UNIT-II

MOVING LOADS AND INFLUENCE LINES

Influence lines for reactions in statically determinate structures – influence lines for member forces in pin-jointed frames – Influence lines for shear force and bending moment in beam sections – Calculation of critical stress resultants due to concentrated and distributed moving loads. Muller Breslau‟s principle – Influence lines for continuous beams and single storey rigid frames – Indirect model analysis for influence lines of indeterminate structures – Beggs deformeter

(DETERMINATE &INDETERMINATE STRUCTURES WITH REDUNDANCY RESTRICTEDTO ONE)

Introduction:

- To statically determinate and statically indeterminate structural analysis under non-moving load (dead load or fixed loads).
- To determination of maximum internal actions at cross-sections of members of statically determinate structured under the effects of moving loads (live loads).
- Common sense tells us that when a load moves over a structure, the deflected shape of the structural will vary.
- In the process, we can arrive at simple conclusion that due to moving load position on the structure, reactions value at the support also will vary.
- From the designer's point of view, it is essential to have safe structure, which doesn't exceed the limits of deformations and also the limits of load carrying capacity of the structure.

Definitions of influence line

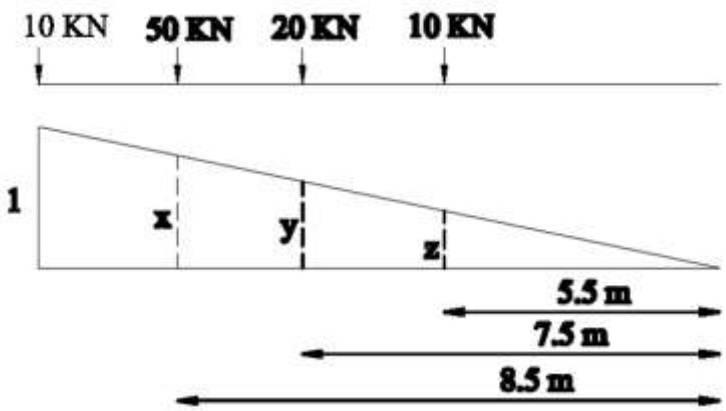
- An influence line is a diagram whose ordinates, which are plotted as a function of distance along the span, give the value of an internal force, a reaction, or a displacement at a particular point in a structure as a unit load move across the structure.
- An influence line is a curve the ordinate to which at any point equals the value of some particular function due to unit load acting at that point.
- An influence line represents the variation of either the reaction, shear, moment, or deflection at a specific point in a member as a unit concentrated force moves over the member.

1) *A system of concentrated load, role beam left to right, s.s beam span of 10m and 10 KN load leading*

Find 1.Absolute max +ve S.F

2. .Absolute max -ve S.F

3..Absolute max BM



Solution

1. **Absolute max +ve S.F**

Using the similar triangle method and we get the x, y & z values

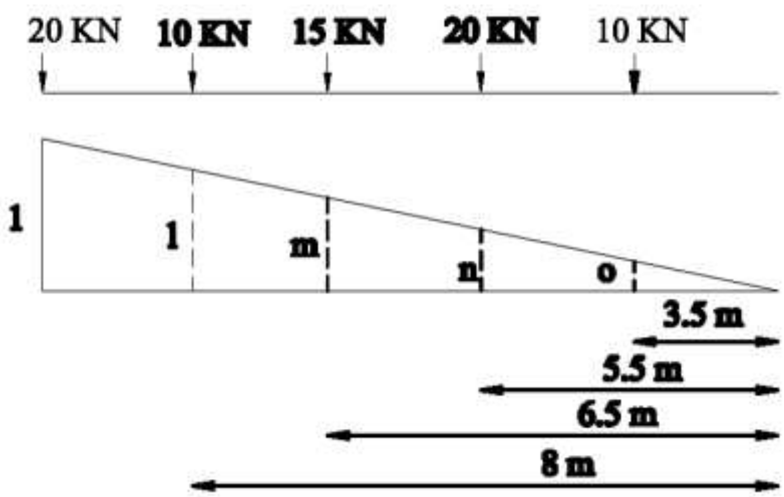
X = 0.85 m

Y = 0.75 m

Z = 0.55 m

S.F = (10×1)+(15×0.83)+(20×0.75)+(10×0.55)

= 43.25 KN



Using the similar triangle method and we get the l, m, n & o values

L=0.8 m

M = 0.65 m

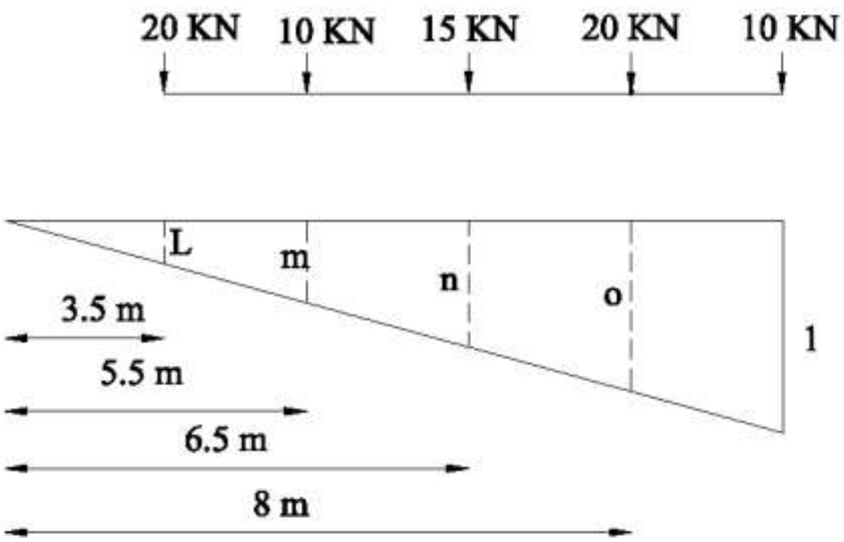
N = 0.55 m

O = 0.35 m

S.F = (20×1)+(10×0.8)+(15×0.65)+(20×0.55)+(10×0.35)

= 52.25 KN

Absolute max -ve S.F



Using the similar triangle method and we get the l, m, n & o values

L = 0.35 m

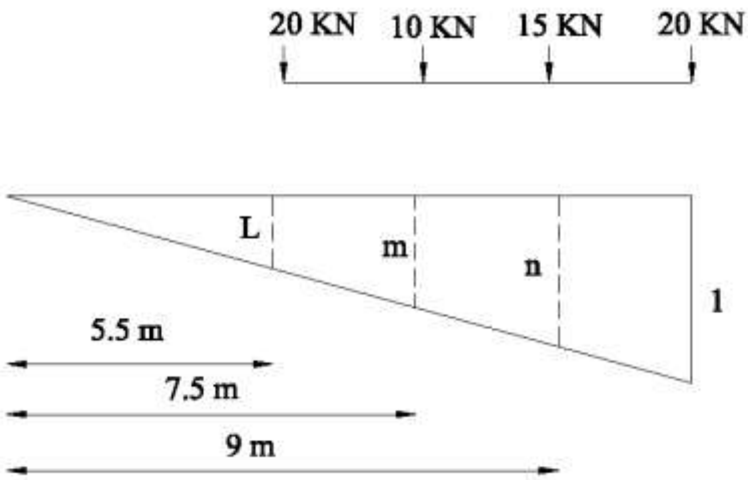
M = 0.55 m

N = 0.7 m

O = 0.8 m

S.F = (10×1)+(20×0.8)+(15×0.7)+(10×0.55)+(20×0.35)

= - 49 KN



Using the similar triangle method and we get the l,m, & n values

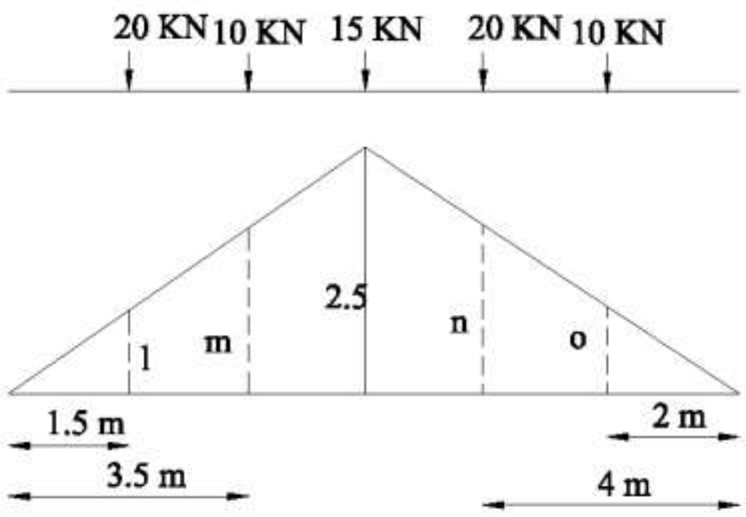
L=0.55 m

M=0.75 m

N=0.85 m

S.F=-((20×1)+(15×0.9)+(10v0.75)+(20×0.55))=-52 KN

(iii) Absolute max BM



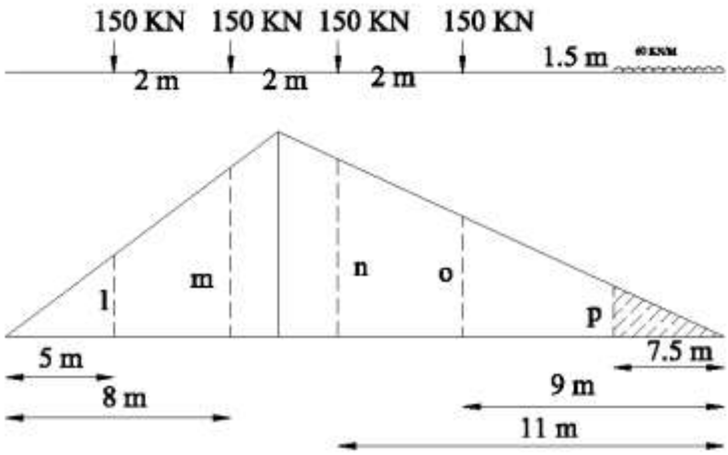
Using the similar triangle method and we get the l, m, n & o values

L	=	0.75 m
M	=	1.75 m
N	=	2 m
O	=	1 m
Max BM	=	(20×0.75) +(10×1.75)+(15×2.5)+(20×2)+(10×1)
	=	22.75 KN

2) ***The four equal loads of 150 KN ,each equally spaced at apart 2m and UDL of 60 KN/m at a distance of 1.5m from the last 150 KN loads cross a girder of 20m from span R to L.Using influence line ,calculate the S.F and BM at a section of 8m from L.H.S support when leading of 150KN 5m from L.H.S.***

Solution

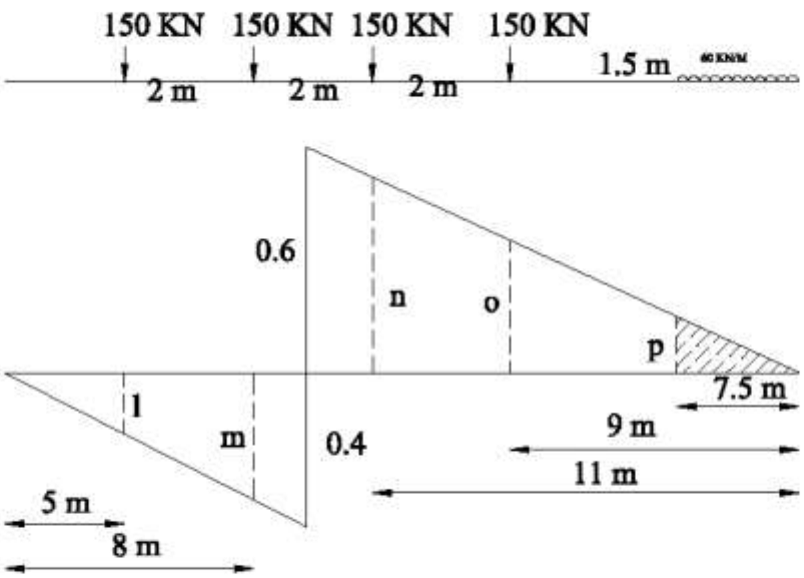
(i) Max BM



L	=	3 m
M	=	4.2 m
N	=	4.4 m
0	=	3.6 m
P	=	3 m
A	=	11.25 m^2
BM	=	(150×3)+(150×4.2)+(150×4.4)+(150×306)+(60×11.25)
	=	2955 KNm

ii) Shear Force

Compute maximum end shear for the given beam loaded with moving loads as shown in Figure



L	=	0.25 m,
M	=	0.3 m,
N	=	0.55 m,
O	=	0.45 m,
P	=	0.375 m
SF	=	((150×0.25)+(150×0.35)+(150×0.55)+(150×0.45)+(60×1.41))
	=	144. KN

Where do you get rolling loads in practice?

- Shifting of load positions is common enough in buildings. But they are more pronounced in bridges and in gantry girders over which vehicles keep rolling.

Name the type of rolling loads for which the absolute maximum bending moment occurs at the midspan of a beam.

- Single concentrated load
- udl longer than the span
- udl shorter than the span
- Also when the resultant of several concentrated loads crossing a span, coincides with a concentrated load then also the maximum bending moment occurs at the centre of the span.

What is meant by absolute maximum bending moment in a beam?

- When a given load system moves from one end to the other end of a girder, depending upon the position of the load, there will be a maximum bending moment for every section.
- The maximum of these bending moments will usually occur near or at the midspan.
- The maximum of maximum bending moments is called the absolute maximum bending moment.

Where do you have the absolute maximum bending moment in a simply supported beam when a series of wheel loads cross it?

- When a series of wheel loads crosses a simply supported beam, the absolute maximum bending moment will occur near midspan under the load Wcr , nearest to midspan (or the heaviest load).
- If Wcr is placed to one side of midspan C, the resultant of the load system R shall be on the other side of C; and Wcr and R shall be equidistant from C.
- Now the absolute maximum bending moment will occur under Wcr .
- If Wcr and R coincide, the absolute maximum bending moment will occur at midspan.

What is the absolute maximum bending moment due to a moving udl longer than the span of a simply supported beam?

- When a simply supported beam is subjected to a moving udl longer than the span, the absolute maximum bending moment occurs when the whole span is loaded.
- Mmax max = $wl^2/ 8$

State the location of maximum shear force in a simple beam with any kind of loading.

- In a simple beam with any kind of load, the maximum positive shear force occurs at the left hand support and maximum negative shear force occurs at right hand support.

What is meant by maximum shear force diagram?

- Due to a given system of rolling loads the maximum shear force for every section of the girder can be worked out by placing the loads in appropriate positions.
- When these are plotted for all the sections of the girder, the diagram that we obtain is the maximum shear force diagram.
- This diagram yields the 'design shear' for each cross section.

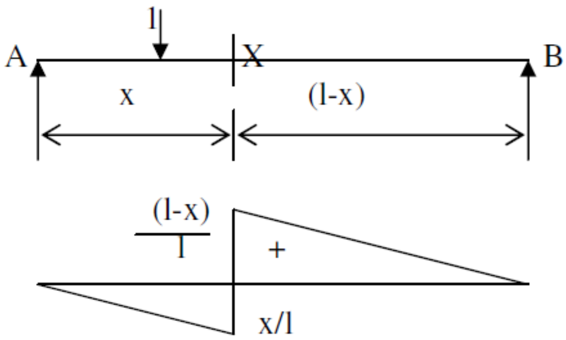
What is meant by influence lines?

- An influence line is a graph showing, for any given frame or truss, the variation of any force or displacement quantity (such as shear force, bending moment, tension, deflection) for all positions of a moving unit load as it crosses the structure from one end to the other.

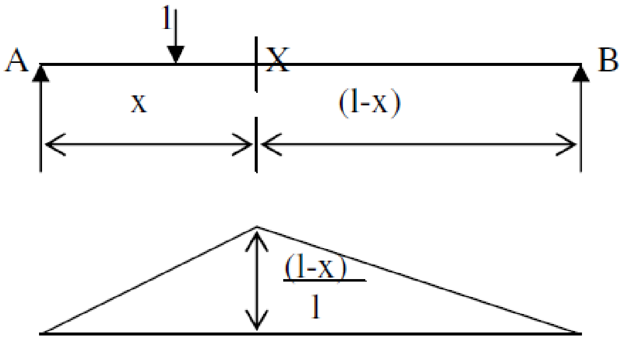
What are the uses of influence line diagrams?

- Influence lines are very useful in the quick determination of reactions, shear force, bending moment or similar functions at a given section under any given system of moving loads and
- Influence lines are useful in determining the load position to cause maximum value of a given function in a structure on which load positions can vary.

Draw the influence line diagram for shear force at a point X in a simply supported beam AB of span 'l' m.



Draw the ILD for bending moment at any section X of a simply supported beam and mark the ordinates.



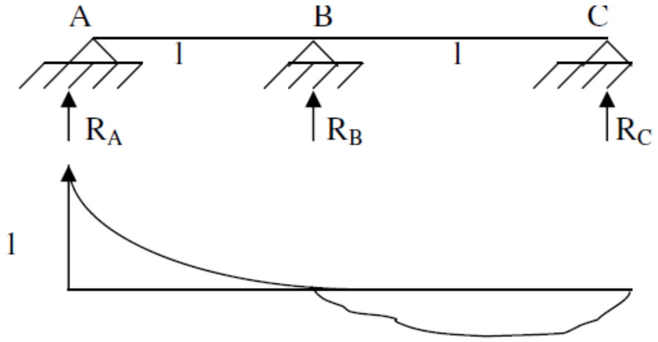
What do you understand by the term reversal of stresses?

- In certain long trusses the web members can develop either tension or compression depending upon the position of live loads.
- This tendancy to change the nature of stresses is called reversal of stresses.

State Muller-Breslau principle.

- Muller-Breslau principle states that, if we want to sketch the influence line for any force quantity (like thrust, shear, reaction, support moment or bending moment) in a structure,
- We remove from the structure the resistant to that force quantity and
- We apply on the remaining structure a unit displacement corresponding to that force quantity.
- The resulting displacements in the structure are the influence line ordinates sought.

State Maxwell-Betti's theorem.



- In a linearly elastic structure in static equilibrium acted upon by either of two systems of external forces, the virtual work done by the first system of forces in undergoing the displacements caused by the second system of forces is equal to the virtual work done by the second system of forces in undergoing the displacements caused by the first system of forces.
- Maxwell Betti's theorem helps us to draw influence lines for structures.

What is the necessity of model analysis?

- When the mathematical analysis of problem is virtually impossible.
- Mathematical analysis though possible is so complicated and time consuming that the model analysis offers a short cut.
- The importance of the problem is such that verification of mathematical analysis by an actual test is essential.

Define similitude.

- Similitude means similarity between two objects namely the model and the prototype with regard to their physical characteristics:

- Geometric similitude is similarity of form
- Kinematic similitude is similarity of motion
- Dynamic and/or mechanical similitude is similarity of masses and/or forces.

State the principle on which indirect model analysis is based.

- The indirect model analysis is based on the Muller Breslau principle.
- Muller Breslau principle has lead to a simple method of using models of structures to get the influence lines for force quantities like bending moments, support moments, reactions, internal shears, thrusts, etc.,
- To get the influence line for any force quantity,
  - (i) remove the resistant due to the force,
  - (ii) apply a unit displacement in the direction
  - (iii) plot the resulting displacement diagram.
- This diagram is the influence line for the force.

What is the principle of dimensional similarity?

- Dimensional similarity means geometric similarity of form.
- This means that all homologous dimensions of prototype and model must be in some constant ratio.

What is Begg's deformeter?

- Begg's deformeter is a device to carry out indirect model analysis on structures.
- It has the facility to apply displacement corresponding to moment, shear or thrust at any desired point in the model.
- In addition, it provides facility to measure accurately the consequent displacements all over the model.

Name any four model making materials.

- Perspex,
- plexiglass,
- acrylic,
- plywood,
- sheet araldite
- bakelite
- Micro-concrete,
- mortar and plaster of paris

What is ‘dummy length’ in models tested with Begg’s deformeter.

- Dummy length is the additional length (of about 10 to 12mm) left at the extremities of the model to enable any desired connection to be made with the gauges.

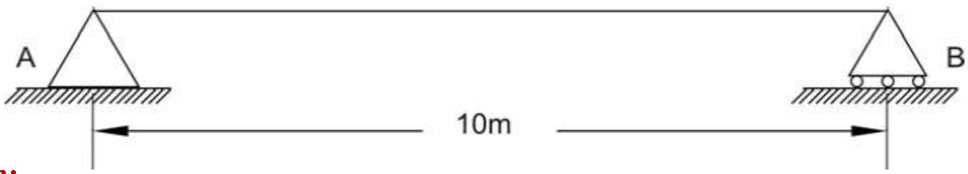
What are the three types of connections possible with the model used with Begg’s deformeter.

- Hinged connection
- Fixed connection
- Floating connection

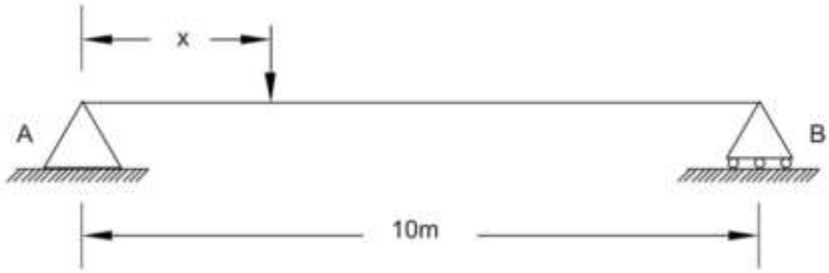
What is the use of a micrometer microscope in model analysis with Begg’s deformeter.

- Micrometer microscope is an instrument used to measure the displacements of any point in the x and y directions of a model during tests with Begg’s deformeter.

Construct the influence line for the reaction at support B for the beam of span 10 m. The beam structure is shown in Figure



Solution:



- A unit load is places at distance x from support A and the reaction value R_B is calculated by taking moment with reference to support A.
- Let us say, if the load is placed at 2.5 m. from support A then the reaction R_B can be calculated as follows

$$\Sigma MA = 0:$$

$$R_B \times 10 - 1 \times 2.5 = 0 \quad \Rightarrow \quad R_B = 0.25$$

- Similarly, the load can be placed at 5.0, 7.5 and 10 m away from support A and reaction R_B can be computed and ta bulated as given below.

X	**R_B**
0	0
2.5	0.25
5	0.5
7.5	0.75
10	1

- Graphical representation of influence line for RB is shown in Figure

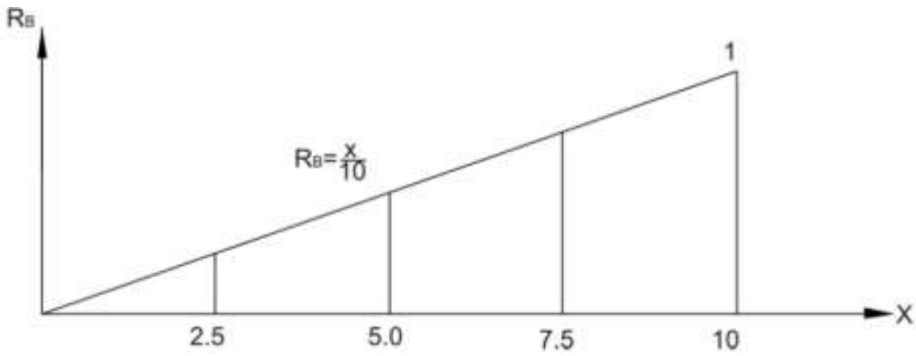
Influence Line Equation:

- When the unit load is placed at any location between two supports from support A at distance x then the equation for reaction RB can be written as

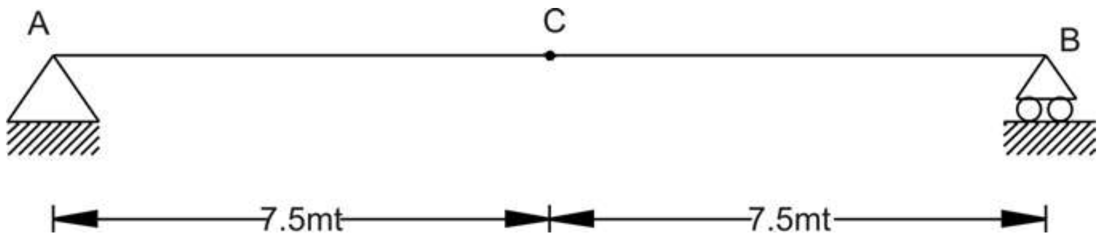
$$\Sigma MA = 0:$$

$$RB \times 10 - x = 0 \quad \Rightarrow \quad RB = x/10$$

Influence line for reaction R_B.

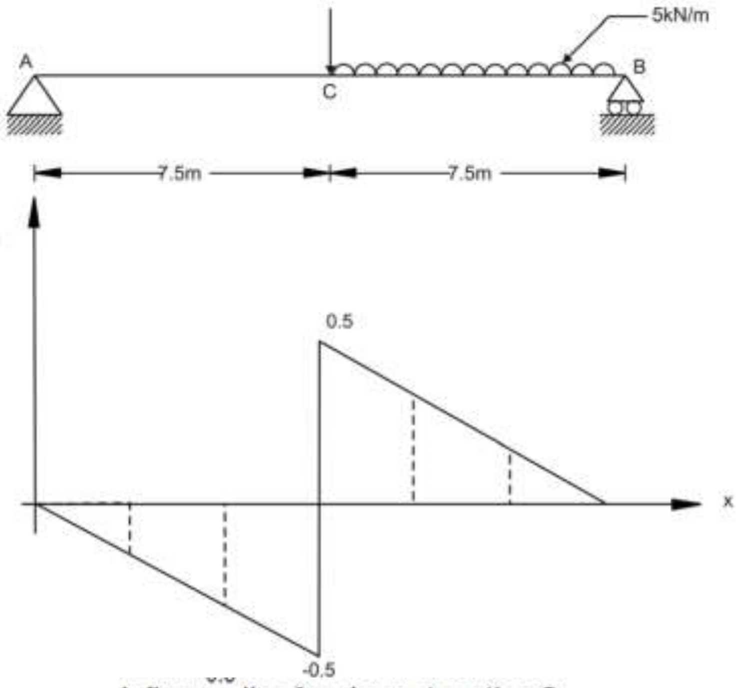


Find the maximum positive live shear at point C when the beam as shown in figure, is loaded with a concentrated moving load of 10 kN and UDL of 5 kN/m.



Concentrated load:

- the maximum live shear force at C will be when the concentrated load 10 kN is located just before C or just after C.
- Our aim is to find positive live shear and hence, we will put 10 kN just after C.
- In that case, Vc = 0.5 x 10 = 5 kN.



Influence line for shear at section C.

UDL:

- the maximum positive live shear force at C willbe when the UDL 5 kN/m is acting between x = 7.5 and x = 15.

Vc = [0.5 x (15 –7.5) (0.5)] x 5 = 9.375

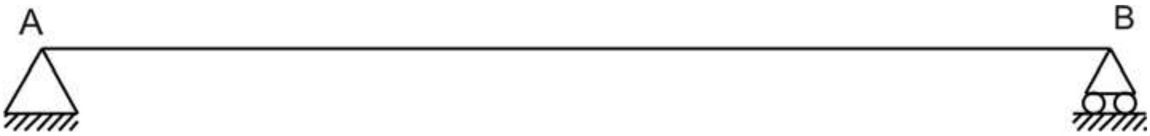
Total maximum Shear at C:

(Vc) max = 5 + 9.375 = 14.375.

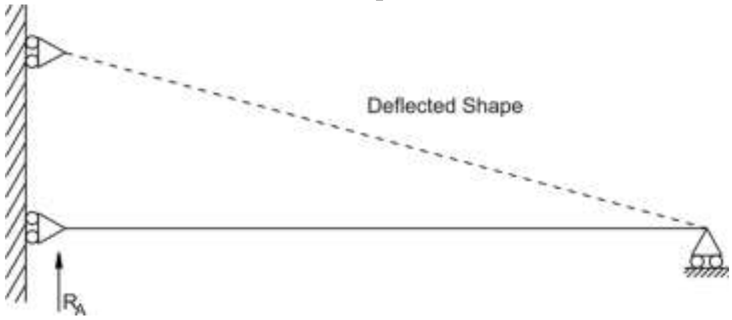
Muller Breslau Principle for Qualitative Influence Lines

- In 1886, Heinrich Müller Breslau proposed a technique to draw influence lines quickly.
- The Müller Breslau Principle states that the ordinate value of an influence line for any function on any structure is proportional to the ordinates of the deflected shape that is obtained by removing the restraint corresponding to the function from the structure and introducing a force that causes a unit displacement in the positive direction.

Procedure:

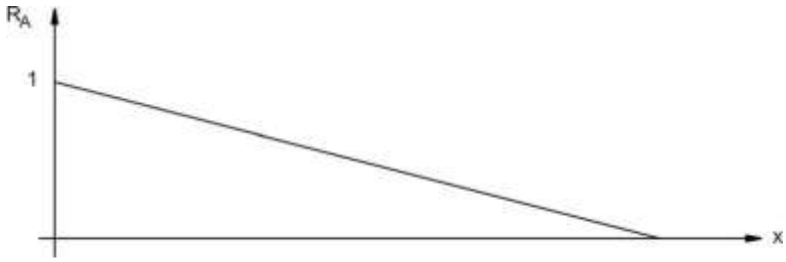


- First of all remove the support corresponding to the reaction and apply a force in the positive direction that will cause a unit displacement in the direction of R_A



Deflected shape of beam

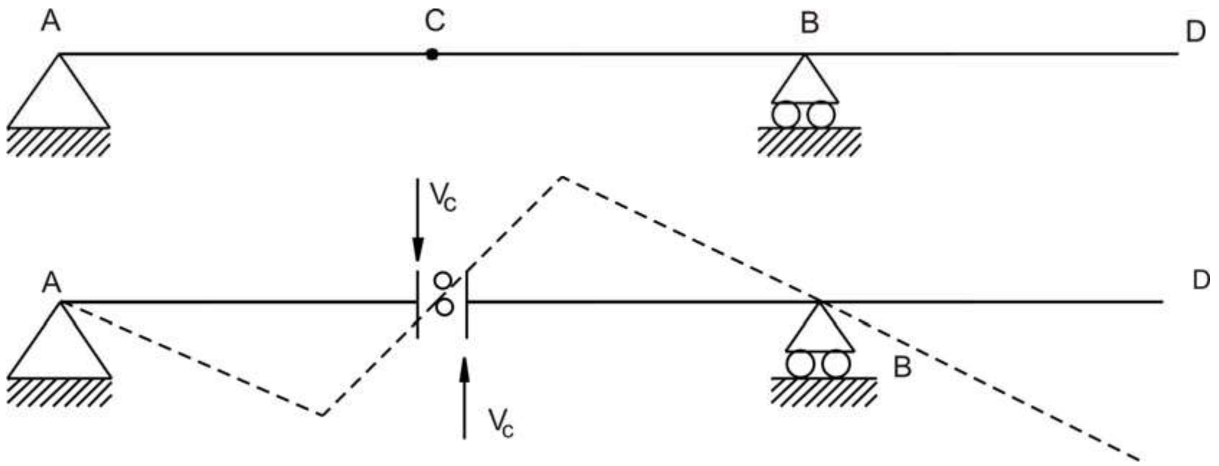
- The resulting deflected shape will be proportional to the true influence line for the support reaction at A.



Influence line for support reaction A

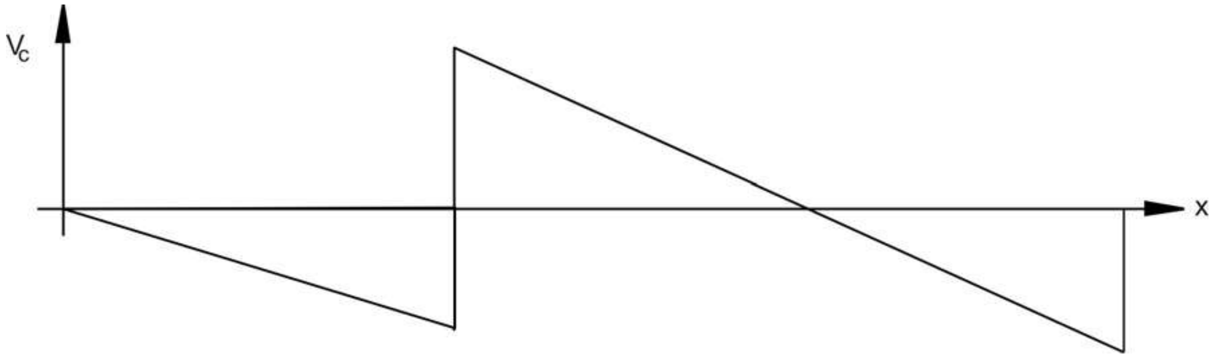
- The deflected shape due to a unit displacement at A is shown in above Figure:1 and matches with the actual influence line shape as shown in Figure 3.
- Note that the deflected shape is linear, i.e., the beam rotates as a rigid body without any curvature. This is true only for statically determinate systems.

Overhang beam



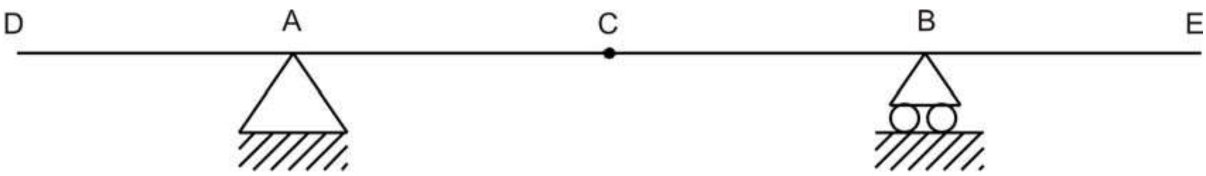
Deflected shape of beam

- Now apply a force in the positive direction that will cause a unit displacement in the direction of V_C.
- The resultant deflected shape is shown above Figure. Again, note that the deflected shape is linear.



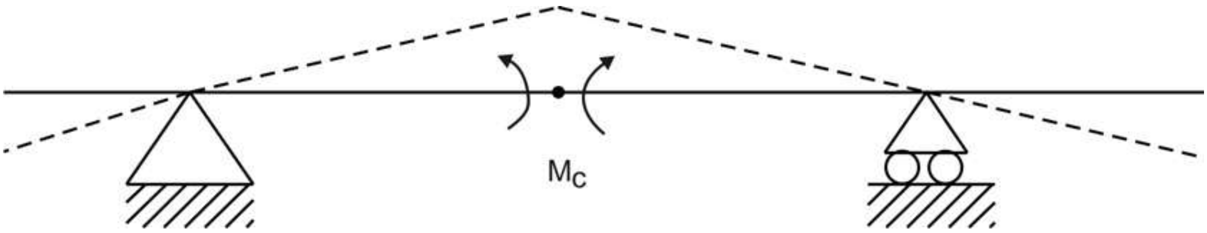
Influence line for shear at section C

Overhang beam - 2

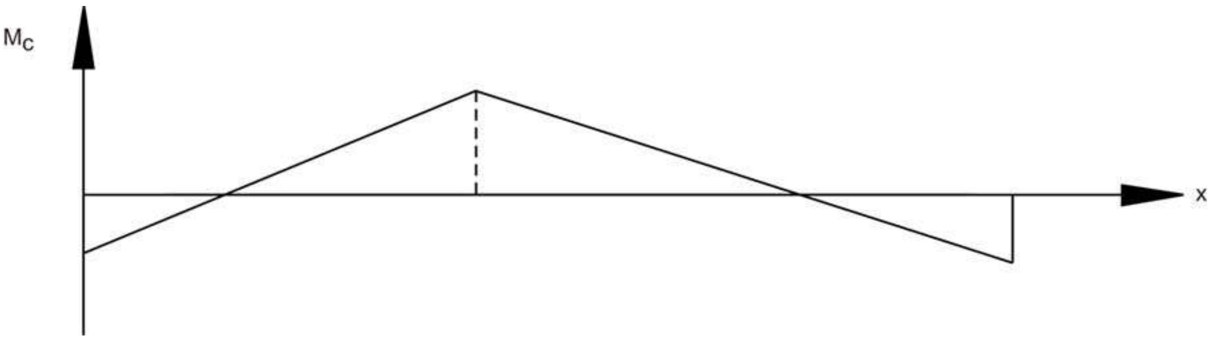


Beam structure

- To construct influence line for moment, we will introduce hinge at C and that will only permit rotation at C.
- Now apply moment in the positive direction that will cause a unit rotation in the direction of Mc.
- The deflected shape due to a unit rotation at C is shown in Figure and matches with the actual shape of the influence line as shown in Figure 3.



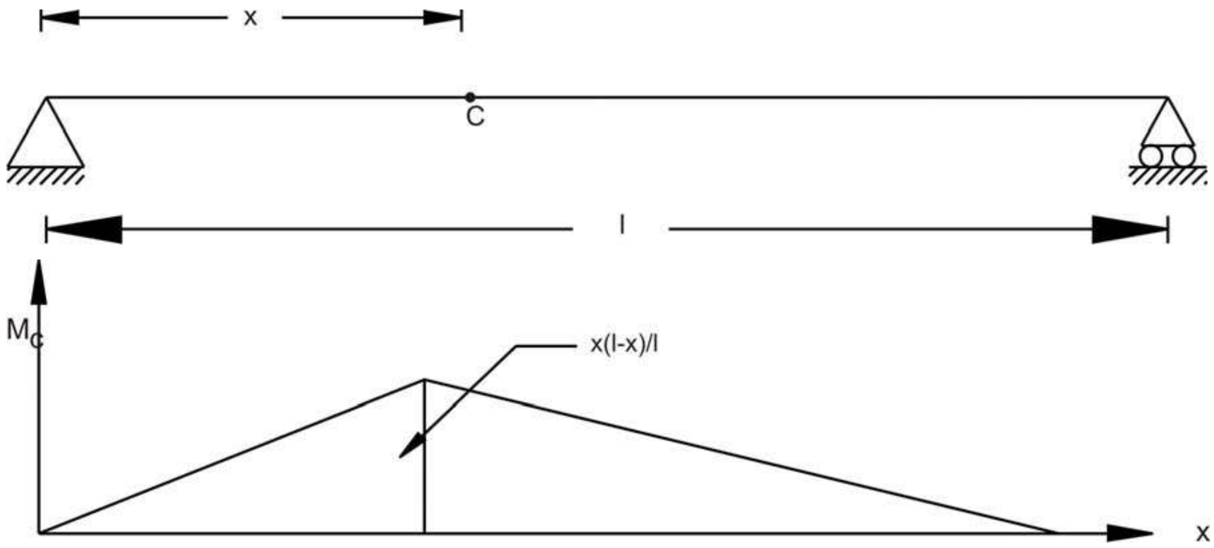
Deflected shape of beam



Influence line for moment at section C

Maximum shear in beam supporting UDLs

UDL longer than the span



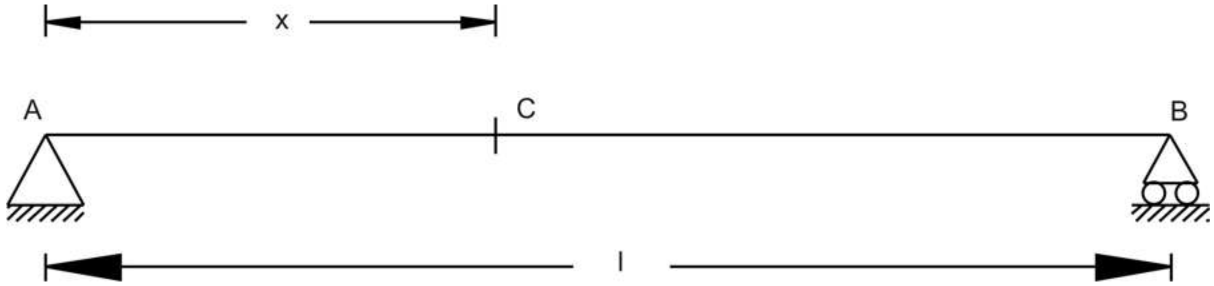
Influence line for moment at section C

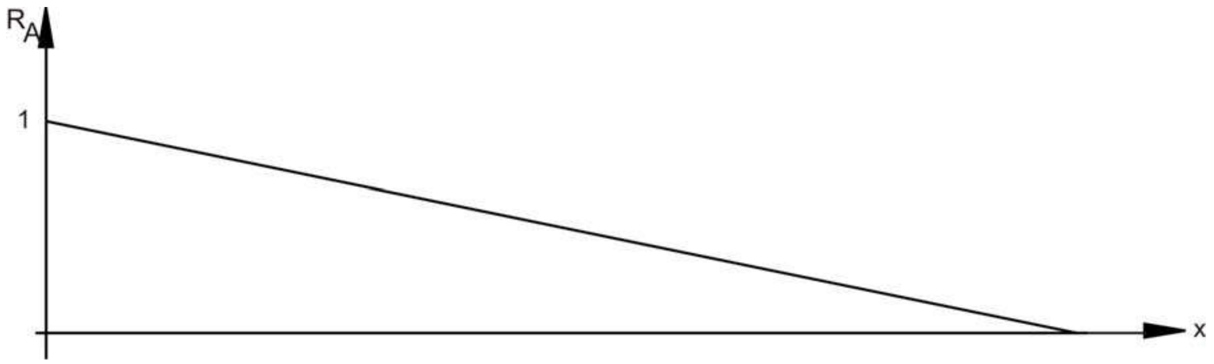
$$w \times \frac{1}{2} \times l \times \frac{x(l-x)}{l} = -\frac{wx(l-x)}{2}$$

Suppose the section C is at mid span, then maximum moment is given by

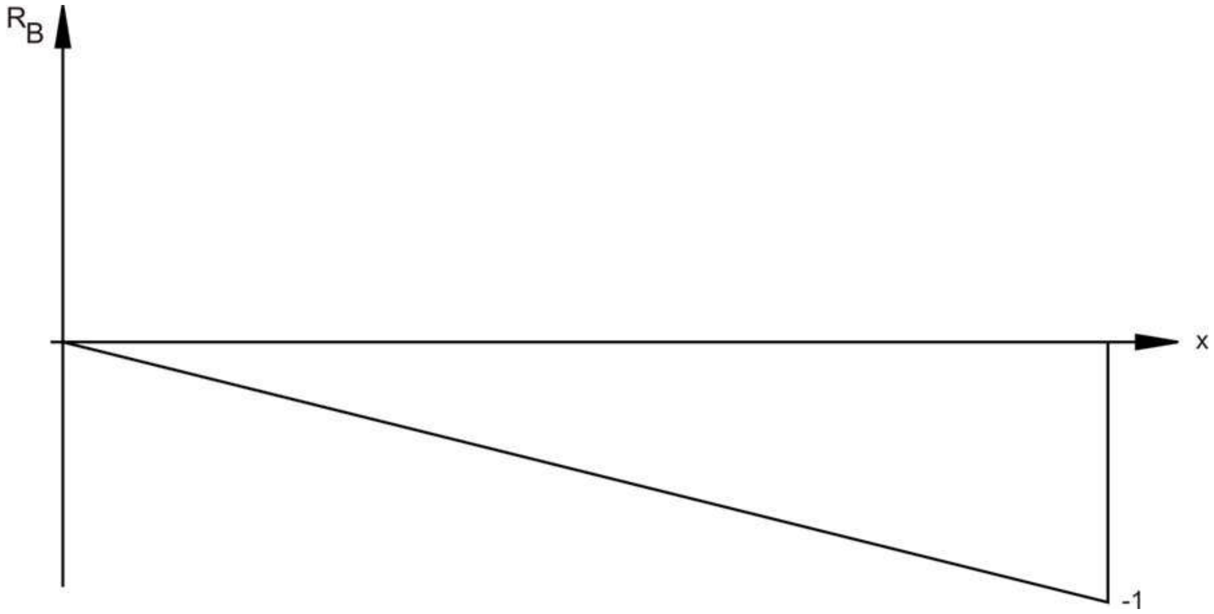
$$\frac{w \times \frac{l}{2} \times \frac{l}{2}}{2} = \frac{wl^2}{8}$$

UDL longer than the span

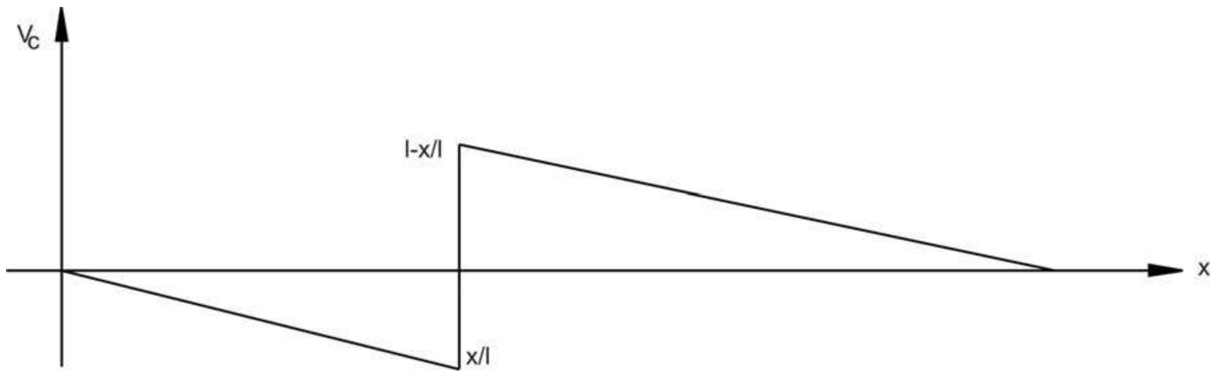




Influence line for support reaction at A



Influence line for support reaction at B



Influence line for shear at section C

$$R_A = w \times \frac{1}{2} \times l \times 1 = \frac{wl}{2}$$

$$R_B = -w \times \frac{1}{2} \times l \times 1 = \frac{-wl}{2}$$

Maximum negative shear is given by

$$= -\frac{1}{2} \times x \times \frac{x}{l} \times w = -\frac{wx^2}{2l}$$

Maximum positive shear is given by

$$= \frac{1}{2} \times \left(\frac{l-x}{l}\right) \times (l-x) \times w = -\frac{w(l-x)^2}{2l}$$