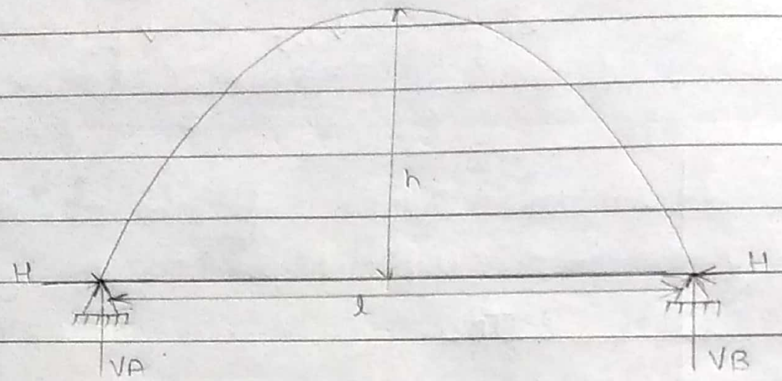
Unit 4 Analysis of Column and Arches [2-3 hinge]

* Analysis of 2 hinge arches



where H = Horizontal Trust

$$H = \int_0^l \frac{My\,dx}{y^2\,dx}$$

where $y = \frac{4hx}{l^2}(l-x)$

$$= \int_0^l \frac{16h^2x^2}{l^4}(l-x)^2\,dx$$

$$= \int_0^l \frac{16h^2x^2}{l^4}(l^2-2lx+x^2)\,dx$$

$$= \frac{16h^2}{l^4}\int_0^l x^2(l^2-2lx+x^2)\,dx$$

$$= \frac{16h^2}{l^2}\int_0^l (x^2l^2-2lx^3+x^4)\,dx$$

$$= \frac{16h^2}{l^4}\; l^2\int_0^l x^2\,dx - 2l\int_0^l x^3\,dx + \int_0^l x^4\,dx$$

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$$= \frac{16h^2}{l^4}\left[l^2 \left(\frac{x^3}{3}\right)_0^l - 2l\left(\frac{x^4}{4}\right)_0^l + \left(\frac{x^5}{5}\right)_0^l \right]$$

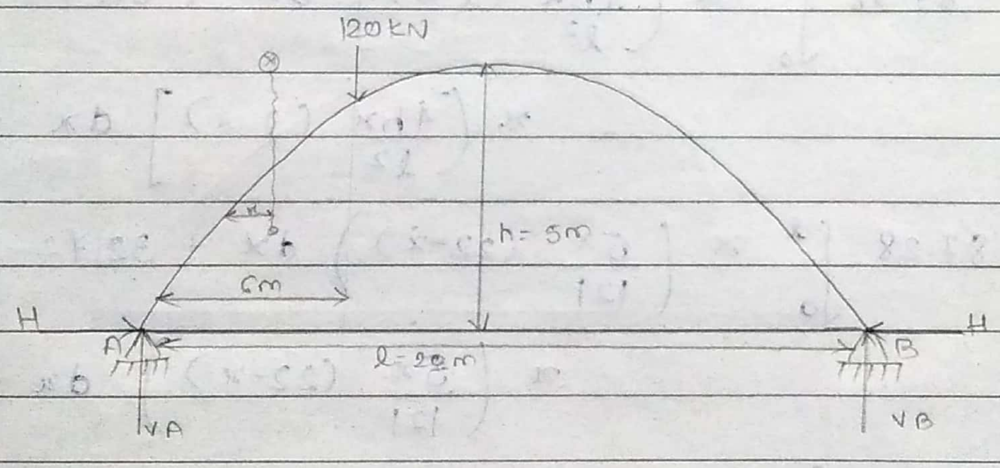
$$= \frac{16h^2}{l^4}\left[l^2 \cdot \frac{l^3}{3} - 2l \times \frac{l^4}{4} + \frac{l^5}{5} \right]$$

$y^2 dx =$

$$= \frac{16h^2}{l^4}\left[\frac{l^5}{3} - \frac{2l^5}{4} + \frac{l^5}{5} \right] = \frac{16h^2 l^5}{l^4}\left(\frac{1}{30}\right) = \frac{8}{15}h^2 l$$

* Numerical *

Q Two hinge parabolic arche of span 22m and the central rise 5m carries the point load of 120 KN, 6m from the left hand support. Analysed the arches and determine the horizontal thrust



$\Sigma F_y = 0$

$V_A + V_B = 120\ KN$

$\Sigma M @ B = 0$

$120 \times 6 - V_A \times 22 = 0$

$V_B = 32.73\ KN$

$V_A = 87.28\ KN$

$y = \frac{4hx}{l^2}(l-x)$

$= \frac{4\times5x}{22^2}(22-x)$

$= \frac{20x}{22^2}(22-x) = \frac{110x-5x^2}{121}$

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Portion	Origin	Limit	M
AC	A	0-6	$87.28x$
BC	B	0-16	$32.78x$

$$H = \int_0^l \frac{My\,dx}{y^2 dx}$$

$$= \int_0^l My\,dx$$

$$= \int_0^l (87.28x)\,y\,dx + \int_0^l (32.78x)\,y\,dx$$

$$= 87.28\int_0^l x\left[\frac{4hx}{l^2}(l-x)\right]dx + 32.72\int_0^l x\left[\frac{4hx}{l^2}(l-x)\right]dx$$

$$= 87.28\int_0^l x\left(\frac{5x}{121}(22-x)\right)dx + 32.72\int_0^l x\left(\frac{5x}{121}(22-x)\right)dx$$

$$= 87.28\int_0^6 x\,(0.90x - 0.04x^2)\,dx + 32.72\int_0^{16} x\,(0.90x - 0.04x^2)\,dx$$

$$= 87.28\left[\int_0^6 0.90x^2\,dx - \int_0^{16} 0.04x^3\,dx\right] + 32.78\left[\int_0^{16} 0.90x^2\,dx - \int_0^{16} 0.04x^3\,dx\right]$$

$$= \int_0^6 87.28x \left(\frac{10x - 5x^2}{121} \right) dx + \int_0^{16} 32.73x \left(\frac{110 - 5x^2}{121} \right) dx$$

$$= \int_0^6 79.34x^2 - 3.6x^3 \, dx + \int_0^{16} 24.75x^2 - 1.33x^3$$

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$$= 87.28 \times 0.90 \left(\frac{x^3}{3} \right)_0^6 - 0.04 \left(\frac{x^4}{4} \right)_0^6 + 32.78 \times 0.90 \left(\frac{x^3}{3} \right)_0^{16} - 0.04 \left(\frac{x^4}{4} \right)_0^{16}$$

$$= 87.28 \times 0.90 \times \left(\frac{6^3}{3} \right) - 0.04 \left(\frac{6^4}{4} \right) + 32.78 \times 0.90 \left(\frac{16^3}{3} \right) - 0.04 \left(\frac{16^4}{4} \right)$$

$$= 87.28 \times 0.90 \times 72 - 0.04 \times 324 + 32.78 \times 0.90 \times 1365.33 - 0.04 \times 16384$$

$$= 4524.39 + 18797.36$$

$$= 23321.95$$

$$\int y^2 \, dx = \frac{8}{15} h^2 \times l$$

$$= \frac{8}{15} \times 5^2 \times 22$$

$$= 293.33$$

$$H = \frac{23321.95}{293.33}$$

$$H = 79.50 \text{ kN}$$

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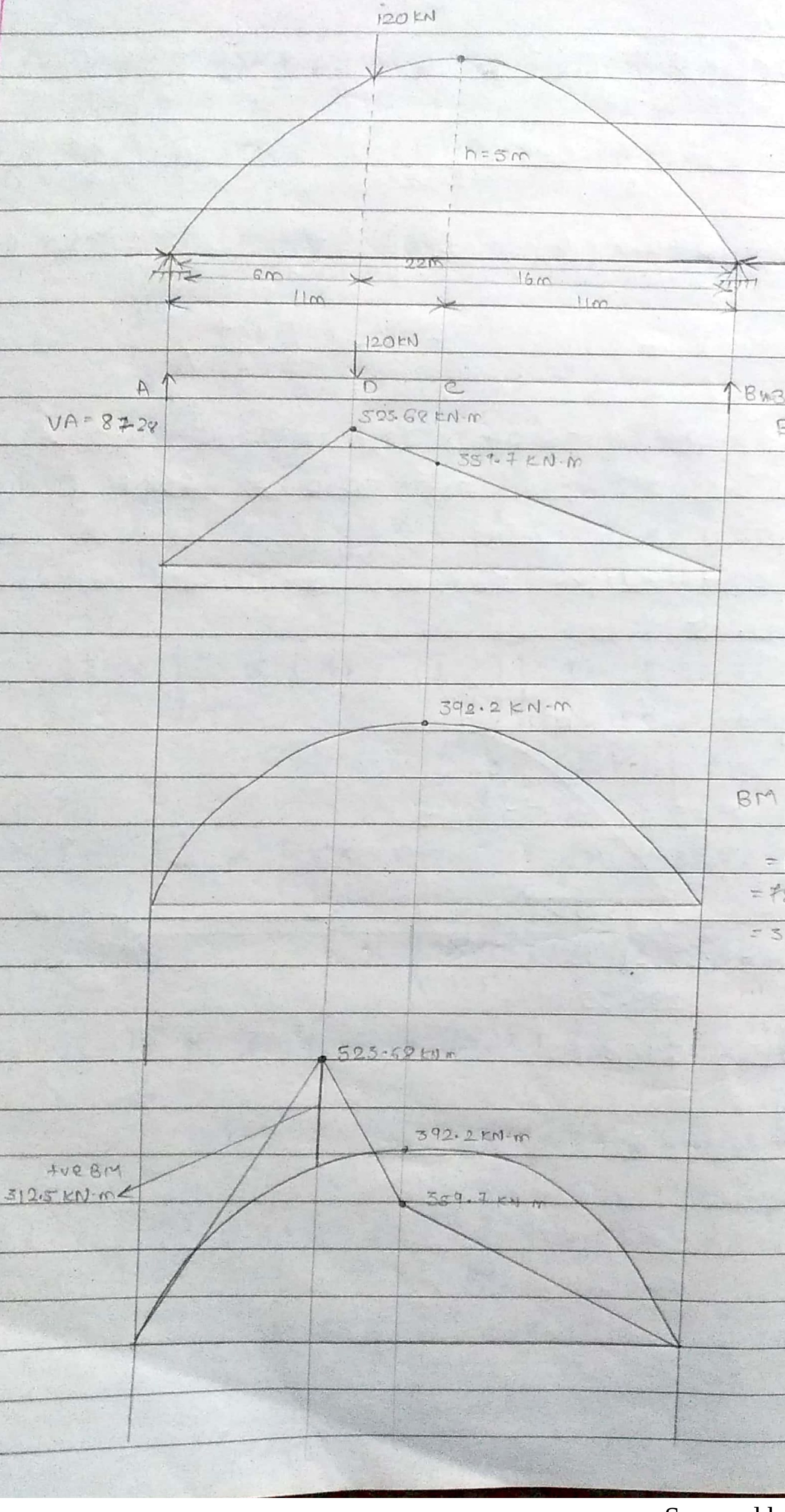


BM for vertical reaction

BM for horizontal reaction

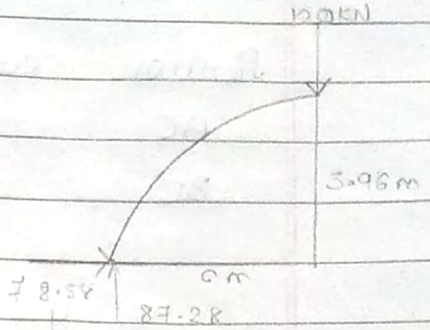
$= H \times h$

$= 78.58 \times 5$

$= 392.2$ kN·m

$$y = \frac{4hx}{l^2}(l-x)$$

$$= \frac{4h \times 6}{22^2}(22-6)$$

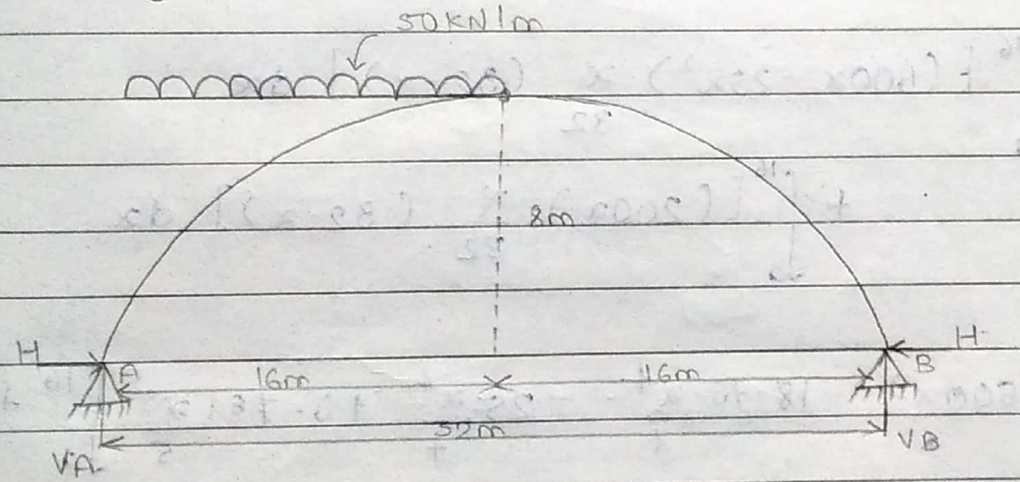


$$y = 3.96\,m$$

Max^m BM $= 87.28 \times 6 - 78.5 \times 3.96$

At A $= 212.5$ KN-m

Q A two hinge parabolic arch of span 22m with a central rise 8m carries the UDL of 50 KN-m over the left half span. Determine the horizontal thrust and plot the BMD profile.



$$\Sigma F_y = 0$$

$$VA + VB - 800 = 0$$

$$VA + VB = 800$$

$$\Sigma M \text{ at } A = 0$$

$$50 \times 16 \times \frac{16}{2} - VB \times 32 = 0$$

$$32\,VB = 6400$$

$$VB = 200\text{ KN}$$

$$VA = 600\text{ KN}$$

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Portion	origin	limit	moment
AC	A	0-16	$600x - \frac{50x^2}{2}$
BC	B	0-16	$200x$

$$y = \frac{4hx}{l^2}(l-x)$$

$$= \frac{4\times 8x}{32^2}(32-x)$$

$$y = \frac{1}{32}x(32-x)$$

$$= \int_0^l My\,dx$$

$$= \int_0^{16}\left[(600x - 25x^2)\frac{x}{32}(32-x)\right]dx$$

$$+ \int_0^{16}\left[(200x)\frac{x}{32}(32-x)\right]dx$$

$$= \left[\frac{600x^3}{3} - 18.75\frac{x^4}{4} - 25\frac{x^4}{4} + 0.781\frac{x^5}{5}\right]_0^{16} +$$

$$\left[200\frac{x^3}{3} - 6.25\frac{x^4}{4}\right]_0^{16}$$

$= 819200 - 307200 - 409.600 + 163787.57 + 273066.67 - 102400$

$= 436854.24$

$$\int y^2\,dx = \frac{8}{15}h^2 l$$

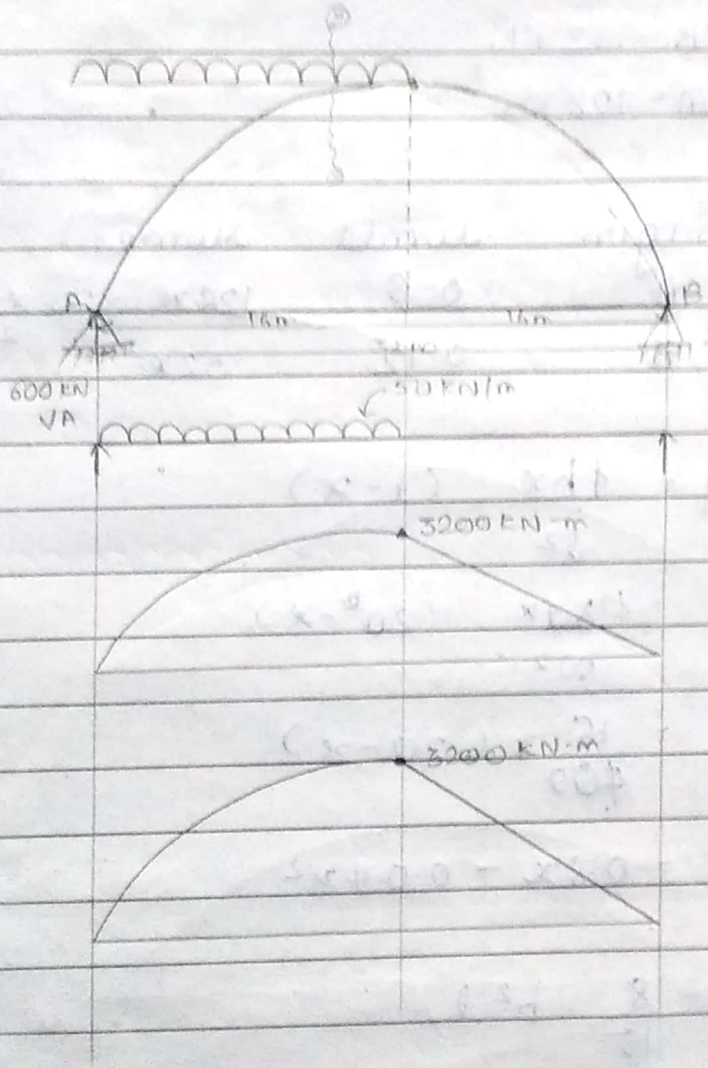
$$= \frac{8}{15}\times 8^2\times 32$$

$= 1092.26$

$$H = \int \frac{My\,dx}{y^2}$$

$$= \frac{436851.24}{1092.26}$$

$$H = 399.95 \text{ KN}$$



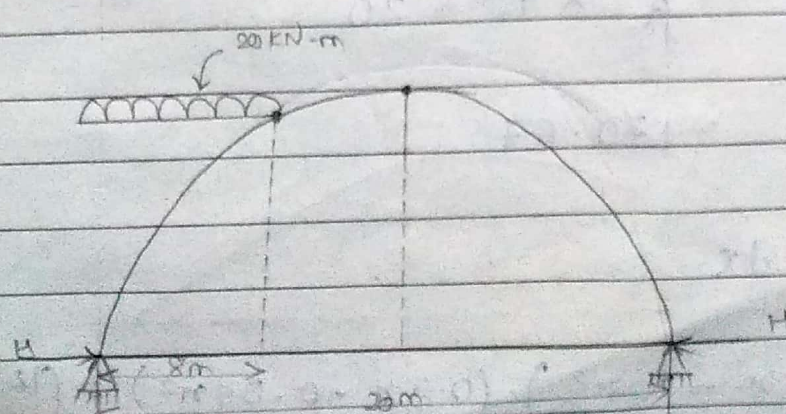
$= 600 \times 16 - 50 \times 16 \times 8$

$= 3200$ kN-m

BM for vertical reaction

BM for horizontal reaction

Q



H = 86.86 KN

$$\Sigma F_y = 0$$

$$V_A + V_B - 20 \times 8 = 0$$

$$V_A + V_B = 160$$

$$\Sigma M @ A = 0$$

$$20 \times 8 \times \frac{8}{2} - V_B \times 20 = 0$$

$$V_B = 32 \text{ KN}$$

$$V_A = 128 \text{ KN}$$

Portion	Origin	Limit	Moment
AC	A	0-8	$128x - 20\frac{x^2}{2}$
BC	B	0-12	$32x.$

$$y = \frac{4hx}{l^2}(l - x)$$

$$= \frac{4 \times 4x}{20^2}(20 - x)$$

$$y = \frac{16}{400}x(20 - x)$$

$$= 0.8x - 0.04x^2$$

$$y^2 = \frac{8}{15}h^2 l$$

$$= \frac{8}{15} \times 4^2 \times 20$$

$$= 170.67$$

$$\int My\,dx$$

$$\int_0^8 \left(128x - \frac{20x^2}{2}\right)(0.8x - 0.04x^2) + \int_0^{12} 32x(0.8x - 0.04x^2$$

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$$\int_0^8 (128-10x^2)(0.8x-0.04x^2) + \int_0^{12} 25.6x^2 - 1.28x^3$$

$$\int_0^8 102.4x^2 - 5.12x^3 - 8x^3 + 0.4x^4 + \int_0^{12} 25.6x^2 - 1.28x^3$$

$$\left[102.4\frac{x^3}{3} - 5.12\times\frac{x^4}{4} - 8\frac{x^4}{4} + 0.4\frac{x^5}{5}\right]_0^8 +$$

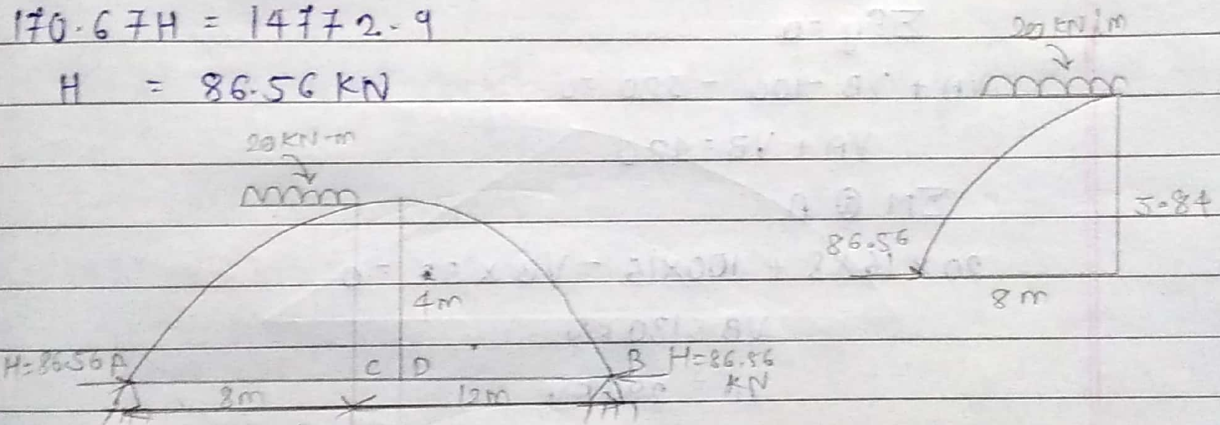
$$\left[25.6\frac{x^3}{3} - 1.28\frac{x^4}{4}\right]_0^{12}$$

$$102.4\times\frac{8^3}{3} - 5.12\times\frac{8^4}{4} - 8\times\frac{8^4}{4} + 0.4\times\frac{8^5}{5} + 25.6\times\frac{12^3}{3} - 1.28\frac{12^4}{4}$$

$$17476.26 - 5242.88 - 8192 + 2621.44 + 14745.6 - 6635.52$$

$$170.67H = 14772.9$$

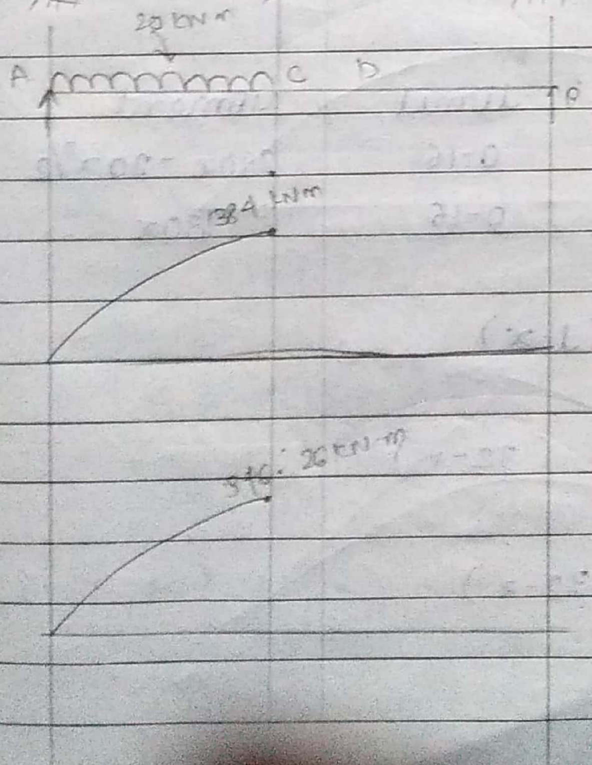
$$H = 86.56\text{ KN}$$



BM at D

$= 128\times 8 - 86.56\times 3.96 - 20\times 8\times 4$

$= 41.22$ kN-m



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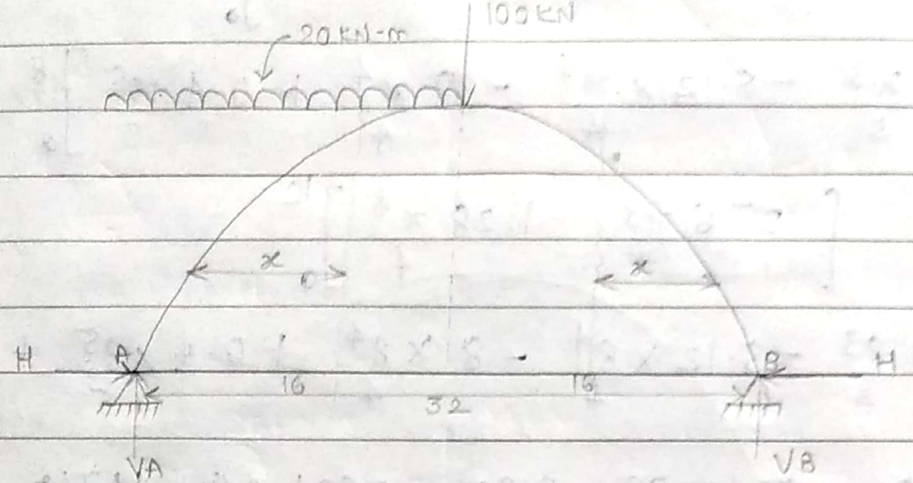
W-16

Q 2 hinge parabolic arch of 32m span and the central rise of 8m as shown in fig calculate the horizontal thrust and draw BMD



$\Sigma Fy = 0$

$VA + VB - 100 - 320 = 0$

$VA + VB = 420$

ΣM @ A

$20 \times 16 \times 8 + 100 \times 16 - VB \times 32 = 0$

$VB = 130$ KN

$VA = 290$ KN

Portion	Origin	limit	Moment
AC	A	0-16	$290x - 20x^2/2$
BC	B	0-16	$130x$

$$y = \frac{4hx}{l^2}(l-x)$$

$$= \frac{4 \times 8 \times x}{32^2}(32-x)$$

$$= \frac{1}{32} x (32-x)$$

$$= \int_0^l M y \, dx$$

$$= \int_0^{16} (290x - 10x^2) \times \frac{1}{32} x(32-x) \, dx + \int_0^{16} 130x \times \frac{1}{32} x (32-x) \, dx.$$

$$= \int_0^{16} (290x - 10x^2)(x - 0.031x^2) + \int_0^{16} 130x(x - 0.031x^2)$$

$$= \int_0^{16} (290x^2 - 8.99x^3 - 10x^3 + 0.31x^4) + \int_0^{16} (130x^2 - 4.03x^3) dx$$

$$= \left[\frac{290x^3}{3} - 8.99\frac{x^4}{4} - \frac{10x^4}{4} + 0.31\frac{x^5}{5} \right]_0^{16} + \left[130\frac{x^3}{3} - 4.03\frac{x^4}{4} \right]_0^{16}$$

$= 149826.218 + 111465.81$

$= 261292.03$

$$\int y^2 \, dx = \frac{8}{15} h^2 l$$

$$= \frac{8}{15} \times 8^2 \times 32$$

$= 1092.26$

$$H = \frac{261292.03}{1092.26}$$

$H = 239.22$

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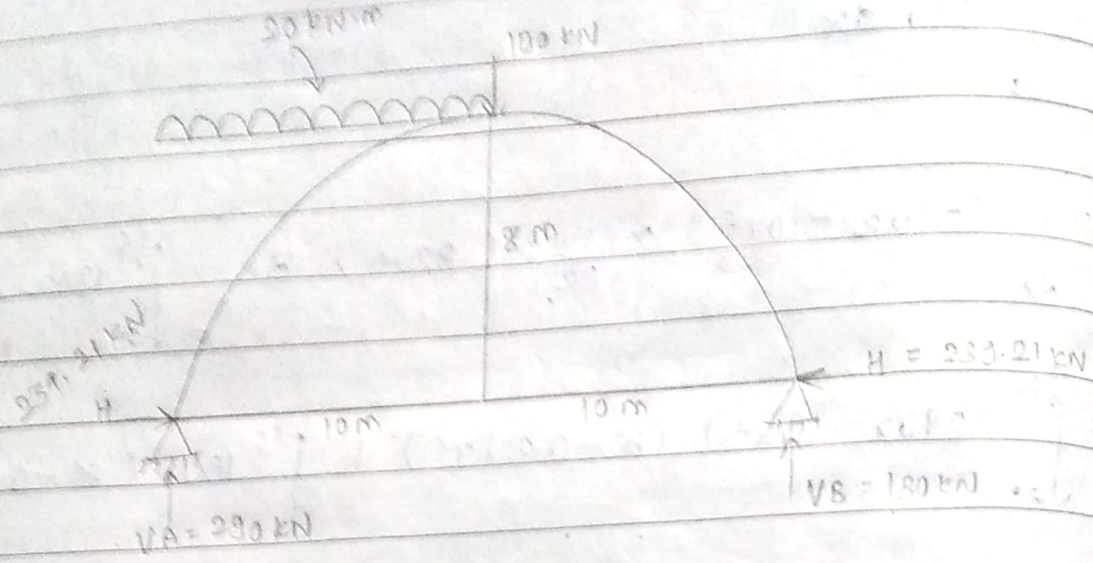
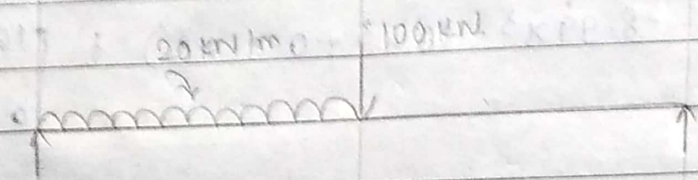
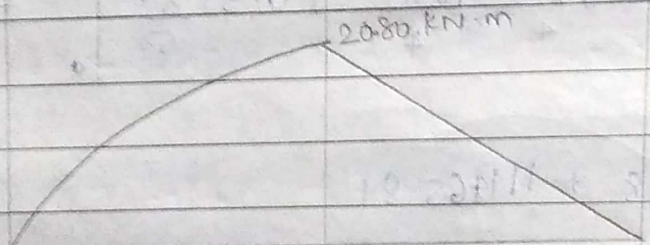
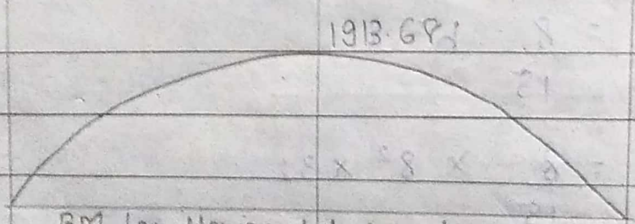


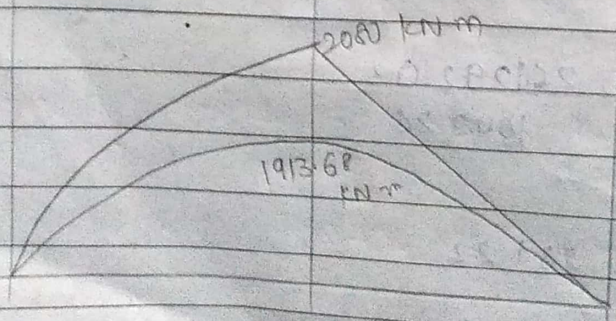




BM for vertical reaction



BM for Horizontal reaction



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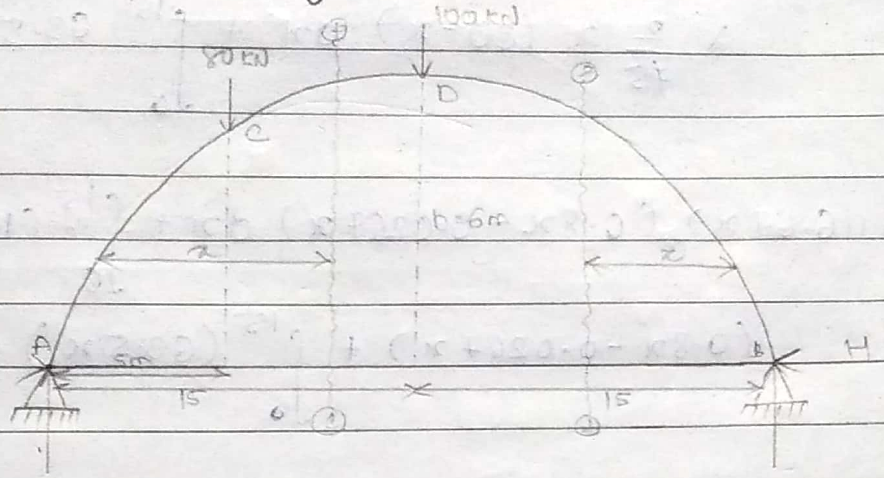
10-15

Q Two hinge parabolic arch of ht 6m carries the two concretrated load of 80KN and 100KN acting at 5m and 15m from the left support calculate the horizontal thrust and BMD the total span of the arch = 30m



$$\Sigma F_y = 0$$

$$V_A + V_B - 80 - 100 = 0$$

$$V_A + V_B = 180$$

$$\Sigma M @ A = 0$$

$$(80\times5) + (100\times15) - V_B \times 30 = 0$$

$$V_B = 63.33 \text{ KN}$$

$$V_A = 116.67 \text{ KN}$$

Portion	Origin	Limit	Moment
AC	A	0-5	$116.67x$
CD	C	5-15	$116.67x - 80(x-5)$
BD	B	0-15	$63.5x$

$$y = \frac{4hx}{l^2}(l-x) = \frac{4\times6\times x}{30^2}(30-x)$$

$$= \frac{2}{75}x(30-x) = 0.8x - 0.0267x^2$$

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$$= \int_0^l My \, dx$$

$$= \int_0^5 (116.67x) \times \frac{2}{75} x(30-x) \, dx + \int_5^{15} (116.67x - 80(x-5)) \times \frac{2}{75} x(30-x) \, dx + \int_0^{15} 63.5x \times \frac{2}{75} x(30-x)$$

$$= \int_0^5 (116.67x)(0.8x - 0.0267x) \, dx + \int_6^{15} (116.67x - 80x - 400)(0.8x - 0.0267x) + \int_0^{15} (63.5x)(0.8x - 0.0267x)$$

$$= \int_0^5 (93.336x^2 - 3.1150x^2) \, dx + \int_5 (93.336x^2 - 3.1150x^2 - 64x^2 - 2.136x^2 - 320x^2 - 10.68x) + \int_0^{15} (50.8x^2 - 1.695x^2)$$

$$= \left(93.336\frac{x^3}{3} - 3.1150\frac{x^3}{3}\right)_0^5 + \left(93.336\frac{x^3}{3} - 3.1150\frac{x^3}{3} - 64\frac{x^3}{3} - 2.136\frac{x^3}{3} - 1320\frac{x^2}{2} - 10.68\frac{x^2}{2}\right)_5^{15} + \left(50.8\frac{x^3}{3} - 1.695\frac{x^3}{3}\right)_0^{15}$$

$$= \left(93.336 \times \frac{5^3}{3} - 3.1150\frac{5^3}{3}\right) + \left(93.336\frac{15^3}{3} - 3.1150\frac{15^3}{3} - 64\times\frac{15^3}{3} - 2.136\frac{15^3}{3} - 1320\frac{15^2}{2} - 10.68\frac{15^2}{2} - 93.336\times\frac{5^3}{3} + 3.1150\frac{5^3}{3} + 64\times\frac{5^3}{3} + 2.136\times\frac{5^3}{3} + 1320\times\frac{5^2}{2} + 10.68\frac{5^2}{2}\right)$$

$$+ \left(50.8 \times \frac{15^3}{3} - 1.695 \times \frac{15^3}{3}\right)$$

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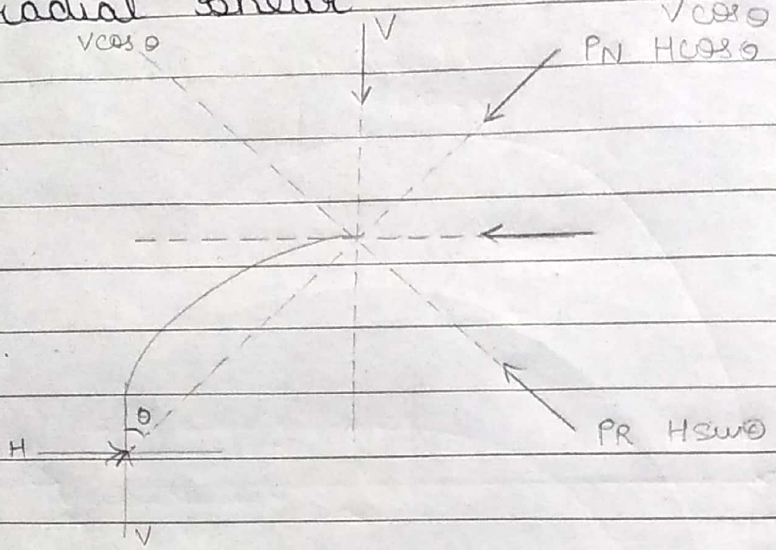
* Two hinge arch subjected to normal thrust and radial shear



$P_R = H\sin\theta - V\cos\theta$

$P_N = H\cos\theta + V\sin\theta$

$$y = \frac{4hx}{l^2}(l-x)$$

$$y = \frac{4hxl}{l^2} - \frac{4hx^2}{l^2}$$

$$= \frac{4h}{l} - \frac{4h \times 2x}{l^2}$$

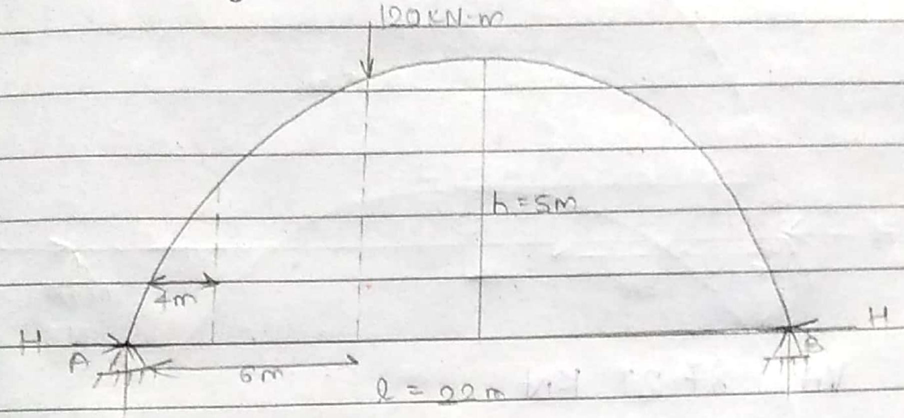
$$= \frac{4h}{l} - \frac{8hx}{l^2}$$

$$\tan\theta = \frac{dy}{dx}$$

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)$$

* Numericals *

Q1 Calculate the normal thrust and radial shear for the given two hinged area at the section 4m from the left support



$$V_A = 87.27 \text{ KN}$$

$$H = 78.58 \text{ KN}$$

$$\frac{dy}{dx} = \frac{4h}{l} = \frac{8hx}{l^2}$$

$$= \frac{4 \times 5}{22} - \frac{8 \times 5 \times 4}{(22)^2}$$

$$\frac{dy}{dx} = 0.578$$

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)$$

$$\theta = \tan^{-1}(0.578)$$

$$\theta = 30.04$$

$$P_R = H \sin\theta - V \cos\theta$$

$$= 78.58 \sin(30.04) - 87.27 \cos(30.04)$$

$$P_R = -36.21 \text{ KN}$$

$$P_N = H \cos\theta + V \sin\theta$$

$$= 78.58 \cos(30.04) - 87.27 \sin(30.04)$$

$$P_N = 111.71 \text{ KN}$$

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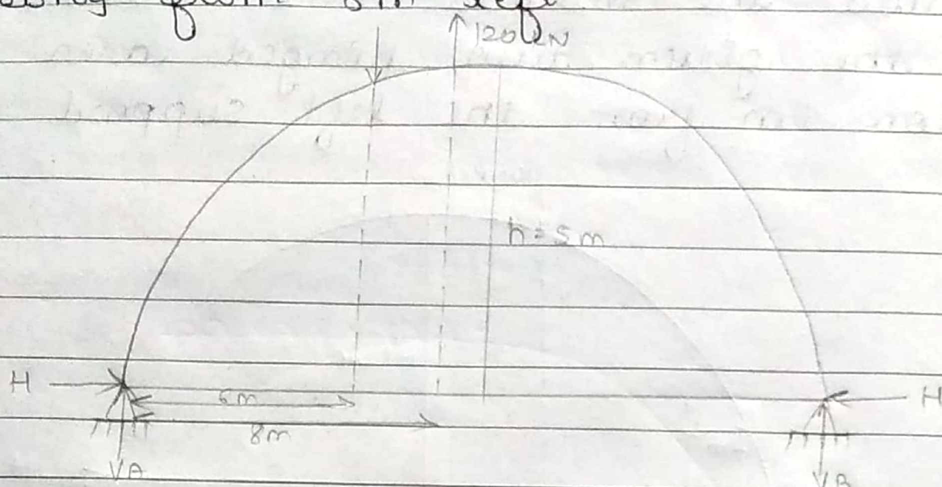
Q. 2 calculate the radial shear and normal reaction for the above numerical section passing from 8 m left



$VA = 87.27$ KN

$H = 78.58$ KN

$V = -120 + 87.27\text{ kN} = -32.73$

$$\frac{dy}{dx} = \frac{4h}{L} - \frac{8hx}{l^2}$$

$$= \frac{4 \times 5}{22} - \frac{8 \times 5 \times 8}{22^2}$$

$$\frac{dy}{dx} = 0.24$$

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)$$

$$= \tan^{-1}(0.24)$$

$$\theta = 13.49^0$$

$PR = 78.58 \sin(13.49) - 32.73 \cos(13.49^0)$

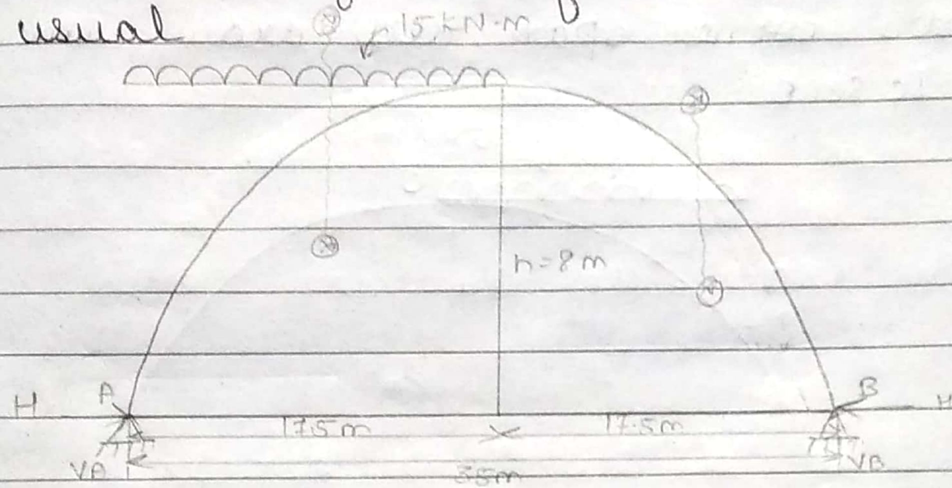
$= -13.49$ KN

$PN = 78.58 \cos\theta + V\sin\theta$

$PN = 68.77$ KN

Q. 3

Q.3 A two hinged parabolic arch with 15m span and 8m rise is subjected to UDL of 15KN-m over the left hand span. Find the reaction at support normal and radial stress at the section 12m from left support. Take I = I sec θ with usual



$\Sigma F_y = 0$

$VA + VB - 262.5 = 0$

$VA + VB = 262.5$

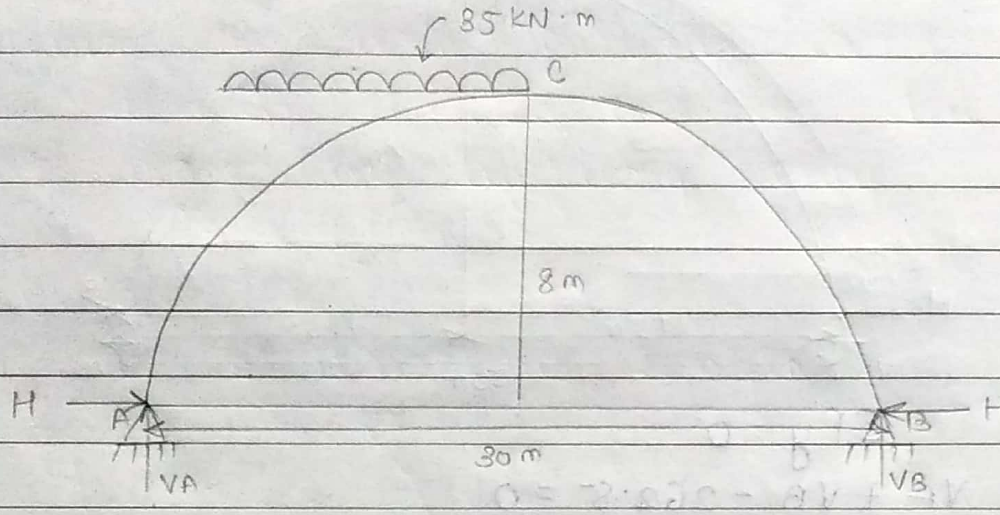
$\Sigma M @ A = 0$

$262.5 \times 8.75 - VB \times 35 = 0$

$VB = 65.625$ KN

$VA = 196.875$ KN

Q A two hinge parabolic arch of span 30m and rise 8m is loaded with the UDL of the intensity 35 KN·m over the left half span. Determine the value of horizontal thrust radial shear and normal thrust at the left centre corner span also draw the BMD use $I = I_c \sec\theta$.



$$VA + VB = 525$$

$$\Sigma M \text{ @ } A = 0$$

$$35 \times 15 \times \frac{15}{2} - VB \times 30 = 0$$

$$VB = 131.25 \text{ KN}$$

$$VA = 393.75 \text{ kN}$$

$$y = \frac{4hx}{l^2}(l - x)$$

$$= \frac{4 \times 8 \times x}{30^2}(30 - x)$$

$$= 1.067x - 0.035x^2$$

$$y^2 = \frac{8}{15} h^2 l$$

$$= \frac{8}{15} \times 8^2 \times 30 = 1024$$

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Portion	Origin	limit	moment
AC	A	0-15	$393.75x - 35\frac{x^2}{2}$
BC	B	0-15	$131.25x$

$$\int My \, dx$$

$$\int_0^{15} (393.75x - 17.5x^2)(1.067x - 0.035x^2)\,dx + \int_0^{15} 131.25x\,(1.067x - 0.035x^2)\,dx$$

$$\int_0^{15} 420.13x^2 - 13.78x^3 - 18.67x^3 + 0.6125x^4\,dx + \int_0^{15} 140.04x^2 - 4.59x^3\,dx$$

$$\left[420.13\frac{x^3}{3} - 13.78\frac{x^4}{4} - 18.67\frac{x^4}{4} + 0.6125\frac{x^5}{5}\right]_0^{15} + \left[140.04 \times \frac{x^3}{3} - 4.59 \times \frac{x^4}{4}\right]_0^{15}$$

$$420.13 \times \frac{15^3}{3} - 13.78 \times \frac{15^4}{4} - 18.67 \times \frac{15^4}{4} + 0.6125 \times \frac{15^5}{5} + 140.04 \times \frac{15^3}{3} - 4.59 \times \frac{15^4}{4}$$

$$472646.25 - 174403.125 - 236292.188 + 93023.438 + 157545 - 58092.188$$

$$\frac{254427.187}{1024}$$

$H = 248.46$

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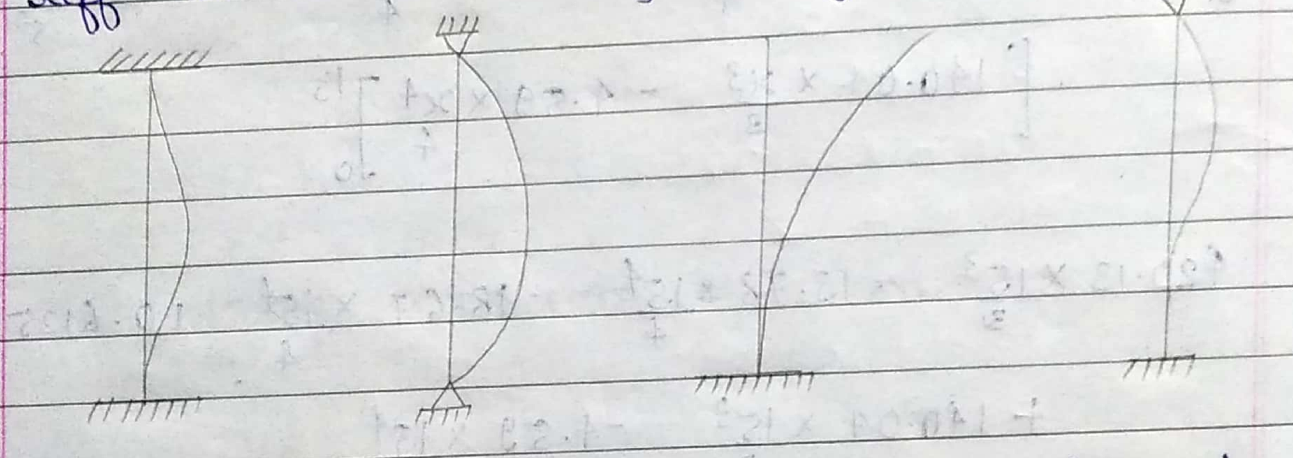
* The load carrying capacity of the column can be estimate by using two theory

1. Euler's Theory

Euler theory is used to determine the load carrying capacity of column which is derived by considering the column as long column and as per this theory the failure of the column takes place by buckling only

$$P = \frac{\pi^2 EI}{l_{eff}^2}$$

l_{eff} is a buckling length



$l_{eff} = \frac{l}{2}$	$l_{eff} = l$	$l_{eff} = 2l$	$l_{eff} = \frac{L}{\sqrt{2}}$
$P = \frac{\pi^2 EI}{(l/2)^2}$	$P = \frac{\pi^2 EI}{l^2}$	$P = \frac{\pi^2 EI}{(2l)^2}$	$P = \frac{\pi^2 EI}{(l/\sqrt{2})^2}$

2. Rankine's Theory

It is the modification of Euler's theory by taking into account the crushing failure along with buckling. Eulers theory only buckling failure is considered

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which is valid for long column only while Rankine's theory both crushing and buckling failures are considered which are applicable for any type of column (long and short column)

$$\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_E}$$

P_R = Rankine's load

R_c = crushing load = $\sigma_c \cdot A$

P_E = Euler's load

$$P_E = \frac{\pi^2 EI}{l_{eff}^2}$$

$$\frac{1}{P_R} = \frac{P_E + P_c}{P_c \cdot P_E}$$

$$= \frac{P_c P_E}{P_E + P_c}$$

$$= \frac{P_c}{\frac{P_E + P_c}{P_E}}$$

$$P_R = \frac{P_c}{1 + \frac{P_c}{P_E}}$$

$$= \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c A}{P_E}}$$

$$= \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c \cdot A}{\frac{\pi^2 EI}{l_{eff}^2}}}$$

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$$= \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c A}{r^2 EI} \times l_{eff}^2}$$

$$= \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c}{r^2 E} \left(\frac{l_{eff}}{I/A} \right)^2}$$

$$= \frac{\sigma_c \cdot A}{1 + \frac{\sigma_c}{r^2 E} \left(\frac{l_{eff}}{r_{min}} \right)^2}$$ ... M and D^n by A

Demo^n

$$P_R = \frac{\sigma_c \cdot A}{1 + \alpha \left(\frac{l_{eff}}{r_{min}} \right)^2}$$

Take $\frac{\sigma_c}{r^2 E} = \alpha =$ constant

$\therefore \sqrt{\frac{I}{A}} = r_{min}$

* The basic equation of Bending

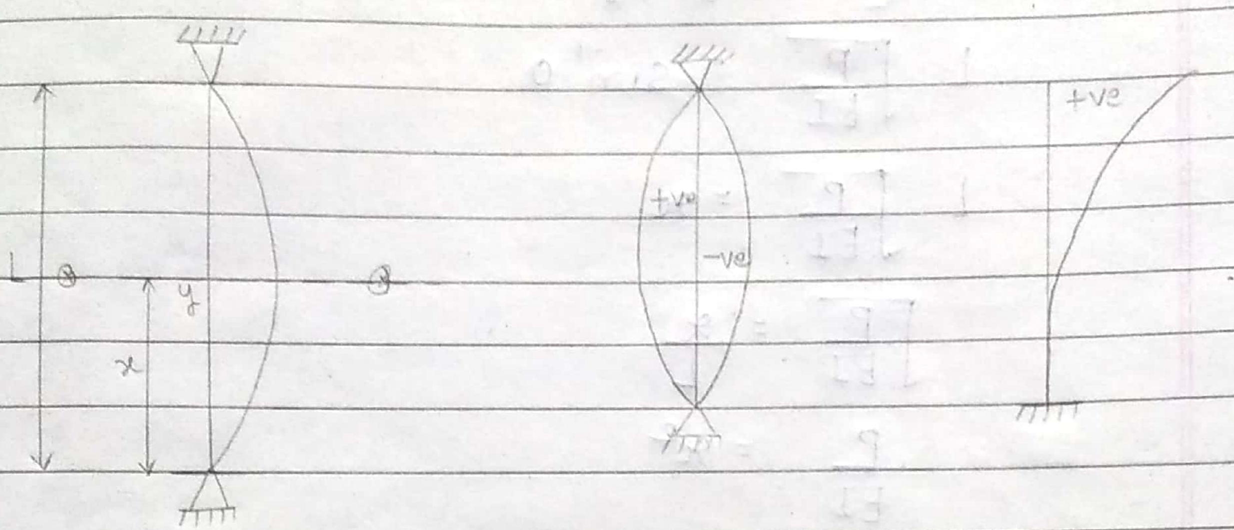
$$\frac{M}{I} = \frac{\sigma}{y}$$

$$M = EI \frac{d^2y}{dx^2}$$

Derive the eq^n for the effective length of column having both the end hinged

OR Derive the eq^n for load carrying capacity of the column having both end hinged

Consider a column AB of length L which is hinged at A and B subjected to the axial load P as shown in fig. Due to the application of load column will buckles as shown.



Consider a section X-X at the distance x from point A where y represents the deflection in column

$$M = -Py$$

$$EI\frac{d^2y}{dx^2} + Py = 0$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = 0$$

The general solution of the above differential eq^n will be written as

$$y = C_1\cos\left(x\sqrt{\frac{P}{EI}}\right) + 2\sin\left(x\sqrt{\frac{P}{EI}}\right)$$

Boundary conditions

1) At $x=0$, $y=0$

$$C_1 = 0$$

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2) At $x = l$ $y = 0$

$$C_2 \sin\left(l\sqrt{\frac{P}{EI}}\right) = 0$$

C_2 can not be equal to zero

$$\sin\left[L\left(\sqrt{\frac{P}{EI}}\right)\right] = 0$$

$$L\sqrt{\frac{P}{EI}} = \sin^{-1} 0$$

$$L\sqrt{\frac{P}{EI}} = \pi$$

$$\sqrt{\frac{P}{EI}} = \frac{\pi}{L}$$

$$\frac{P}{EI} = \pi^2$$

$$P = \frac{\pi^2 EI}{l^2}$$

$$= \frac{\pi^2 EI}{l_{eff}^2}$$

comparing the basic eq^n of eulers theo

$$l_{eff} = l.$$