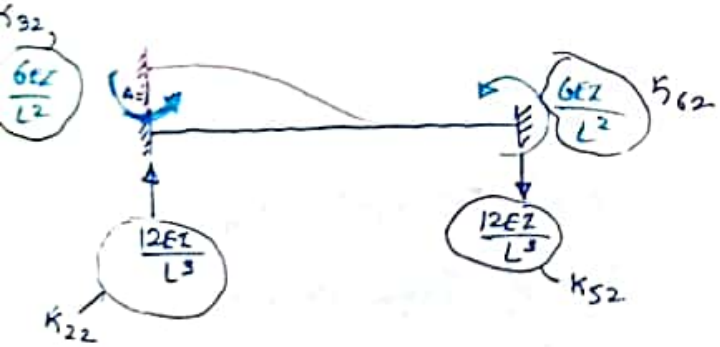
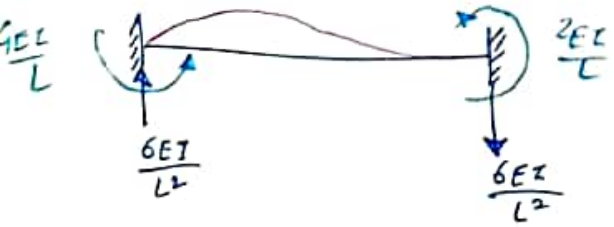
① Derive the stiffness matrix for a member of portal frame Considering the effect of axial deformation

(i) Apply unit Horizontal Displacement in direction of 1

(ii) Apply unit Displacement in Direct^{n} of (2) →

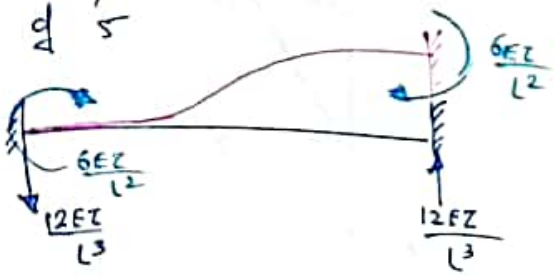


(iii) Apply unit rotation in Direction of 3 →



(iv) Apply unit Displacement in Direct^{n} of 4.

(v) Apply unit Displacement in Direct^{n} of 5



(vi) Apply unit rotation in Direct of 6 ⟶

$$K = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

2) Formulation of member stiffness matrix

$$SM = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

FORMULATION OF MEMBER STIFFNESS MATRIX FOR BEAM

V	R	V	R	
$12EI/L^3$	$6EI/L^2$	$-12EI/L^3$	$6EI/L^2$	V
$6EI/L^2$	$4EI/L$	$-6EI/L^2$	$2EI/L$	R
$-12EI/L^3$	$-6EI/L^2$	$12EI/L^3$	$-6EI/L^2$	V
$6EI/L^2$	$2EI/L$	$-6EI/L^2$	$4EI/L$	R

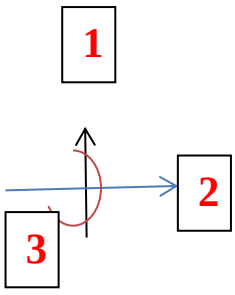
SIMPLIFIED FORM

TAKE EI/L AS COMMON FROM BRACKET

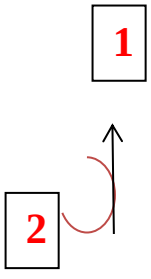
$12/L^2$	$6/L$	$-12/L^2$	$6/L$
$6/L$	4	$-6/L$	2
$-12/L^2$	$-6/L$	$12/L^2$	$-6/L$
$6/L$	2	$-6/L$	4

STEP BY STEP PROCEDURE FOR ANALYSIS OF BEAM SING DIRECET STIFNESS METHOD

1) IDENTIFICATION OF DOKI (DEGREE OF KINAMETIC INDETERMENANCY) FROM FIG.
2) MARKING OF UNKNOWN AND KNOWN DOF AT EACH NODAL POINT USING SYMBOLS 1, 2, 3 ETC
3) REMEMBER, ALWAYS START WRITING THE SYMBOLS WITH UNKOWN DOF FIRST THEN KNOWN DOF AND IN THAT REPRESENTAION IS LIKE THIS



ALSO AS IN BEAM AXIAL DEFORMATION IS NEGLECTED SO SYMBOL 2 WILL BE SKIP THEN WILL GET



4) FORMULATION OF MEMBER STIFFNESS MATRIX FOR BEAM OF 2 NODAL POINTS AT A TIME.

V	R	V	R	
$12EI/L^3$	$6EI/L^2$	$-12EI/L^3$	$6EI/L^2$	V
$6EI/L^2$	$4EI/L$	$-6EI/L^2$	$2EI/L$	R
$-12EI/L^3$	$-6EI/L^2$	$12EI/L^3$	$-6EI/L^2$	V
$6EI/L^2$	$2EI/L$	$-6EI/L^2$	$4EI/L$	R

SIMPLIFIED FORM

TAKE EI/L AS COMMON FROM BRACKET

$12/L^2$	$6/L$	$-12/L^2$	$6/L$
$6/L$	4	$-6/L$	2
$-12/L^2$	$-6/L$	$12/L^2$	$-6/L$
$6/L$	2	$-6/L$	4

5) FORMULATION OF GLOBAL STIFFNESS MATRIX

K MATRIX BASED ON UNKOWN DOF

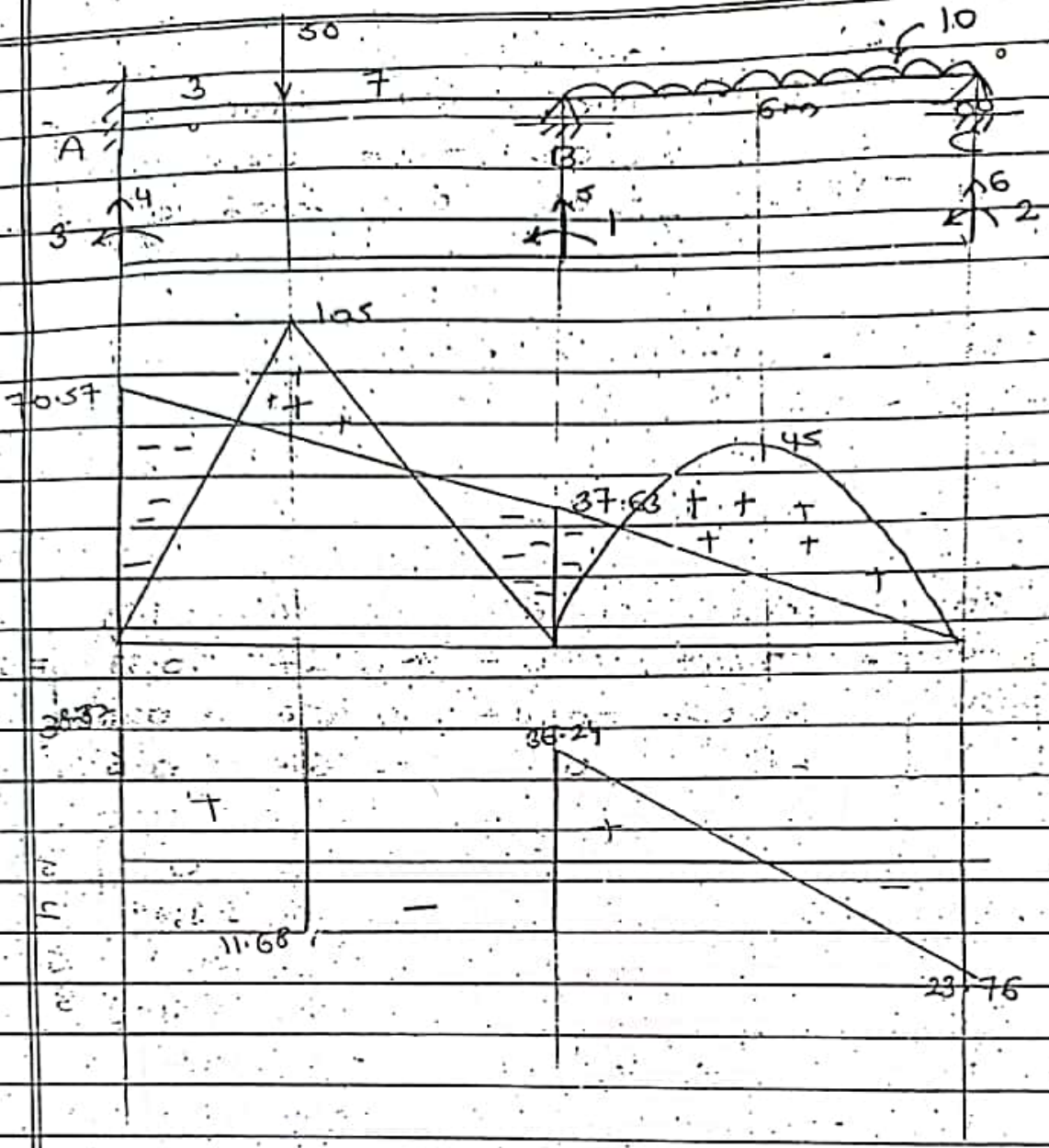
6) CALCULATION OF GLOBAL LOAD MATRIX

7) DISPLACEMENT CALCULATION
(K) (D.F) = (A.FC)

8) CALCULATION OF FINAL MEMBER FORCES
(A.M) = (AML) + (SM) (D.M)

9) PLOTING OF BMD AND SFD

2)



1) DOKI = 2

2) Formulation of member stiffness matrix

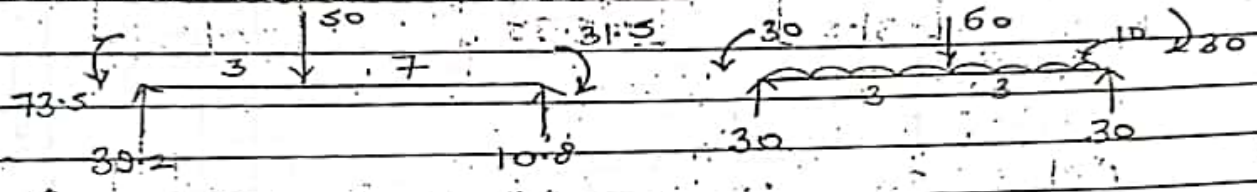
	4	3	5	1	
$SM_1 =$	0.012	0.06	-0.012	0.06	4
	0.06	0.4	-0.06	0.2	3
	-0.012	-0.06	0.012	-0.06	5
	0.06	0.2	-0.06	0.4	1

	5	1	6	2	
$SM_2 =$	0.056	0.167	−0.056	0.167	5
	0.167	0.67	+0.167	0.33	1
	−0.056	−0.167	0.056	−0.167	6
	0.167	0.33	−0.167	0.67	2

3) Global stiffness matrix:

$$(K)_{2\times 2} = \begin{bmatrix} 1.07 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

4) Global load matrix:



$$AML_1 = \begin{bmatrix} 39.2 \\ 73.5 \\ 10.8 \\ -31.5 \end{bmatrix} \qquad AML_2 = \begin{bmatrix} 30 \\ 30 \\ 30 \\ -30 \end{bmatrix}$$

$$AFc = \begin{bmatrix} +1.5 \\ +30 \end{bmatrix}$$

5) Displacement calculations

$$(K)(DF) = AFc$$

$$\begin{bmatrix} 1.07 & 0.33 \\ 0.33 & 0.67 \end{bmatrix} EI \begin{bmatrix} DF_1 \\ DF_2 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 30 \end{bmatrix}$$

$$DF_1 = \frac{-14.63}{EI} \qquad DF_2 = 51.98 \approx \frac{52}{EI}$$

6) Final member forces:

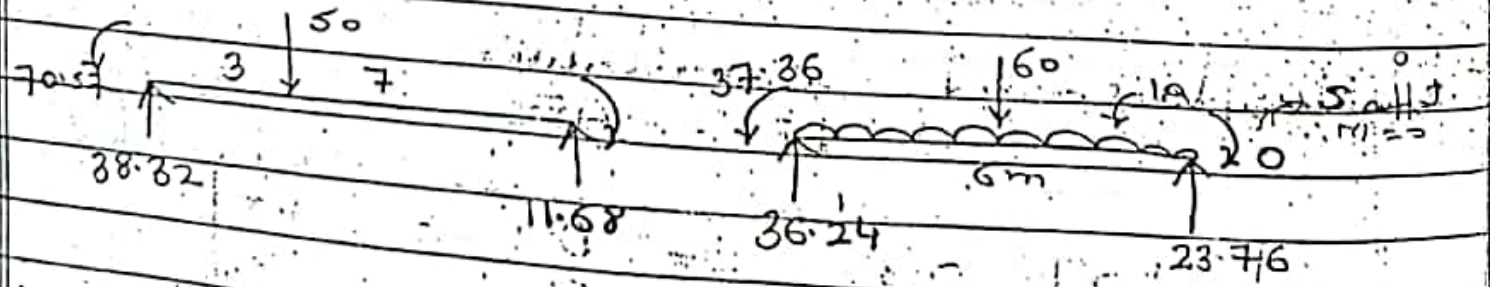
$AM_1 = (AML_1) + (SM_1) \quad (DM_1)$ are reactions

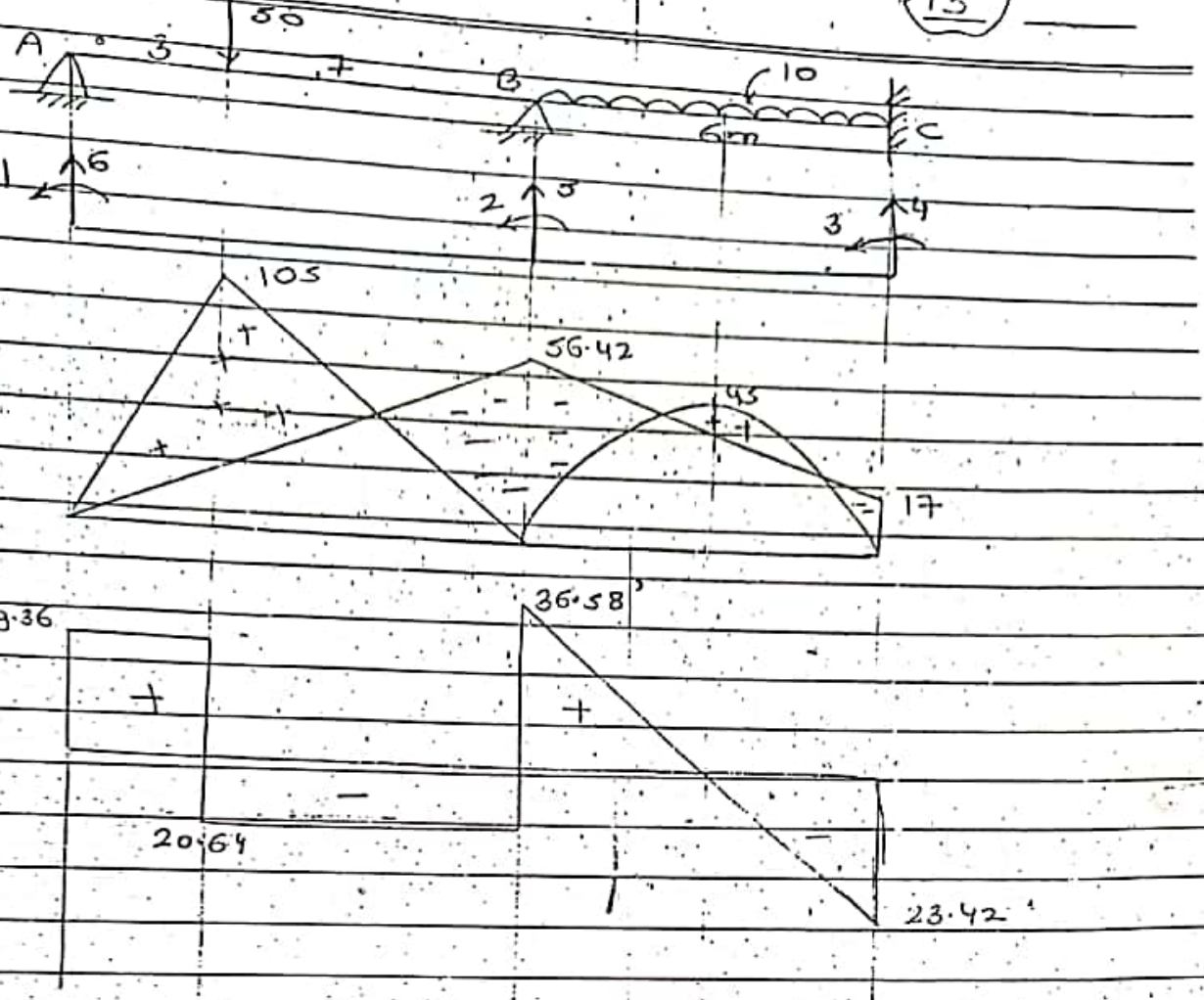
$$AM_1 = \begin{bmatrix} 39.2 \\ 73.5 \\ 10.8 \\ -31.5 \end{bmatrix} + \begin{bmatrix} 0.012 & 0.06 & -0.012 & 0.06 \\ 0.06 & 0.04 & -0.06 & 0.2 \\ -0.012 & -0.06 & 0.012 & -0.06 \\ 0.06 & 0.2 & -0.06 & 0.4 \end{bmatrix} EJ \begin{bmatrix} 0 \\ 0 \\ 0 \\ -14.63 \end{bmatrix} \begin{matrix} 4 \\ 3\frac{1}{6} \\ 5 \\ 1 \end{matrix}$$

$$= \begin{bmatrix} 39.2 \\ 73.5 \\ 10.8 \\ -31.5 \end{bmatrix} + \begin{bmatrix} -0.88 \\ -2.93 \\ +0.88 \\ -5.85 \end{bmatrix} = \begin{bmatrix} 38.32 \\ 70.57 \\ 11.68 \\ -37.35 \end{bmatrix}$$

$$AM_2 = \begin{bmatrix} 30 \\ 30 \\ 30 \\ -30 \end{bmatrix} + \begin{bmatrix} 0.056 & 0.167 & -0.056 & 0.167 \\ 0.167 & 0.67 & -0.167 & 0.33 \\ -0.056 & -0.167 & 0.056 & -0.167 \\ +0.167 & 0.33 & -0.167 & 0.67 \end{bmatrix} EJ \begin{bmatrix} 0 \\ -14.63 \\ 0 \\ 52 \end{bmatrix} \begin{matrix} 5 \\ 1\frac{1}{6} \\ 6 \\ 2 \end{matrix}$$

$$= \begin{bmatrix} 30 \\ 30 \\ 30 \\ -30 \end{bmatrix} + \begin{bmatrix} 6.24 \\ 7.36 \\ -6.24 \\ 30 \end{bmatrix} = \begin{bmatrix} 36.24 \\ 37.36 \\ 23.76 \\ 0 \end{bmatrix}$$





1) DOKI = 2

2) Formulation of member stiffness matrix:

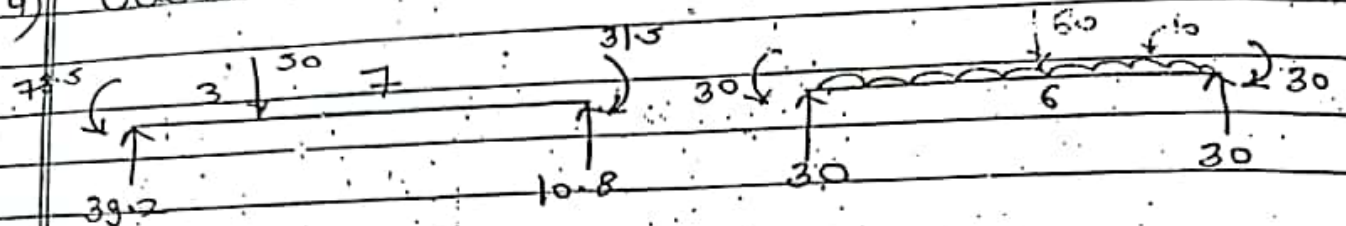
$$SM_1 = EI \begin{bmatrix} 6 & 1 & 5 & 2 \\ 0.012 & 0.06 & -0.012 & 0.06 \\ 0.06 & 0.4 & -0.06 & 0.2 \\ -0.012 & -0.06 & 0.012 & -0.06 \\ 0.06 & 0.2 & -0.06 & 0.4 \end{bmatrix} \begin{matrix} 6 \\ 1 \\ 5 \\ 2 \end{matrix}$$

$$SM_2 = \begin{bmatrix} 5 & 2 & 4 & 3 \\ 0.056 & 0.167 & -0.056 & 0.167 \\ 0.167 & 0.67 & -0.67 & 0.33 \\ -0.056 & -0.167 & 0.056 & -0.167 \\ 0.167 & 0.33 & -0.167 & 0.67 \end{bmatrix} \begin{matrix} 5 \\ 2 \\ 4 \\ 3 \end{matrix}$$

3) Global stiffness matrix

$$(K)_{2\times2} = \begin{bmatrix} 0.04 & 0.2 \\ 0.2 & 1.07 \end{bmatrix}$$

4) Global load matrix



$$AML_1 = \begin{bmatrix} 39.2 \\ 73.5 \\ 10.8 \\ -31.5 \end{bmatrix} \qquad AML_2 = \begin{bmatrix} 30 \\ 30 \\ 30 \\ -30 \end{bmatrix}$$

$$AFc = \begin{bmatrix} -73.5 \\ +1.5 \end{bmatrix}$$

5) Displacement calculations

$$(K)(DF) = AFc$$

$$\begin{bmatrix} 0.04 & 0.2 \\ 0.2 & 1.07 \end{bmatrix} \begin{bmatrix} DF_1 \\ DF_2 \end{bmatrix} = \begin{bmatrix} -73.5 \\ 1.5 \end{bmatrix}$$

$$DF_1 = \frac{-203.46}{EI} \qquad DF_2 = \frac{39.43}{EI}$$

6) Final member forces:

$$(AM_1) = (AML_1) + (SM_1)(DM_1)$$

Ffi Page 14 Date

$AM_1 =$

39.2		0.012	0.06	−0.012	0.06		0	6
73.5	+	0.06	0.4	−0.06	0.2	E/I +	−203.46	1 1/4
10.8		−0.012	−0.06	0.012	−0.06		0	5
−31.5		0.06	0.2	−0.06	0.4		39.43	2

$=$

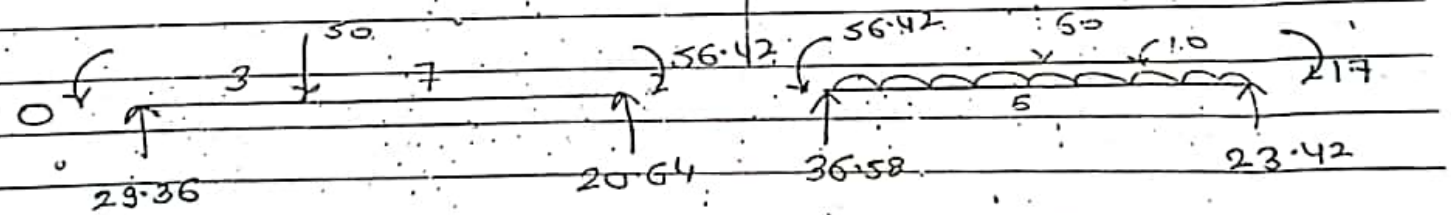
39.2		−9.84		29.36
73.5	+	−73.5	=	0
10.8		9.84		20.64
−31.5		−24.92		−56.42

$AM_2 =$

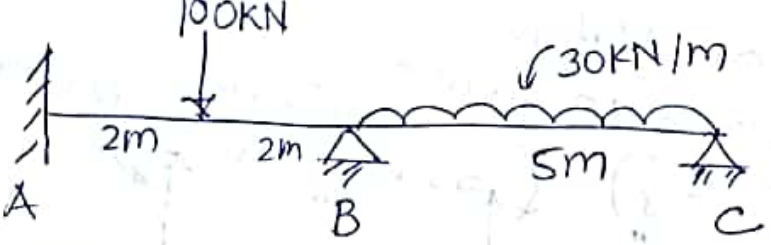
30		0.056	0.167	−0.056	0.167		0	5
30	+	0.167	0.67	−0.167	0.33	E/I	39.43	2 1/4
30		−0.056	−0.167	0.056	−0.167		0	4
−30		0.167	0.33	−0.167	0.67		0	3

$=$

30		6.58		36.58
30	+	26.42	=	56.42
30		−6.58		23.42
−30		13.01		−17



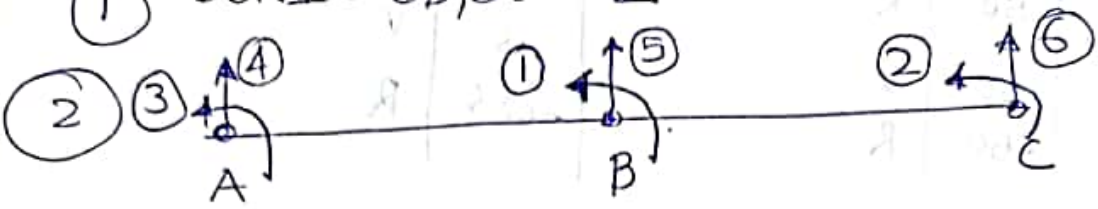
S/19



Take EI as constant. Find unknown & Draw SFD BMD

Soln:-

(1) DOKI = $\theta_B, \theta_C = 2$.



(3) Formation of stiffness matrix for AB.

$$SM_1 = EI \begin{bmatrix} 12/L^3 & 6/L^2 & -12/L^3 & 6/L^2 \\ 6/L^2 & 4/L & -6/L^2 & 2/L \\ -12/L^3 & -6/L^2 & 12/L^3 & -6/L^2 \\ 6/L^2 & 2/L & -6/L^2 & 4/L \end{bmatrix} \begin{matrix} ④ \\ ③ \\ ⑤ \\ ① \end{matrix}$$

(④ ③ ⑤ ①)

$$= EI \begin{bmatrix} 0.18 & 0.37 & -0.18 & 0.37 \\ 0.37 & 1 & -0.37 & 0.50 \\ -0.18 & -0.37 & 0.18 & -0.37 \\ 0.37 & 0.50 & -0.37 & 1 \end{bmatrix} \begin{matrix} ④ \\ ③ \\ ⑤ \\ ① \end{matrix}$$

(④ ③ ⑤ ①)

IIrly for $SM_2 = EI$

$$\begin{bmatrix} 0.096 & 0.24 & -0.096 & 0.24 \\ 0.24 & 0.80 & -0.24 & 0.4 \\ -0.096 & -0.24 & 0.096 & -0.24 \\ 0.24 & 0.40 & -0.24 & 0.80 \end{bmatrix} \begin{matrix} ⑤ \\ ① \\ ⑥ \\ ② \end{matrix}$$

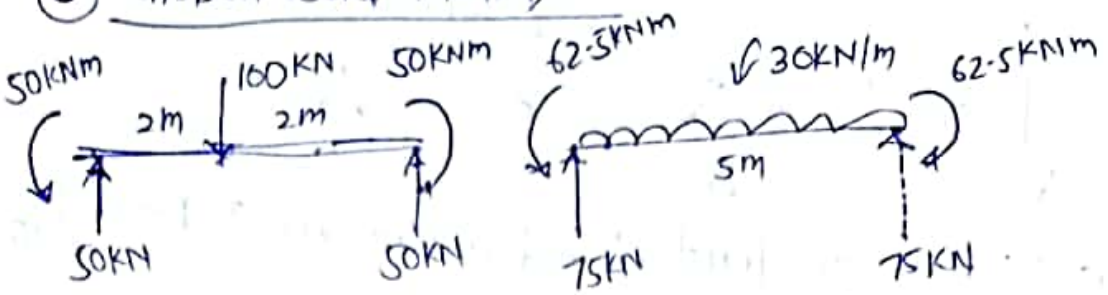
(⑤ ① ⑥ ②)

(4) Global stiffness matrix :-

$$(K)_{2\times 2} = \begin{bmatrix} 1.80 & 0.40 \\ 0.4 & 0.80 \end{bmatrix} \begin{matrix} ① \\ ② \end{matrix}$$

(① ②)

⑤ Global load Matrix



$$AML_1 = \begin{bmatrix} 50 \\ 50 \\ 50 \\ -50 \end{bmatrix} \begin{matrix} V \\ R \\ V \\ R \end{matrix} \qquad AML_2 = \begin{bmatrix} 75 \\ 62.5 \\ 75 \\ -62.5 \end{bmatrix} \begin{matrix} V \\ R \\ V \\ R \end{matrix}$$

AFc = Action at free coordinates.

$$AF_c = \begin{bmatrix} -12.5 \\ \hline 62.5 \end{bmatrix} \begin{matrix} ① \\ \hline ② \end{matrix}$$

⑥ Displacement calculation

$$\{K\}\{DF\} = AFC$$

$$EI \begin{bmatrix} 180 & 0.40 \\ 0.40 & 0.80 \end{bmatrix} \begin{Bmatrix} DF_1 \\ DF_2 \end{Bmatrix} = \begin{Bmatrix} -12.5 \\ 62.5 \end{Bmatrix}$$

$DF_1 = -875/32 = -27.34/EI$

$DF_2 = 5875/64 = 91.79/EI$

⑦ Final Reaction & Moment calculation.

$$AM_1 = \begin{bmatrix} 50 \\ 50 \\ 50 \\ -50 \end{bmatrix} + \begin{bmatrix} 0.18 & 0.37 & -0.18 & 0.37 \\ 0.37 & 1 & -0.37 & 0.50 \\ -0.18 & -0.37 & 0.18 & -0.37 \\ 0.37 & 0.50 & -0.37 & 1 \end{bmatrix} \begin{matrix} ④ \\ ③ \\ ⑤ \\ ① \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -27.34 \end{bmatrix}$$

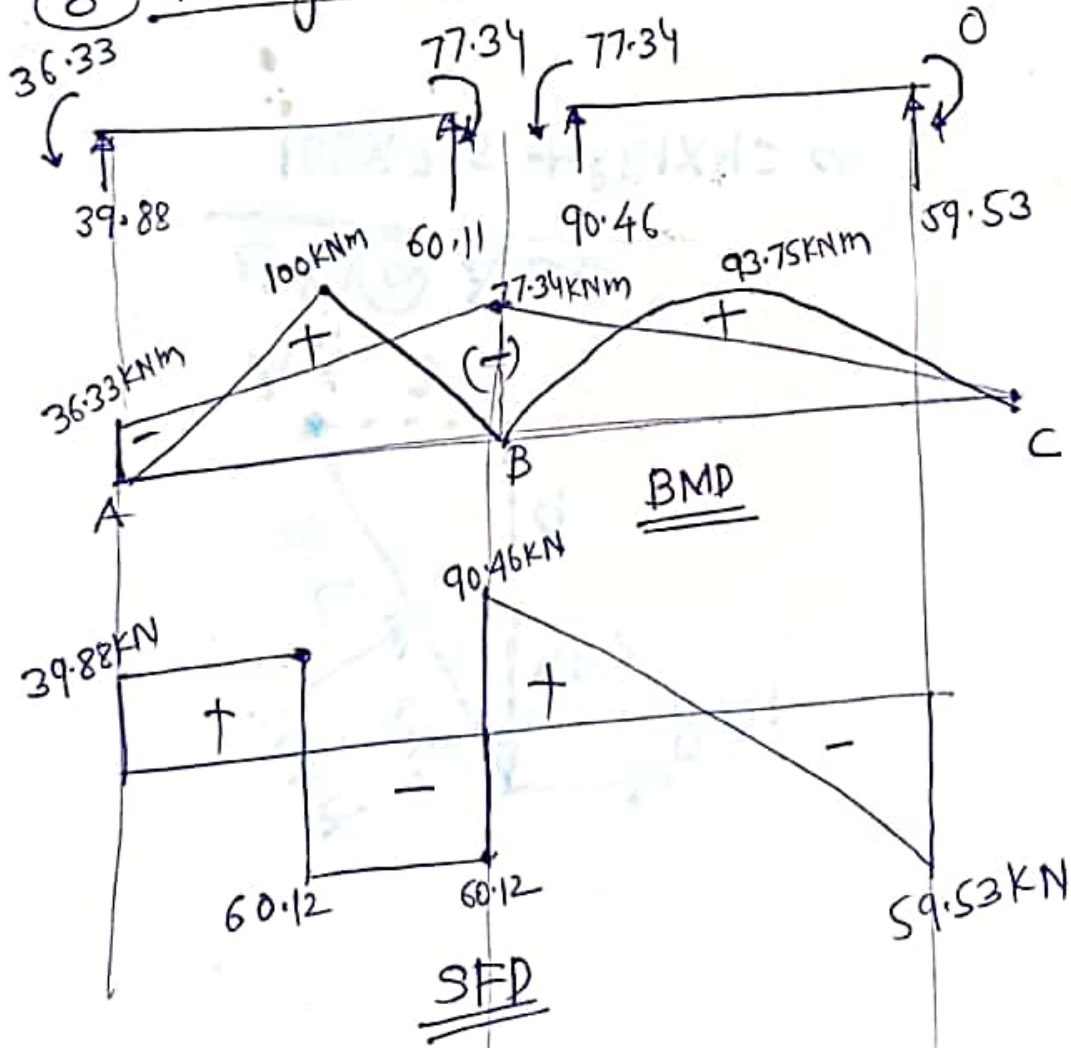
$$AM_1 = \begin{bmatrix} 39.88 \\ 36.33 \\ 60.11 \\ -77.34 \end{bmatrix} \begin{matrix} V \\ R \\ V \\ R \end{matrix}$$

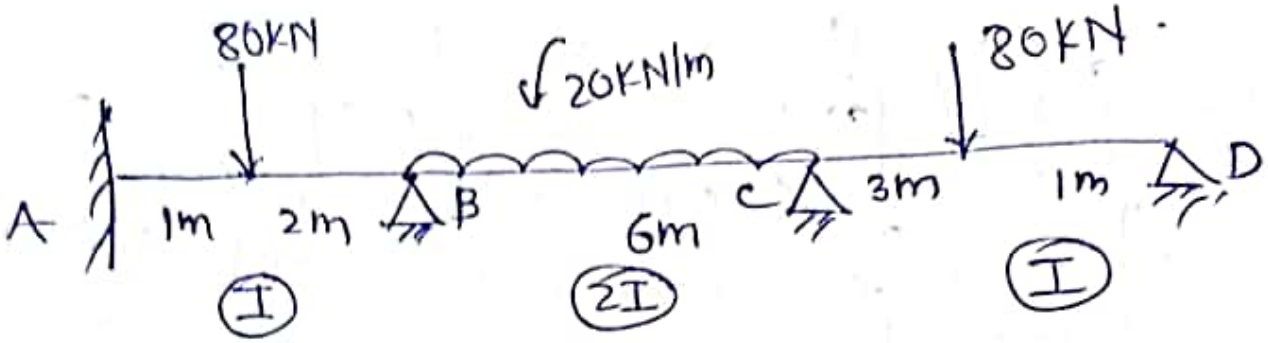
Hrly

$$AM_2 = \begin{bmatrix} 75 \\ 62.5 \\ 75 \\ -62.5 \end{bmatrix} + \begin{bmatrix} 0.096 & 0.24 & -0.096 & 0.24 \\ 0.24 & 0.80 & -0.24 & 0.4 \\ -0.096 & -0.24 & 0.096 & -0.24 \\ 0.24 & 0.40 & -0.24 & 0.80 \end{bmatrix} \begin{matrix} (5) \\ (1) \\ (6) \\ (2) \end{matrix} \begin{bmatrix} 0 \\ -27.34 \\ 0 \\ 91.79 \end{bmatrix} \begin{matrix} (5) \\ (1) \\ (6) \\ (2) \end{matrix}$$

$$= \begin{bmatrix} 90.46 \\ 77.34 \\ 59.53 \\ -4 \times 10^{-3} \end{bmatrix} \begin{matrix} V \\ R \\ V \\ R \end{matrix}$$

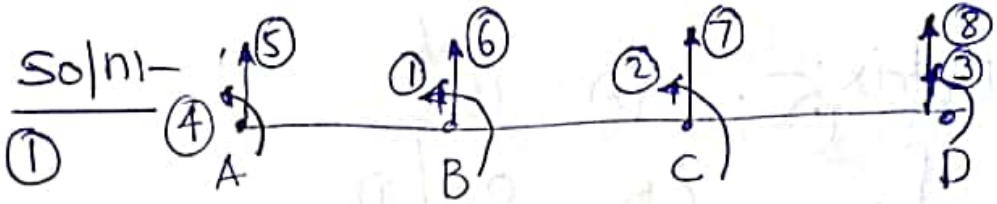
(8) Plotting of BMD & SFD





Here DOKI = $\odot_B, \odot_C, \odot_D = 3$

Soln-



unknown DOF OR DOKI is at 1, 2, 3

(2) $SM_1 = EI$

	⑤	④	⑥	①	
	0.44	0.66	-0.44	0.66	⑤
	0.66	1.3	-0.66	0.66	④
	-0.44	-0.66	0.44	-0.66	⑥
	0.66	0.66	-0.66	1.3	①

$SM_2 = EI \times I$

	⑥	①	⑦	②	
	0.055	0.166	-0.055	0.166	⑥
	0.166	0.66	-0.166	0.33	①
	-0.055	-0.166	0.055	-0.166	⑦
	0.166	0.33	-0.166	0.66	②

$SM_3 = EI$

	⑦	②	⑧	③	
	0.18	0.37	-0.18	0.37	⑦
	0.37	1	-0.37	0.50	②
	-0.18	-0.37	0.18	-0.37	⑧
	0.37	0.50	-0.37	1	③

As Member BC contains 2I. therefore SM_2 will be

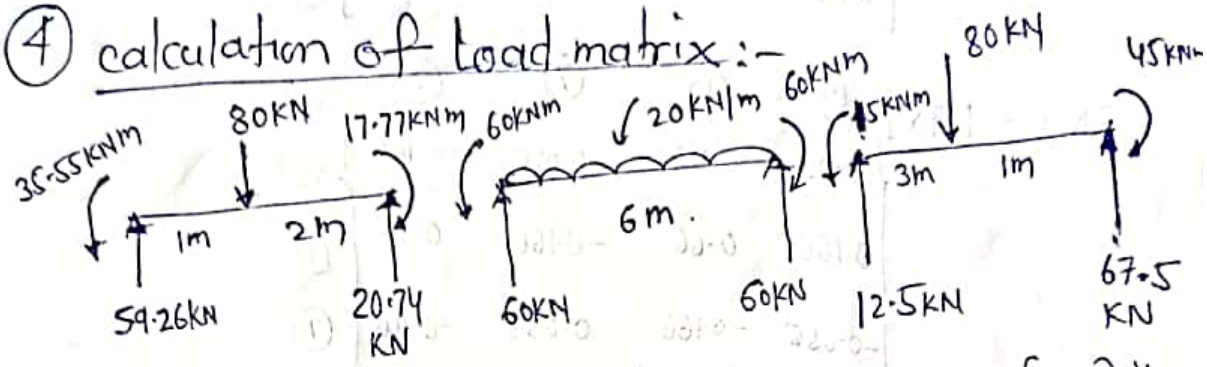
$SM_2 = EI$

	⑥	①	⑦	②	
	0.11	0.33	-0.11	0.33	⑥
	0.33	1.32	-0.33	0.66	①
	-0.11	-0.33	0.11	-0.33	⑦
	0.33	0.66	-0.33	1.32	②

③ Global Matrix :-

$$K_{3\times3} = EI\begin{bmatrix} 2.62 & 0.66 & 0 \\ 0.66 & 1.32 & 0.50 \\ 0 & 0.50 & 1 \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \end{matrix}$$

④ calculation of load matrix :-



$$AML_1 = \begin{bmatrix} 59.26 \\ 35.55 \\ 20.74 \\ -17.77 \end{bmatrix} \begin{matrix} V \\ R \\ V \\ R \end{matrix} \qquad AML_2 = \begin{bmatrix} 60 \\ 60 \\ 60 \\ -60 \end{bmatrix} \begin{matrix} V \\ R \\ V \\ R \end{matrix} \qquad AML_3 = \begin{bmatrix} 12.5 \\ 15 \\ 67.5 \\ -45 \end{bmatrix} \begin{matrix} V \\ R \\ V \\ R \end{matrix}$$

⑤ Displacement calculation

$$\{k\}\{DF\} = \{AFC\}$$

$$\therefore EI\begin{bmatrix} 2.62 & 0.66 & 0 \\ 0.66 & 1.32 & 0.50 \\ 0 & 0.50 & 1 \end{bmatrix} \begin{bmatrix} DF_1 \\ DF_2 \\ DF_3 \end{bmatrix} = \begin{bmatrix} -42.23 \\ 45 \\ 45 \end{bmatrix}$$

$$AF_C = \begin{bmatrix} -42.23 \\ 45 \\ 45 \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \end{matrix}$$

$DF_1 = -25.35/EI$

$DF_2 = 36.66/EI$

$DF_3 = 26.66/EI$

(6) Final reaction & moment calculation

$$AM_1 = \{AML_1\} + \{SM_1\}\{DF\}$$

$$AM_1 = \begin{Bmatrix} 59.26 \\ 35.55 \\ 20.74 \\ -17.77 \end{Bmatrix} + \begin{bmatrix} 0.44 & 0.66 & -0.44 & 0.66 \\ 0.66 & 1.3 & -0.66 & 0.66 \\ -0.44 & -0.66 & 0.44 & -0.66 \\ 0.66 & 0.66 & -0.66 & 1.3 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -25.35 \end{Bmatrix}$$

(⑤ ④ ⑥ ①)

$$AM_1 = \begin{Bmatrix} 42.52 \\ 18.81 \\ 37.47 \\ -50.72 \end{Bmatrix} \begin{matrix} V \\ R \\ V \\ R \end{matrix}$$

$$AM_2 = \{AML_2\} + \{SM_2\}\{DF\}$$

$$AM_2 = \begin{Bmatrix} 60 \\ 60 \\ 60 \\ -60 \end{Bmatrix} + \begin{bmatrix} 0.11 & 0.33 & -0.11 & 0.33 \\ 0.33 & 1.32 & -0.33 & 0.66 \\ -0.11 & -0.33 & 0.11 & -0.33 \\ 0.33 & 0.66 & -0.33 & 1.32 \end{bmatrix} \begin{Bmatrix} 0 \\ -25.35 \\ 0 \\ 36.66 \end{Bmatrix}$$

(⑥ ① ⑦ ②)

$$AM_2 = \begin{Bmatrix} 63.73 \\ 50.73 \\ 56.26 \\ -28.33 \end{Bmatrix} \begin{matrix} V \\ R \\ V \\ R \end{matrix}$$

$$AM_3 = \{AML_3\} + [SM_3]\{DF\}$$

$$= \begin{Bmatrix} 12.5 \\ 15 \\ 67.5 \\ -45 \end{Bmatrix} + \begin{bmatrix} 0.18 & 0.37 & -0.18 & 0.37 \\ 0.37 & 1 & -0.37 & 0.50 \\ -0.18 & -0.37 & 0.18 & -0.37 \\ 0.37 & 0.50 & -0.37 & 1 \end{bmatrix} \begin{matrix} \textcircled{7} \\ \textcircled{2} \\ \textcircled{8} \\ \textcircled{3} \end{matrix} \begin{Bmatrix} 0 \\ 36.66 \\ 0 \\ 26.66 \end{Bmatrix}$$

STIFFNESS MATRIX

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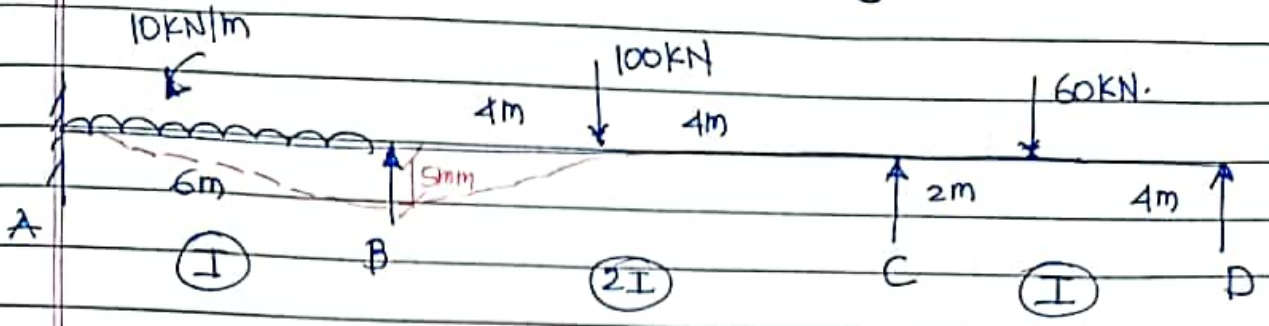
2D Beam numerical Based on sinking of support.



Support B is sink by 5mm

E = 200 GPA.

I = 3×10^8 mm^4.

Soln:-

E = 200×10^9 N/m^2

Step(1) = $200 \times 10^3 / 10^9$

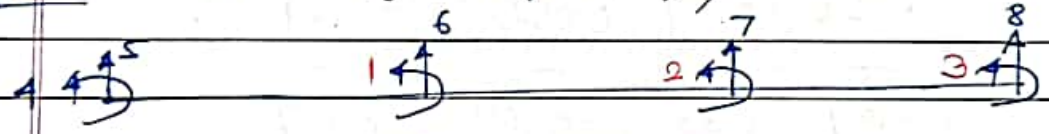
$\therefore$ E = 200×10^5 KN/m^2

Also I = $3 \times 10^8 \times 10^{-12}$

I = 3×10^{-4} m^4.

} Initially we can convert the EI unit in the form of unit given in diagram i.e for loading KN & span (m).

Step(2) DOKI = 3. (i.e at pt B, C, D)



unknown to be find at 1, 2, 3

Step(3) Formation of stiff matrix for members.

SM_1 =

	⑤	④	⑥	①	
	0.056	0.167	-0.056	0.167	⑤
EI	0.167	0.67	-0.167	0.33	④
	-0.056	-0.167	0.056	-0.167	⑥
	0.167	0.33	-0.167	0.67	①

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$SM_2 =$ EI

⑥	①	⑦	⑦	
0.047	0.19	-0.047	0.19	⑥
0.19	1	-0.19	0.5	①
-0.047	-0.19	0.047	-0.19	⑦
0.19	0.5	-0.19	1	②

$SM_3 =$ EI

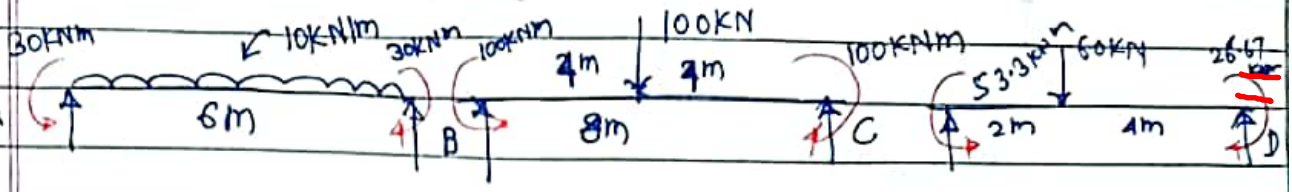
⑦	②	⑧	③	
0.056	0.167	-0.056	0.167	⑦
0.167	0.67	-0.167	0.33	②
-0.056	-0.167	0.056	-0.167	⑧
0.167	0.33	-0.167	0.67	③

Step 4 Global Matrix

K(3x3)

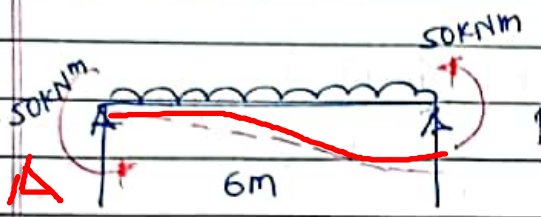
①	②	③	
1.67	0.5	0	①
0.5	1.67	0.33	②
0	0.33	0.67	③

Steps Global Load matrix

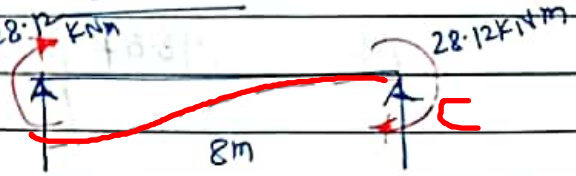


Now here due to sinking of support we have to consider sinking effect on support B for left & right case.

Case I



Case II



Always remember when right support sink sign convention will be clockwise & vise versa for left support sinking.

calculation of moment due to sinking.

$$= \frac{6EI\Delta}{l^2}$$

$$= \frac{6 \times 200 \times 10^6 \times 3 \times 10^{-4} \times 5 \times 10^{-3}}{6^2}$$

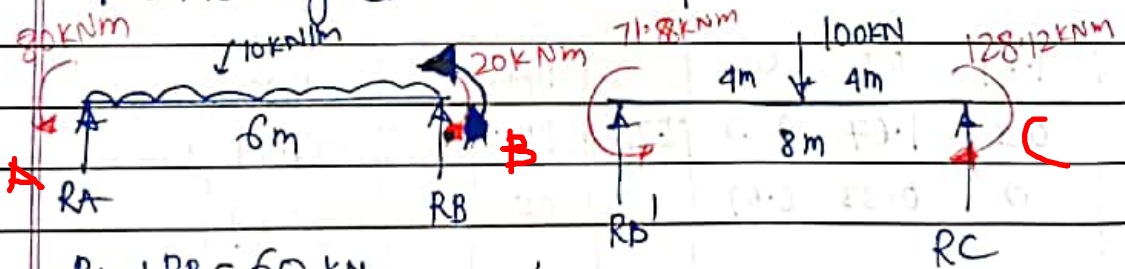
$$= 50\ KNm.$$

$$= 6EI\Delta / l^2$$

$$= \frac{6 \times 200 \times 10^5 \times 3 \times 10^{-4} \times 5 \times 10^{-3}}{8^2}$$

$$= 28.12\ KNm$$

Final loading & moment on spans.



$R_A + R_B = 60\ KN$

$\sum m @ A = 0$

$-80 + 10 \times 6 \times 3 - 20 = R_B \times 6$

$R_B = 13.33$ ↑ KN.

$R_A = 46.67$ ↑ KN.

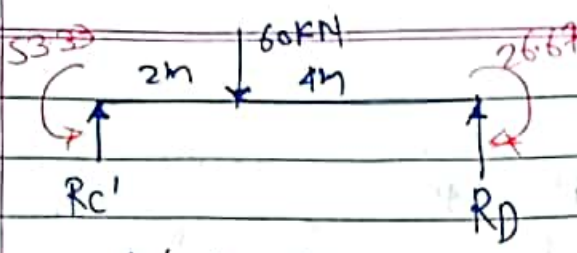
$R_{B'} + R_C = 100\ KN$

$\sum m @ B' = 0$

$-71.88 + 100 \times 4 + 128.12 = R_C \times 8$

$R_C = 57.03\ KN$ ↑

& $R_{B'} = 42.97\ KN$ ↑



$R_C' + R_D = 60$

Em@ c' = 0

$-53.33 + 60 \times 2 + 26.67 = R_D \times 6$

$R_D = 15.55$ KN ✓

$R_C' = 44.44$ KN. ✓

$$AML_1 = \begin{Bmatrix} 46.67 \\ 80 \\ 13.33 \\ 20 \end{Bmatrix} \quad AML_2 = \begin{Bmatrix} 42.97 \\ 71.88 \\ 57.03 \\ -128.12 \end{Bmatrix} \quad AML_3 = \begin{Bmatrix} 44.44 \\ 53.33 \\ 15.55 \\ -26.67 \end{Bmatrix}$$

Step 6 Displacement coordinate:-

$$AF_C = \begin{Bmatrix} -91.88 \\ 74.79 \\ 26.67 \end{Bmatrix}$$

Step 7 Displacement calculation

$$[K]\{DF\} = \{AF_C\}$$

$$\begin{Bmatrix} 1.67 & 0.5 & 0 \\ 0.5 & 1.67 & 0.33 \\ 0 & 0.33 & 0.67 \end{Bmatrix} EI \begin{Bmatrix} DF_1 \\ DF_2 \\ DF_3 \end{Bmatrix} = \begin{Bmatrix} -91.88 \\ 74.79 \\ 26.67 \end{Bmatrix}$$

$DF_1 = -74.67/EI$ ✓

$DF_2 = 65.66/EI$ ✓

$DF_3 = 7.46/EI$ ✓

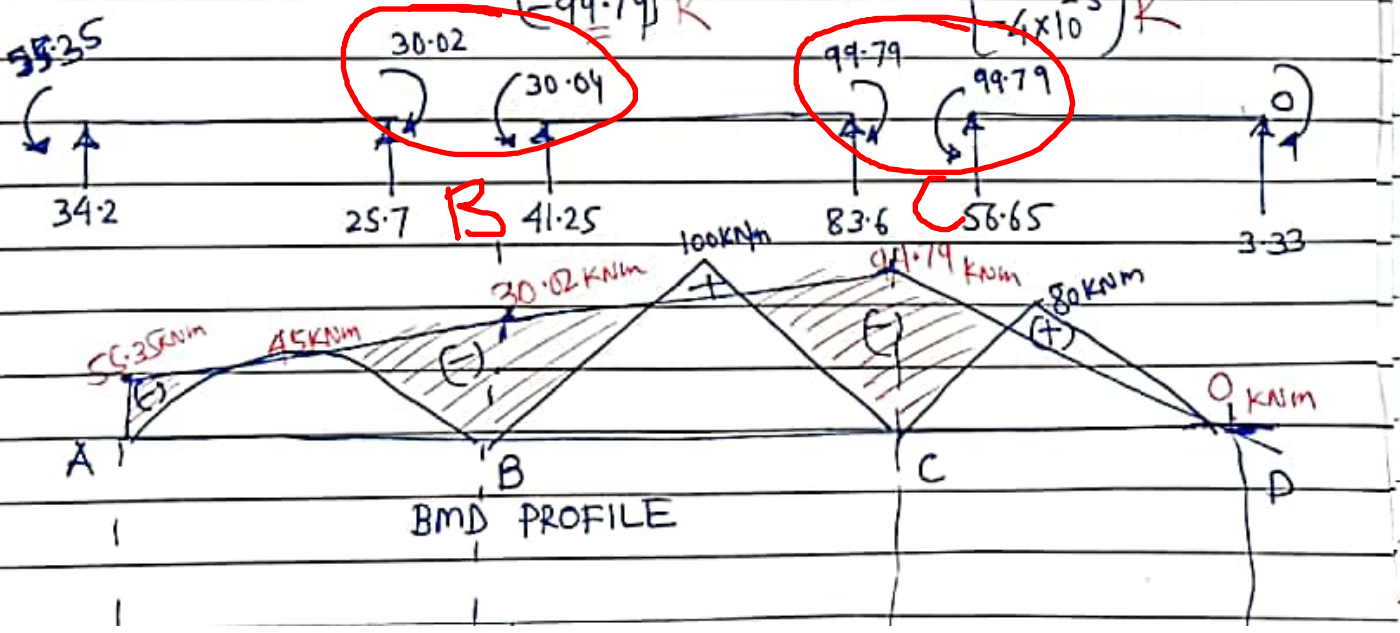
Step 8 Final member forces

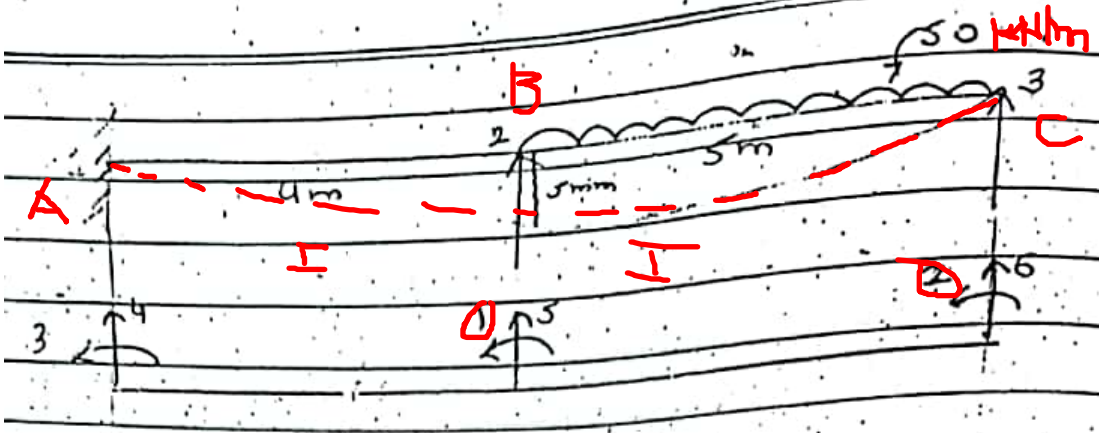
$$AM = AML + (SM)(DF)$$

$$AM_1 = \begin{bmatrix} 46.67 \\ 80 \\ 13.33 \\ 20 \end{bmatrix} + \begin{bmatrix} 0.056 & 0.167 & -0.056 & 0.167 \\ 0.167 & 0.67 & -0.167 & 0.33 \\ -0.056 & -0.167 & 0.056 & -0.167 \\ 0.167 & 0.33 & -0.167 & 0.67 \end{bmatrix} \begin{matrix} ⑤ \\ ④ \\ ⑥ \\ ① \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -74.67 \end{bmatrix} \begin{matrix} ⑤ \\ ④ \\ ⑥ \\ ① \end{matrix} \quad EI$$

$$AM_1 = \begin{bmatrix} 34.2 \\ 55.35 \\ 25.7 \\ -30.02 \end{bmatrix} \begin{matrix} V \\ R \\ V \\ R \end{matrix}$$

Similarly $AM_2 = \begin{bmatrix} 41.25 \\ 30.04 \\ 83.6 \\ -99.79 \end{bmatrix} \begin{matrix} V \\ R \\ V \\ R \end{matrix}$ & $A_3 = \begin{bmatrix} 56.65 \\ 99.79 \\ 3.33 \\ -4\times10^{-3} \end{bmatrix} \begin{matrix} V \\ R \\ V \\ R \end{matrix}$





$E_s = 25.5 \times 10^6 \ kN/m^2$

$b = 0.3m$, $d = 0.6m$

$I = \frac{bd^3}{12} = 5.4 \times 10^{-3} \ m^4$

$EI = 137.7 \times 10^3 \ kN/m^2$

DOKJ = 2

Formulation of stiffness matrix:

	2	4	3	5	1	
$SM_1 =$	0.19	0.375	-0.19	0.375	4	
	0.375	1	-0.375	0.5	3	EI
	-0.19	-0.375	0.19	-0.375	5	
	0.375	0.5	-0.375	1	1	

	5	1	6	2	
SM_2 =	0.096	0.24	−0.096	0.24	5
	0.24	1.25 (0.8)	−0.24	6.25 (0.4)	1
	−0.096	−0.24	0.096	−0.24	6
	0.24	2.5 (0.4)	−0.24	1.25 (0.8)	2

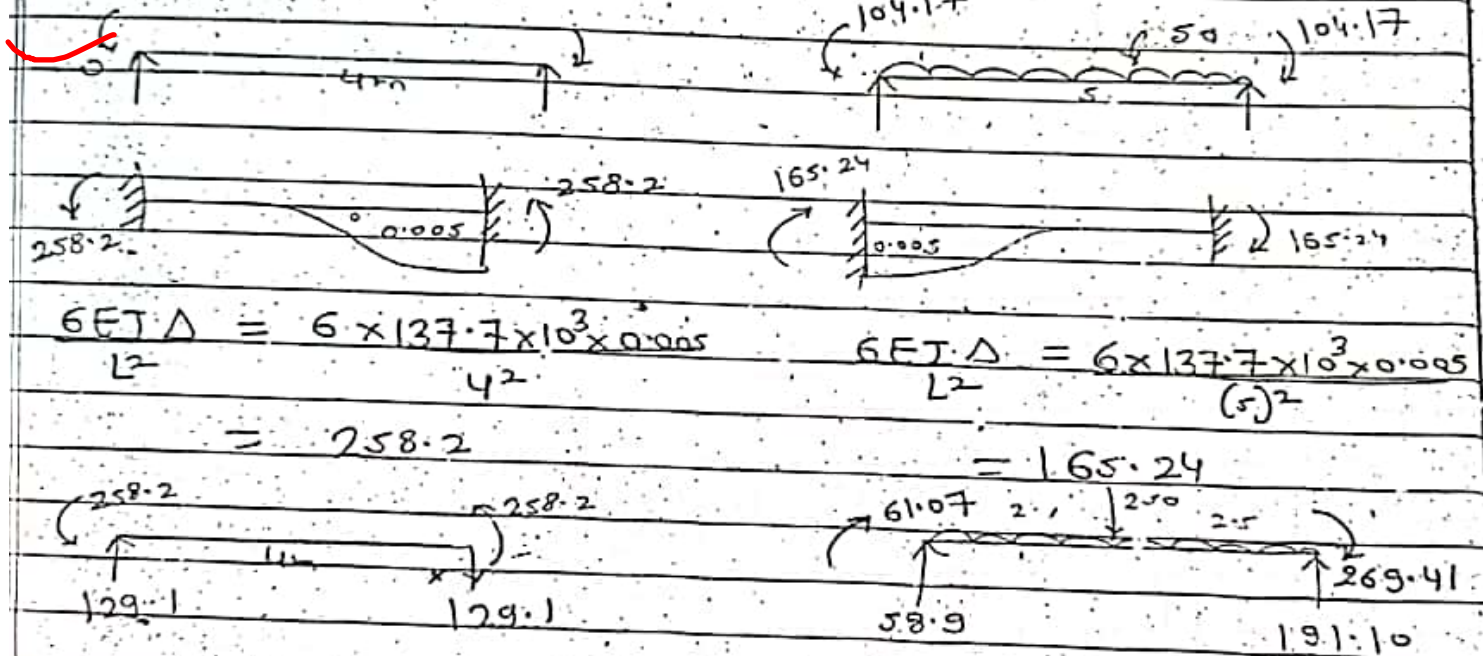
EI

Global stiffness matrix:

$[K]_{2\times2}$ =

	1	2	
	1.8	0.4	1
	0.4	0.8	2

Global load matrix:



$\frac{6EI\Delta}{L^2} = \frac{6\times137.7\times10^3\times0.005}{4^2} = 258.2$

$\frac{6EI\Delta}{L^2} = \frac{6\times137.7\times10^3\times0.005}{(5)^2} = 165.24$

AML_1 =			AML_2 =		
	129.1	V		58.9	V
	+258.2	R		−61.07	R
	−129.1	V		191.10	V
	+258.2	R		−269.41	R

AFC =	
	−197.13
	269.41

Displacement calculation:

(K) (DF) = (AFC)

1.8	0.4		DF1	=	-197.13
0.4	0.8		DF2		269.41

$DF_1 = \frac{-207.4}{EI}$ $DF_2 = \frac{440.46}{EI}$

$DF_1 =$ $DF_3 =$

6) Final member forces:

$AM = AML_1 + (SM)(DF)$

AM_1

4 3 5 1

$AM_1 =$	129.1	+	0.19	0.375	-0.19	0.375	0 4
	258.2		0.375	1	-0.375	0.5	0 3
	-129.1		-0.19	-0.375	0.19	-0.375	0 5
	258.2		0.375	0.5	-0.375	1	-207.4 1

=

51.32	V
154.5	R
-51.32	V
50.8	R

5 1 6 2

$AM_2 =$	58.9	+	0.096	0.24	-0.096	0.24	0 5
	-61.07		0.24	0.8	-0.24	0.4	-207.4 1
	191.10		-0.096	-0.24	0.096	-0.24	0 6
	-269.41		0.24	0.4	0.24	0.8	440.46 2

=

114.83	V
-50.81	R
135.16	V
0	R