ 18.21 \end{bmatrix}$$

$\therefore \; P_1 = V_c = 13.50$ KN (↑) $\quad P_2 = H_c = 18.21$ KN (←)

$M_b = 13.5 \times 4 - 40 \times 2 = -26.0$ (hogging)

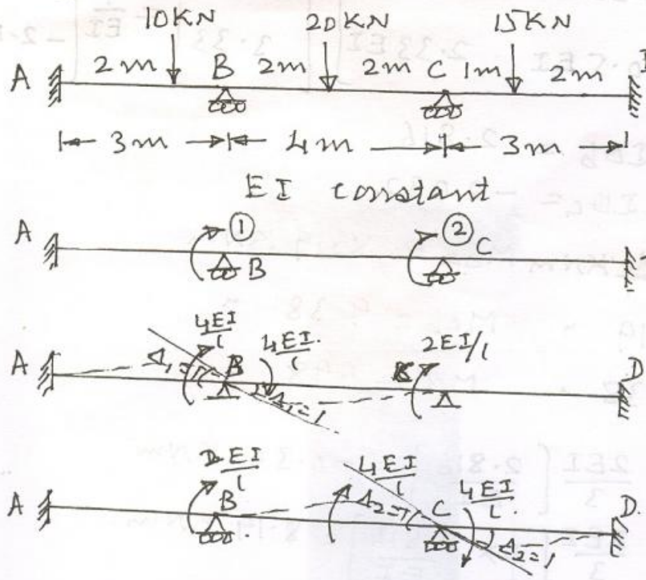
$M_a = 13.5 \times 4 + 18.21 \times 5 - 40 \times 2 - 50 \times 2$

$= -34.95$ (hogging)



LECTURE No. 9

Ex-①



$D_K = 2$

θ_b, θ_c.

coord. ① & ②

co-ord

unit displ. at ①.

unit displ. at ②.

F.E.M: AB : $\overline{M}_{ab} = -\frac{10 \times 2 \times 1^2}{3^2} = -2.22$ KNm

$\overline{M}_{ba} = +\frac{10 \times 2^2 \times 1}{3^2} = 4.44$ "

BC : $\overline{M}_{bc} = -\frac{20 \times 4}{8} = -10.00$ "

$\overline{M}_{cb} = + \text{ - - - } +10.00$ "

CD $\overline{M}_{cd} = -\frac{15 \times 2^2 \times 1}{3^2} = -6.67$ "

$\overline{M}_{dc} = +\frac{15 \times 1^2 \times 2}{3^2} = 3.33$ ".

Locking forces: $P_1' = 4.44 - 10 = -5.56$

$P_2' = +10 - 6.67 = 3.33$

Stiffness Matrix :- $[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$

first coln: unit displ. at co-ord ①

$K_{11} = \left(\frac{4EI}{l}\right)_{AB} + \left(\frac{4EI}{l}\right)_{BC} = \frac{4EI}{3} + \frac{4EI}{4} = 2.33\,EI.$

$K_{21} = \left(\frac{2EI}{l}\right)_{BC} = \frac{2EI}{4} = 0.50\,EI.$

2nd coln: unit displ. at co-ord ②

$K_{12} = K_{21} = \left(\frac{2EI}{l}\right)_{BC} = 0.50\,EI.$

$K_{22} = \left(\frac{4EI}{l}\right)_{BC} + \left(\frac{4EI}{l}\right)_{CD} = \frac{4EI}{4} + \frac{4EI}{3} = 2.33\,EI$

∴ No Ext. loads at the co-ord. $P_1 = P_2 = 0$.

$[\Delta] = [K]^{-1}[\{P\} - \{P'\}]$ $\quad [P] = 0.$

∴ $[\Delta] = -[K]^{-1}[P']$.

$$\therefore \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = - \begin{bmatrix} 2.33\,EI & 0.5\,EI \\ 0.5\,EI & 2.33\,EI \end{bmatrix}^{-1} \begin{bmatrix} -5.56 \\ 3.33 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 2.816 \\ -2.032 \end{bmatrix}.$$

$\therefore$ $EI\Delta_1 = EI\theta_b = 2.816$

$EI\Delta_2 = EI\theta_c = -2.032$

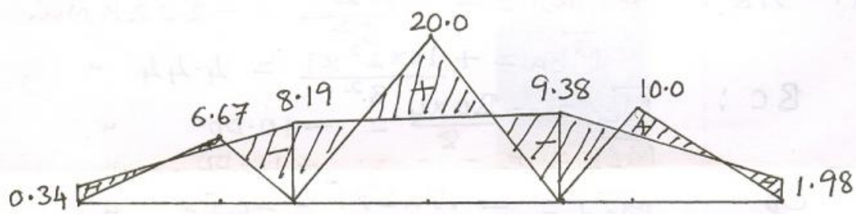
$\therefore$ $M_{ab} = -0.34$ KNm $\quad M_{ba} = 8.19$ KNm

$M_{bc} = -8.19$ " $\quad M_{cb} = 9.38$ "

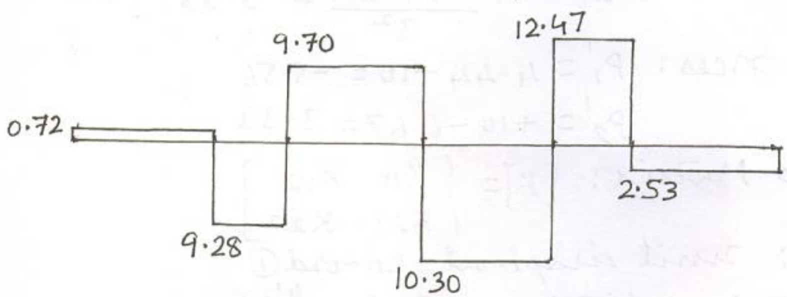
$M_{cd} = -9.38$ " $\quad M_{dc} = 1.98$ "

$$M_{ab} = -2.22 + \frac{2EI}{3}\left[\frac{2.816}{EI}\right] = -0.34 \text{ KNm}$$

$$M_{ba} = 4.44 + \frac{2EI}{3}\left[2 \times \frac{2.816}{EI}\right] = 8.19 \text{ KNm.}$$



BM Diagram



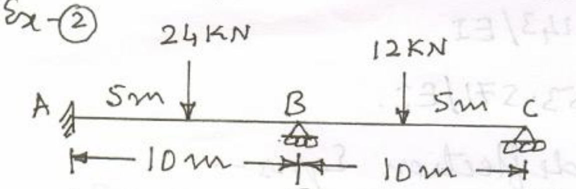
Reactions:-

$R_a = 0.72$ KN $\qquad R_c = 22.77$ KN

$R_b = 18.98$ KN $\qquad R_d = 2.53$ KN.

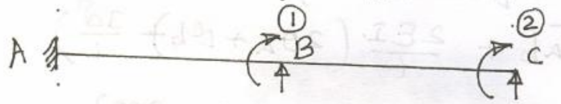
Supports B & C sinks down by $\frac{200}{EI}$ & $\frac{100}{EI}$ units.

Ex-(2)

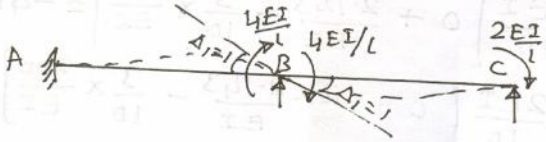


$DK = 2$

$\theta_b,\ \theta_c$.



co-ord. (1) & (2)

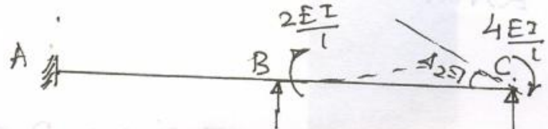


unit displ. at (1)

$K_{11} = \left(\frac{4EI}{l}\right)_{AB} + \left(\frac{4EI}{l}\right)_{BC}$

$= 0.8EI.$

$K_{21} = \left(\frac{2EI}{l}\right)_{BC} = 0.2EI.$



$K_{12} = K_{21} = 0.2EI$

$K_{22} = \left(\frac{4EI}{l}\right) = 0.4EI.$

unit displ. at (2)

F.E.Ms : Locking forces :

FEM = (FEM) loading + (FEM) sinking.

F.E.M. - due to sinking -

$\overline{M}_{ab} = -\frac{24\times10}{8} - \frac{6EI}{10^2}\times\frac{200}{EI}$

$= -30 - 12 = -42\ KNm$

$\overline{M}_{ba} = +30 - 12 = +18$ "

$\overline{M}_{bc} = -\frac{12\times10}{8} + \frac{6EI}{10^2}\times\frac{100}{EI}$

$= -15 + 6 = -9.0\ KNm$

$\overline{M}_{cb} = +15 + 6 = 21$ "

Locking force at (1) $P_1' = 18 - 9 = +9.0$

" " (2) $P_2' = 21.0\ KNm.$

∴ No. Ext forces at co-ord (1) & (2) ∴ $P_1 = P_2 = 0$.

$\therefore \{\Delta\} = -[K]^{-1}\{P'\}$

$$\therefore \begin{bmatrix}\Delta_1\\ \Delta_2\end{bmatrix} = -\begin{bmatrix}0.8EI & 0.2EI\\ 0.2EI & 0.4EI\end{bmatrix}^{-1}\begin{bmatrix}9.0\\ 21.0\end{bmatrix} = \begin{bmatrix}2.143/EI\\ -53.571/EI\end{bmatrix}.$$

$\therefore \Delta_1 = \theta_b = 2.143/EI.$

$\Delta_2 = \theta_c = -53.571/EI.$

$\therefore$ using slope deflection Eqns —

$$M_{ab} = \bar{M}_{ab} + \frac{2EI}{l}\left(2\theta_a + \theta_b - \frac{3\delta}{l}\right) \quad \delta = -\frac{200}{EI}$$

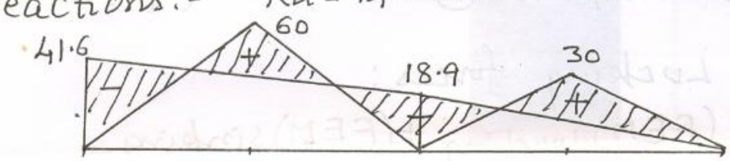
$$\therefore M_{ab} = -30 + \frac{2EI}{10}\left[0 + \frac{2.143}{EI} - \frac{3}{10} \times \frac{200}{EI}\right] = -41.6 \text{ KNm}$$

$$M_{ba} = +30 + \frac{2EI}{10}\left[0 + 2 \times \frac{2.143}{EI} - \frac{3}{10} \times \frac{200}{EI}\right] = 18.9 \text{ KNm}$$

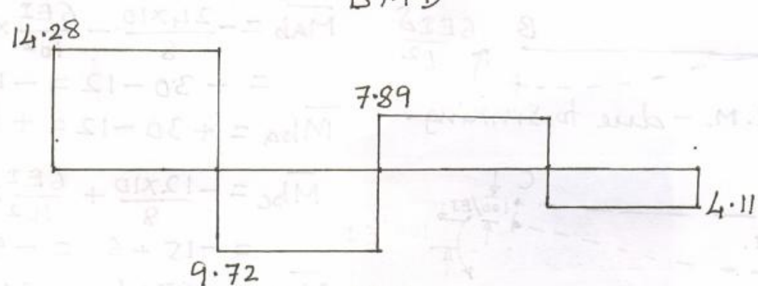
$|||^{ly}$ $M_{bc} = -18.9$ KNm

$M_{cb} = 0.$

Reactions:- $R_a = 14.28$ KN $\quad R_b = 17.61$ KN $\quad R_c = 4.11$ KN.



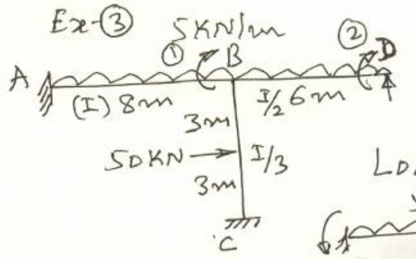
BMD



SFD

LECTURE No. 10

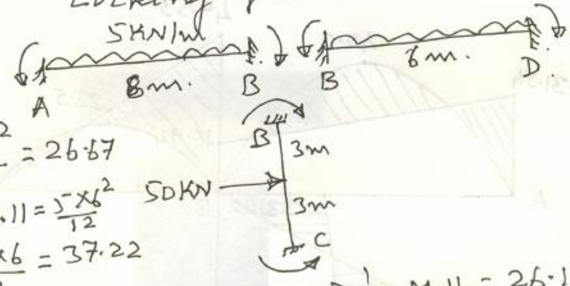
Ex-③



DK = 2.

θb, θd are the displ. co-ord. ①&②

Locking forces -



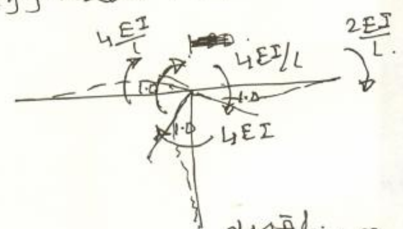
$\therefore M_{ab} = M_{ba} = \frac{5\times8^2}{12} = 26.67$

$-M_{bd} = M_{db} = 26.11 = \frac{5\times6^2}{12}$

$M_{bc} = -M_{cb} = \frac{50\times6}{8} = 37.22$

$\therefore P_1' = 26.67 - 26.11 + 37.22 = 38.06 \qquad P_2' = M_{db} = 26.11$

Stiffness Matrix:-



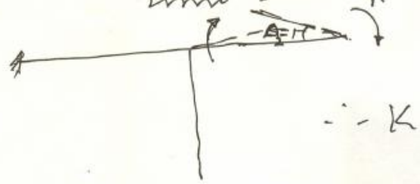
$\therefore M = \left(\frac{4EI}{l}\right)$ for AB, BD, BC.

$\therefore K_{11} = \frac{4E\times I}{8} + \frac{4EI/2}{6} + \frac{4EI/3}{6}$

$= 1.05EI$

$K_{21} = \left(\frac{2EI}{l}\right)_{BD} = \frac{2E\times I/3}{6} = 0.167EI$

unit displ. at ①



$K_{22} = \left(\frac{4EI}{l}\right)_{BD} = \frac{4E\times I/3}{6} = 0.33EI$

$K_{12} = K_{21} = 0.167EI.$

$\therefore K = EI\begin{bmatrix} 1.05 & 0.167 \\ 0.167 & 0.33 \end{bmatrix}.$

$\therefore$ Using Eql. Eqn -

$\{P\} = 0$ = Net forces.

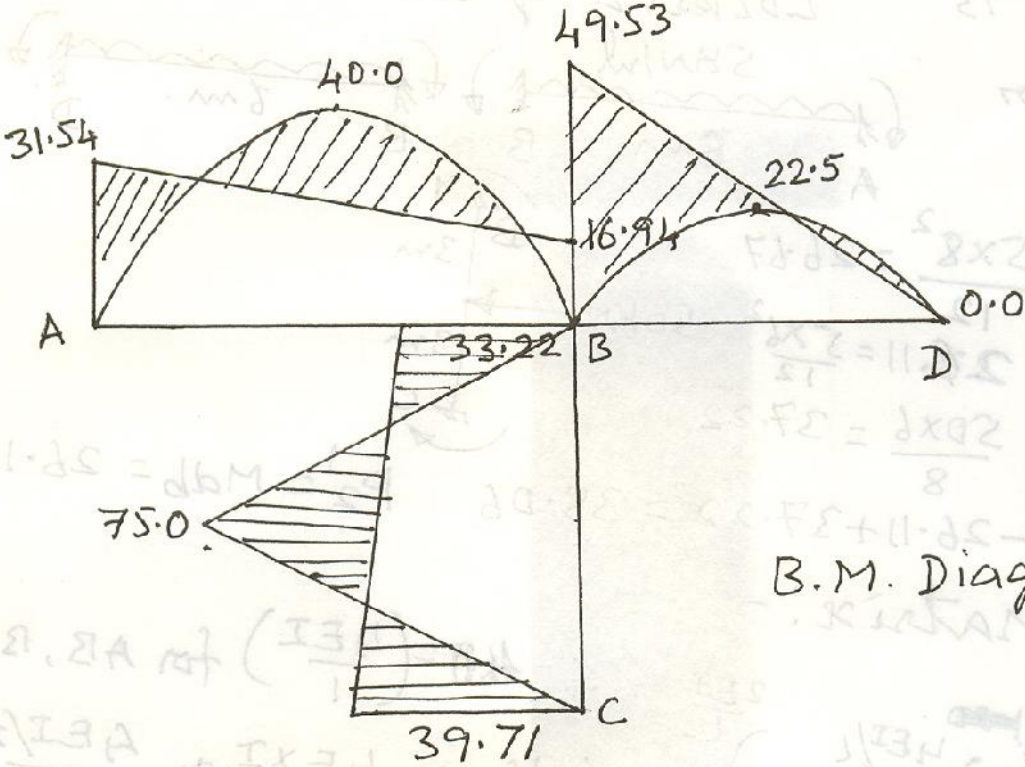
$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = +[K]^{-1}\left\{[P]-[P']\right\}$

$= -\frac{1}{EI}\begin{bmatrix} 1.05 & 0.167 \\ 0.167 & 0.33 \end{bmatrix}^{-1}\begin{bmatrix} 38.06 \\ 26.11 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -19.47 \\ -101.55 \end{bmatrix}$

$\therefore EI\Delta_1 = EI\theta_b = -19.47, \qquad EI\Delta_2 = EI\theta_d = -101.55$

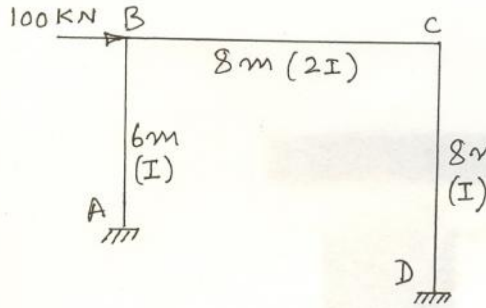
$M_{ab} = -31.54 \quad M_{bd} = -49.53 \quad M_{bc} = +33.22$ Draw B.M.D.

$M_{ba} = +16.94 \quad M_{db} = 0 \quad M_{cb} = -39.71$



B.M. Diagram.

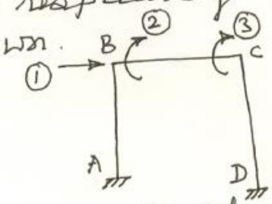
Ex - (4)



DK = 3

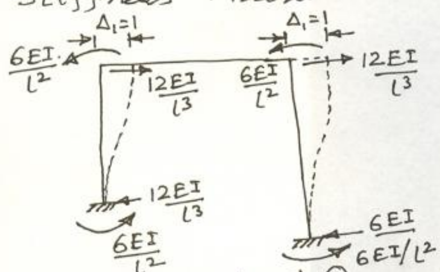
δ, θb, θc are the unknown displacements

Assign co-ord. (1), (2) & (3) respectively as shown.



co-ord.

Stiffness Matrix:



unit displ. at (1)

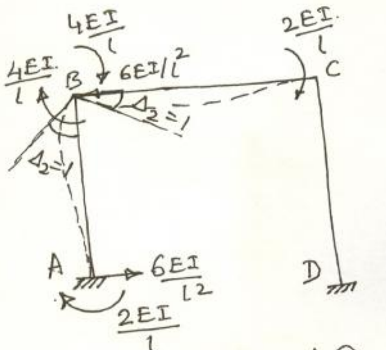
$\therefore K_{11} = \left(\frac{12EI}{l^3}\right)_{AB} + \left(\frac{12EI}{l^3}\right)_{CD}$

$= \frac{12EI}{6^3} + \frac{12EI}{8^3}$

$= 0.08\,EI.$

$K_{21} = -\left(\frac{6EI}{l^2}\right)_{AB} = -0.167\,EI.$

$K_{31} = -\left(\frac{6EI}{l^2}\right)_{CD} = -0.094\,EI.$



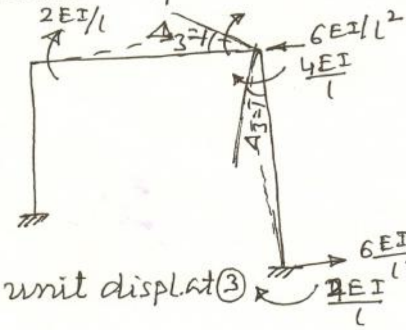
unit displ. at (2)

$\therefore K_{12} = -\left(\frac{6EI}{l^2}\right)_{AB} = K_{21} = -0.167\,EI.$

$K_{22} = \left(\frac{4EI}{l}\right)_{AB} + \left(\frac{4EI}{l}\right)_{BC}$

$= \frac{4EI}{6} + \left(\frac{4E \times 2I}{8}\right) = 1.667\,EI$

$K_{32} = \left(\frac{2EI}{l}\right)_{BC} = \frac{2E \times 2I}{8} = 0.5\,EI$



unit displ. at (3)

$K_{13} = K_{31} = -0.094\,EI.$

$K_{23} = K_{32} = 0.5\,EI.$

$K_{33} = \left(\frac{4EI}{l}\right)_{BC} + \left(\frac{4EI}{l}\right)_{CD}$

$= \frac{4E \times 2I}{8} + \frac{4EI}{8}$

$= 1.5\,EI.$

Locking forces — F.E.Ms. — are all zero.

$\therefore P_1' = P_2' = P_3' = 0$

Ext. forces — at the co-ordinates

$P_1 = 100$ KN. $P_2 = P_3 = 0$.

$$\therefore \{\Delta\} = \{K\}^{-1}\{[P] - [P']\}.$$

$$\therefore \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 0.08 & -0.167 & -0.094 \\ -0.167 & 1.667 & 0.5 \\ -0.094 & 0.5 & 1.5 \end{bmatrix}^{-1}\left\{\begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right\}$$

$$= \frac{1}{EI}\begin{bmatrix} 16.17 & 1.46 & 0.526 \\ 1.46 & 0.80 & -0.175 \\ 0.526 & -0.175 & 0.758 \end{bmatrix}\begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 1617 \\ 146 \\ 52.6 \end{bmatrix}$$

$\therefore EI\Delta_1 = EI\delta = 1617$

$EI\Delta_2 = EI\theta_b = 146$

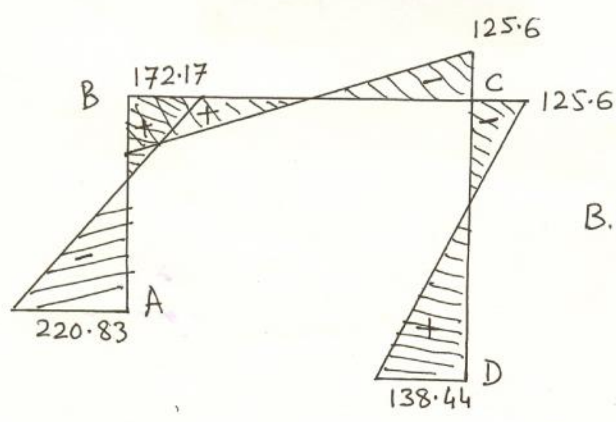
$EI\Delta_3 = EI\theta_c = 52.6$

From slope deflection Eqns :-

$M_{ab} = -220.83$ KNm $\quad M_{ba} = -172.17$ KNm

$M_{bc} = +172.3$ KNm. $\quad M_{cb} = +125.6$ KNm.

$M_{cd} = -125.3$ " $\quad M_{dc} = -138.44$ ".



B.M. Diagram.