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Unit V. Approximate Method

* Approximate value *

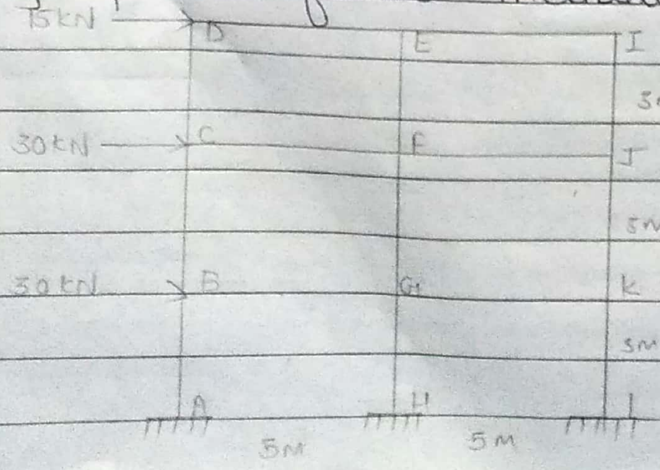
* Types of method

1. Portal frame method — applicable for 25 storey buildings
2. Cantilever bay method — applicable for 25-55 storey buildings
3. Substitute frame method (Factor method)

- Frames are divided into different portal frames which act individually at each storeyed
- The point of contraflexure are assumed to the midpoint of the column and the girder / beam
- Total horizontal shear at each storey level are distributed among the column of that storey in direct proportion to width of the bay supported by column

* Numericals *

19 Q Analysis the portal frame as shown in fig
M using portal frame method



axial
antaxial
lateral

in every corner Σ of moment should be zero
symmetrical axis has axial force zero because
it is at center

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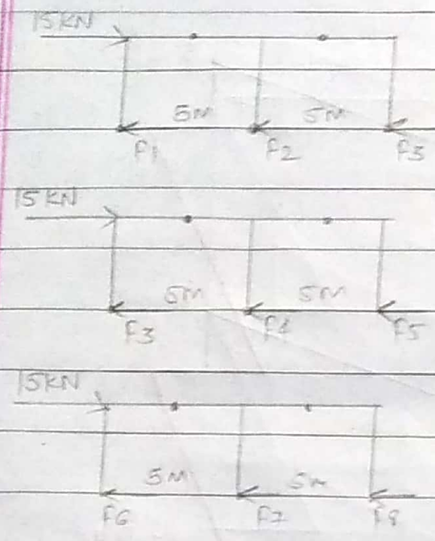


$$\frac{f_1 : f_2 : f_3}{15} = \frac{5}{2} : \frac{5+5}{2} : \frac{5}{2} \quad (10)$$

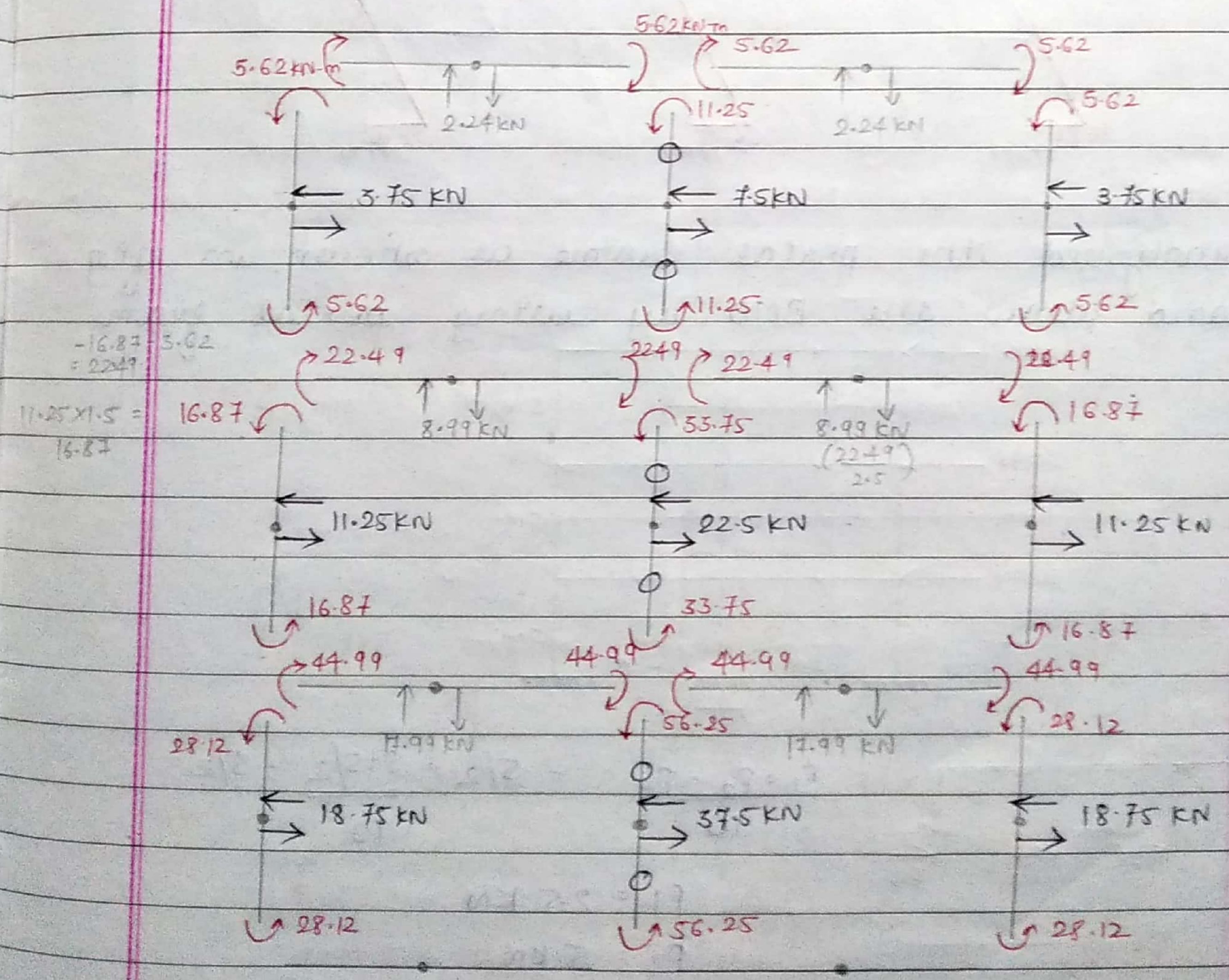
$f_1 = 3.75\ kN$, $f_2 = 7.5\ kN$, $F_3 = 3.75\ kN$

$$\frac{f_4 : f_5 : f_6}{45} = \frac{5}{2} : \frac{5+5}{2} : \frac{5}{2} \quad (10)$$

$F_4 = 11.25\ kN$, $F_5 = 22.5\ kN$, $F_6 = 11.25\ kN$

$$\frac{F_7 : F_8 : F_9}{75} = \frac{5}{2} + \frac{5+5}{2} : \frac{5}{2} \quad (10)$$

$F_7 = 18.75\ kN$, $F_8 = 37.5\ kN$, $F_9 = 18.75\ kN$



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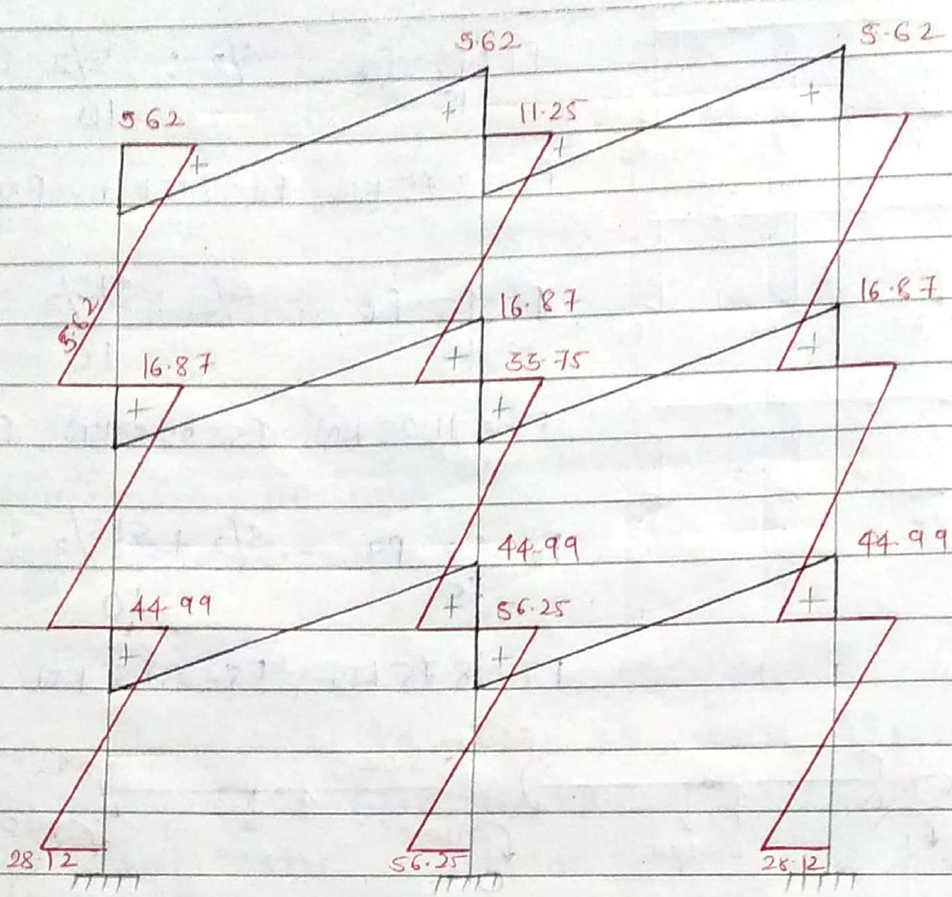
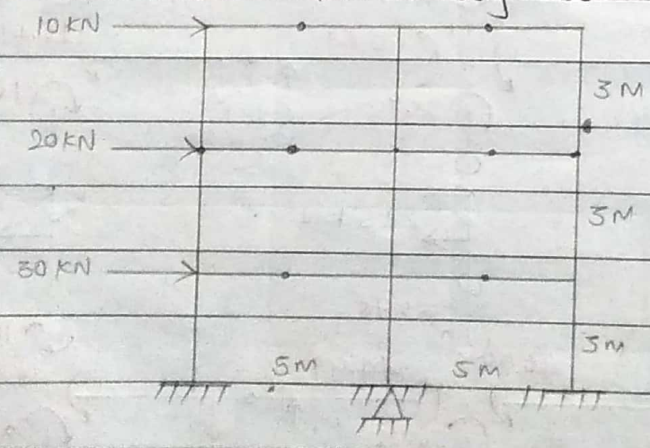
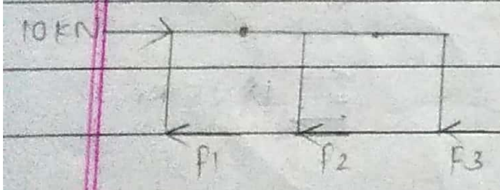


Q Analysized the portal frame as shown in fig and draw the BMD by using portal frame.



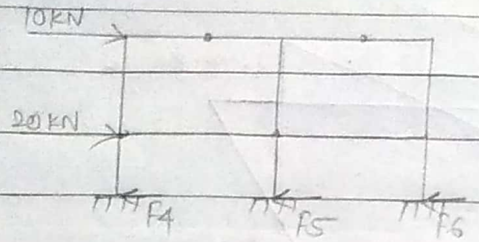


$$\frac{F_1 : F_2 : F_3}{10} = 5/2 : \frac{5+5/2}{10} : 5/2$$

$F_1 = 2.5$ KN

$F_2 = 5$ KN

$F_3 = 2.5$ KN



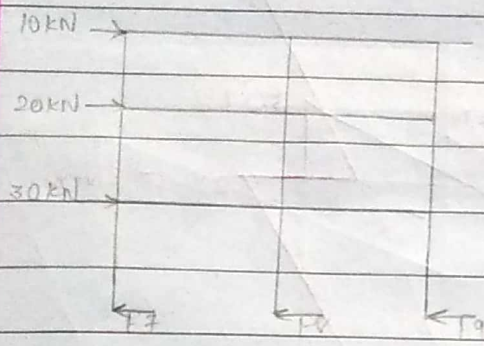
$$f_4 : f_5 : f_6 = \frac{5}{2} : \frac{5+5}{2} : \frac{5}{2}$$

30   10

$f_4 = 7.5$ KN

$f_5 = 15$ KN

$f_6 = 7.5$ KN



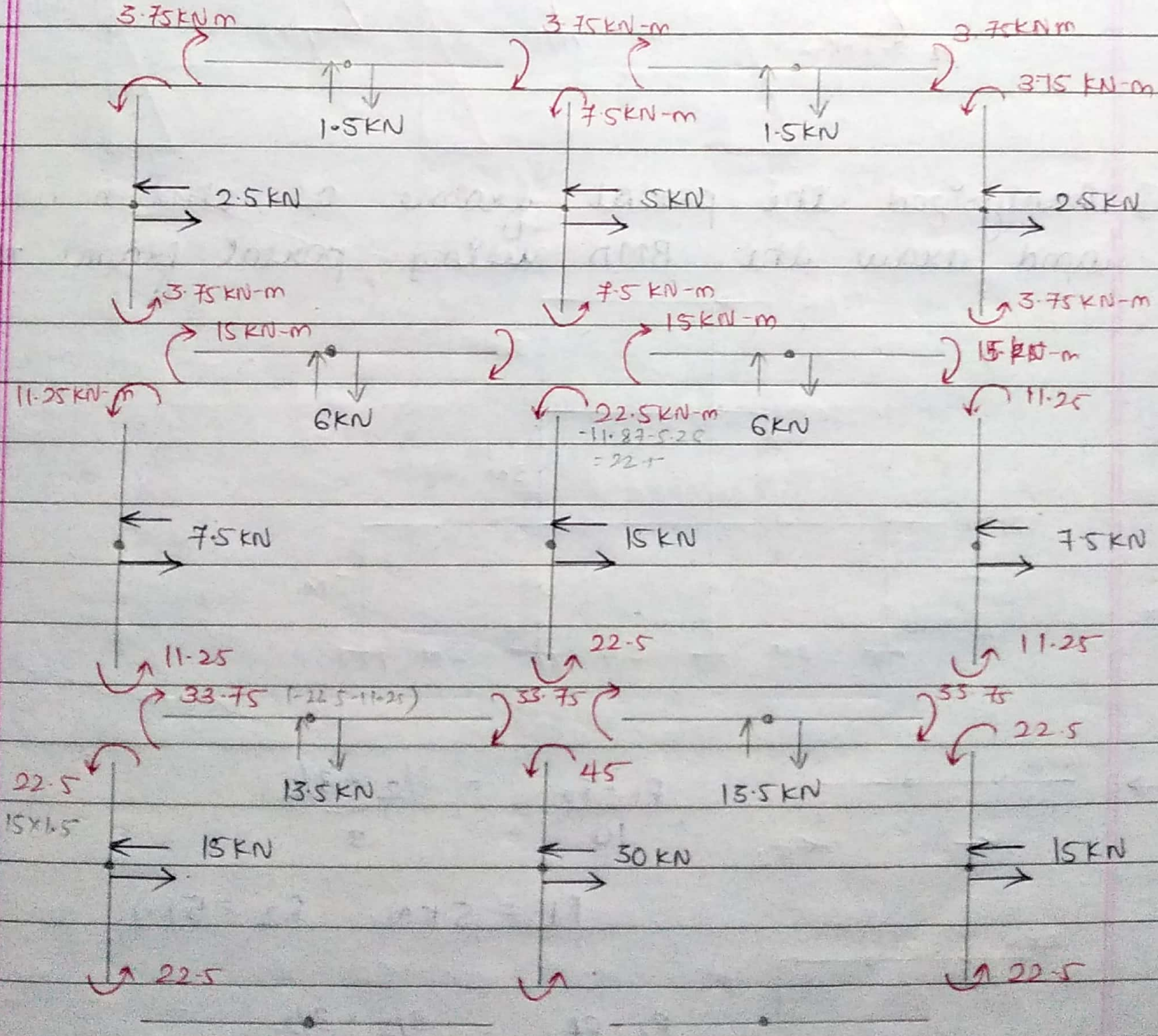
$$f_7 : f_8 : f_9 = \frac{5}{2} : \frac{5+5}{2} : \frac{5}{2}$$

60   10

$f_7 = 15$ KN

$F_8 = 30$ KN

$F_9 = 15$ KN



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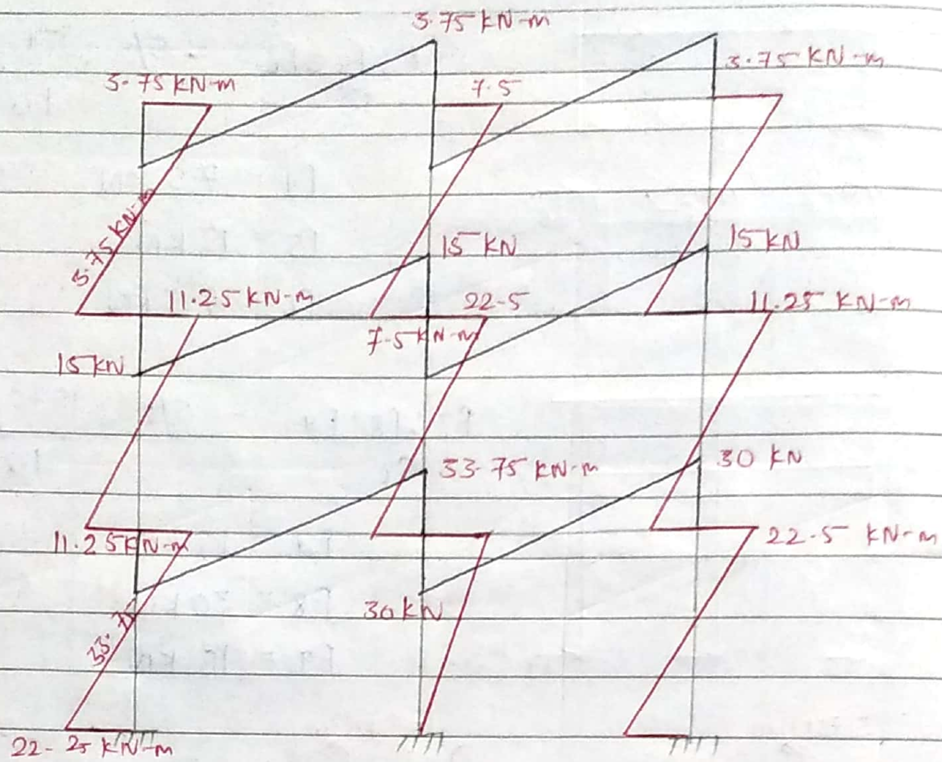
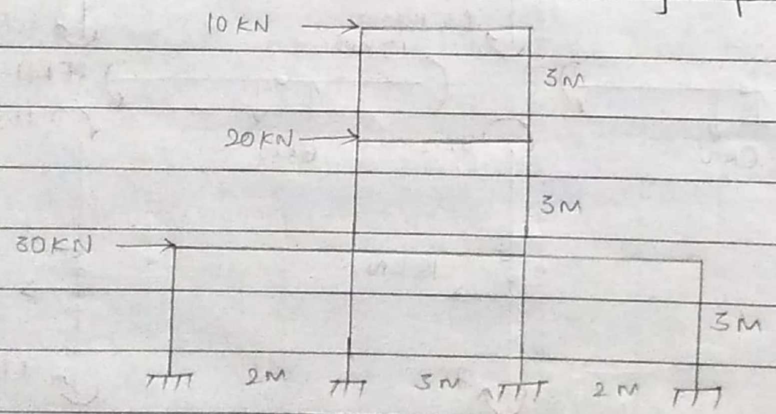
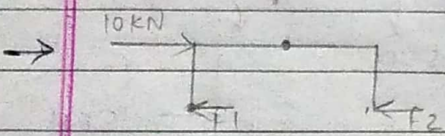


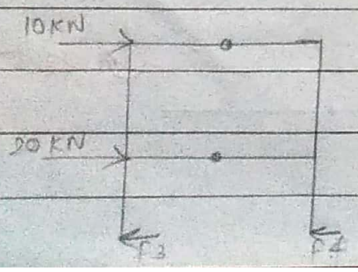
Q Analysized the portal frame as shown in fig and draw the BMD using portal frame method





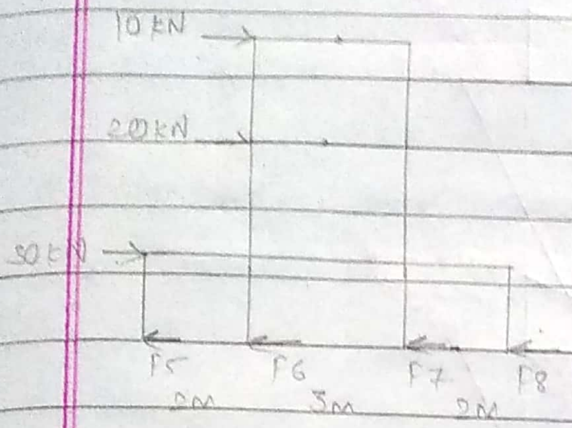
$$\frac{F_1 : F_2}{10} = \frac{3/2 : 3/2}{3}$$

$$F_1 = 5\,kN, \quad F_2 = 5\,kN$$



$$\frac{F_3 : F_4}{30} = \frac{3/2 : 3/2}{3}$$

$$F_3 = 15\,kN, \quad F_4 = 15\,kN$$



$F_5 : F_6 : F_7 : F_8 = 2/2 : \frac{2+3}{2} : \frac{2+3}{2} : 2/2$

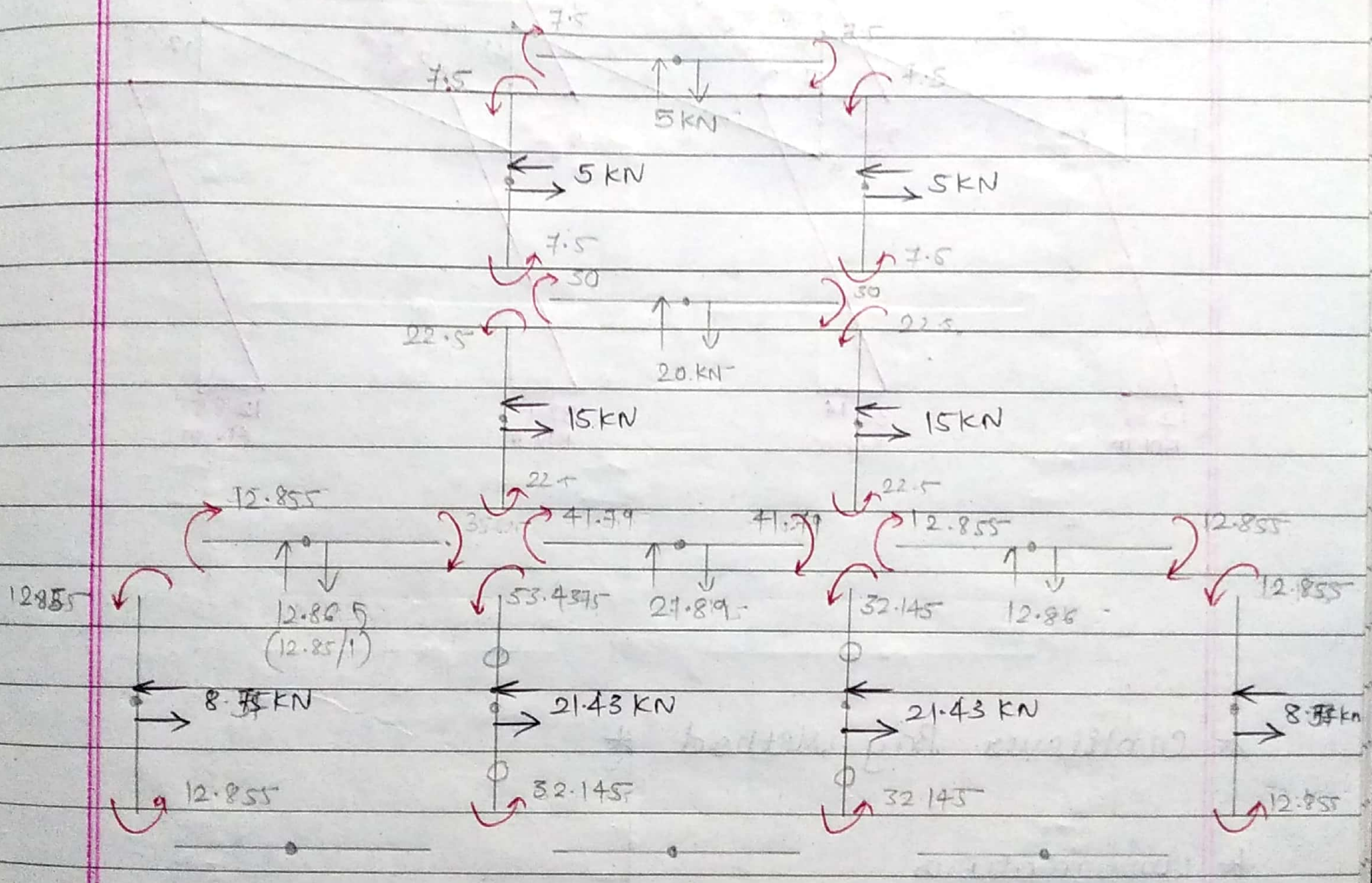
60   7

$F_5 = 8.75$ kN

$F_6 = 21.43$ kN

$F_7 = 21.43$ kN

$F_8 = 8.75$ kN



Tension Compression

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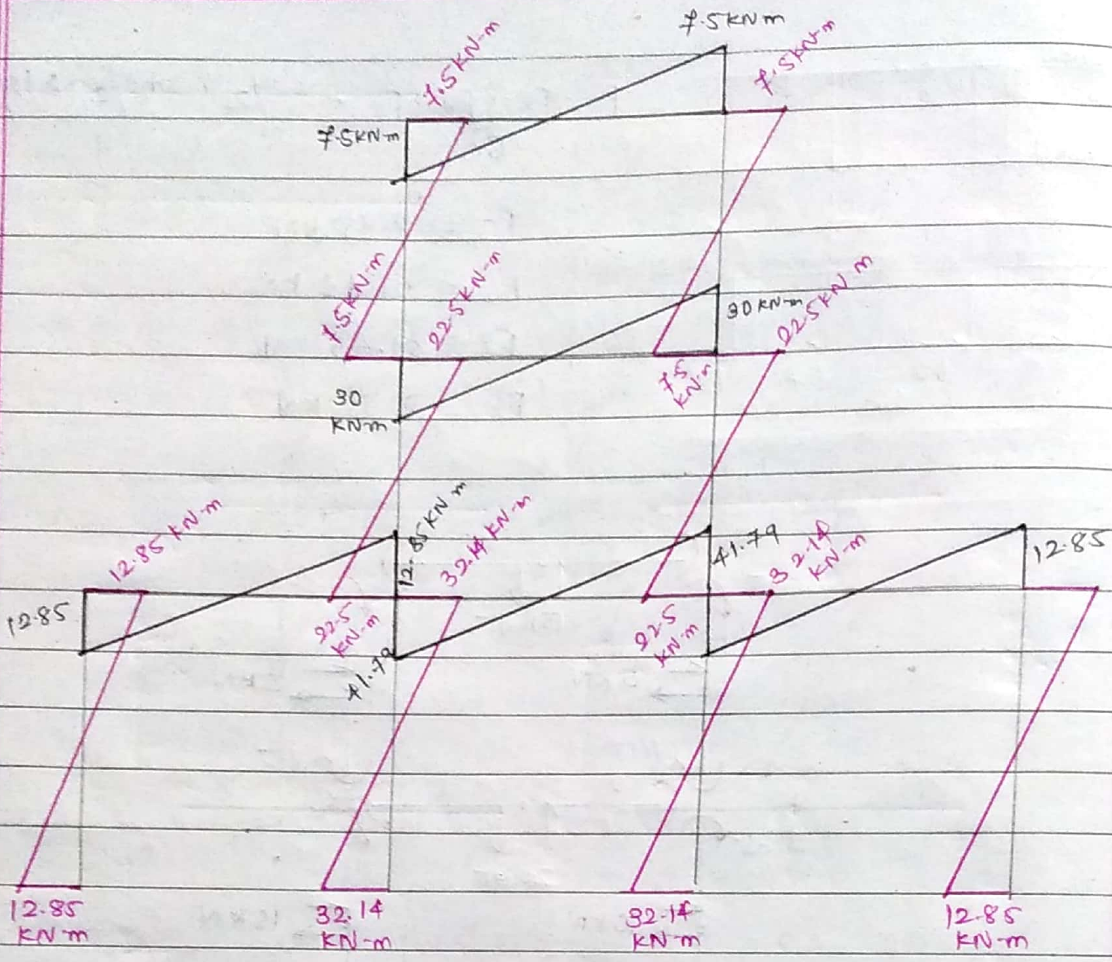

* Cantilever Bay Method *

* Assumptions

1. Point of contraflexure lies at the midpoint of column and beam.

2. Direct stress in column is directly proportional to its centroidal distance to the CG of frame

3. cross sectional area of all column are assumed

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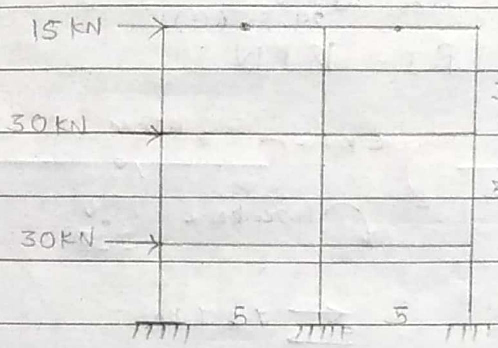
as same

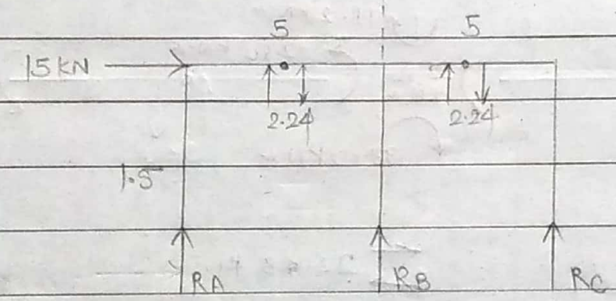
4. Moment due to external lateral load are calculated at the middle level of each frame

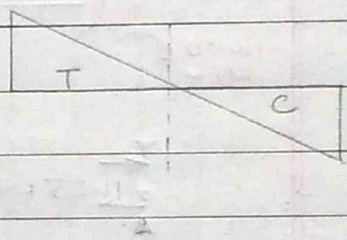


$$\bar{X} = \frac{(A \times 5) + (A \times 10)}{A + A + A}$$

$$\bar{X} = \frac{15A}{3A} = 5m$$

$$R_n = \frac{M}{I} \times x$$





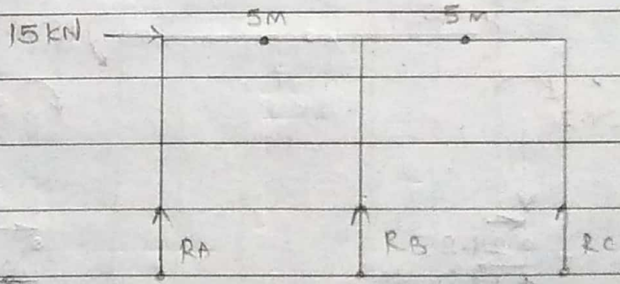
$$R_A = \frac{15 \times 1.5}{50} \times 5$$

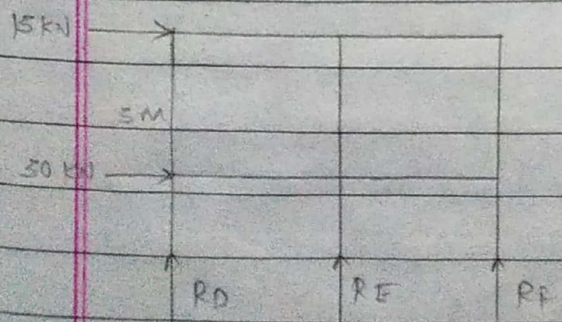
$R_A = 22.5$ (T)

$R_B = 0$

$$R_C = \frac{15 \times 1.5}{50} \times 5$$

$R_C = 22.5$ (C)





$$R_D = \frac{15 \times 4.5 + 30 \times 1.5}{50} \times 5$$

$R_D = 11.25$ ∴ $R_D = 11.25 - 2.25 = 9$ kN

$R_E = 0$

$R_F = 11.25 - 2.25 = 9$ kN

Portal method and cantilever method give good results only when

> 5×B > B

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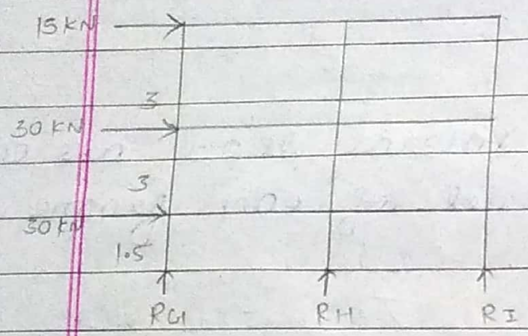


$$R_G = \frac{15\times7.5 + 30\cdot4.5 + 30\times1.5 \times 5}{50}$$

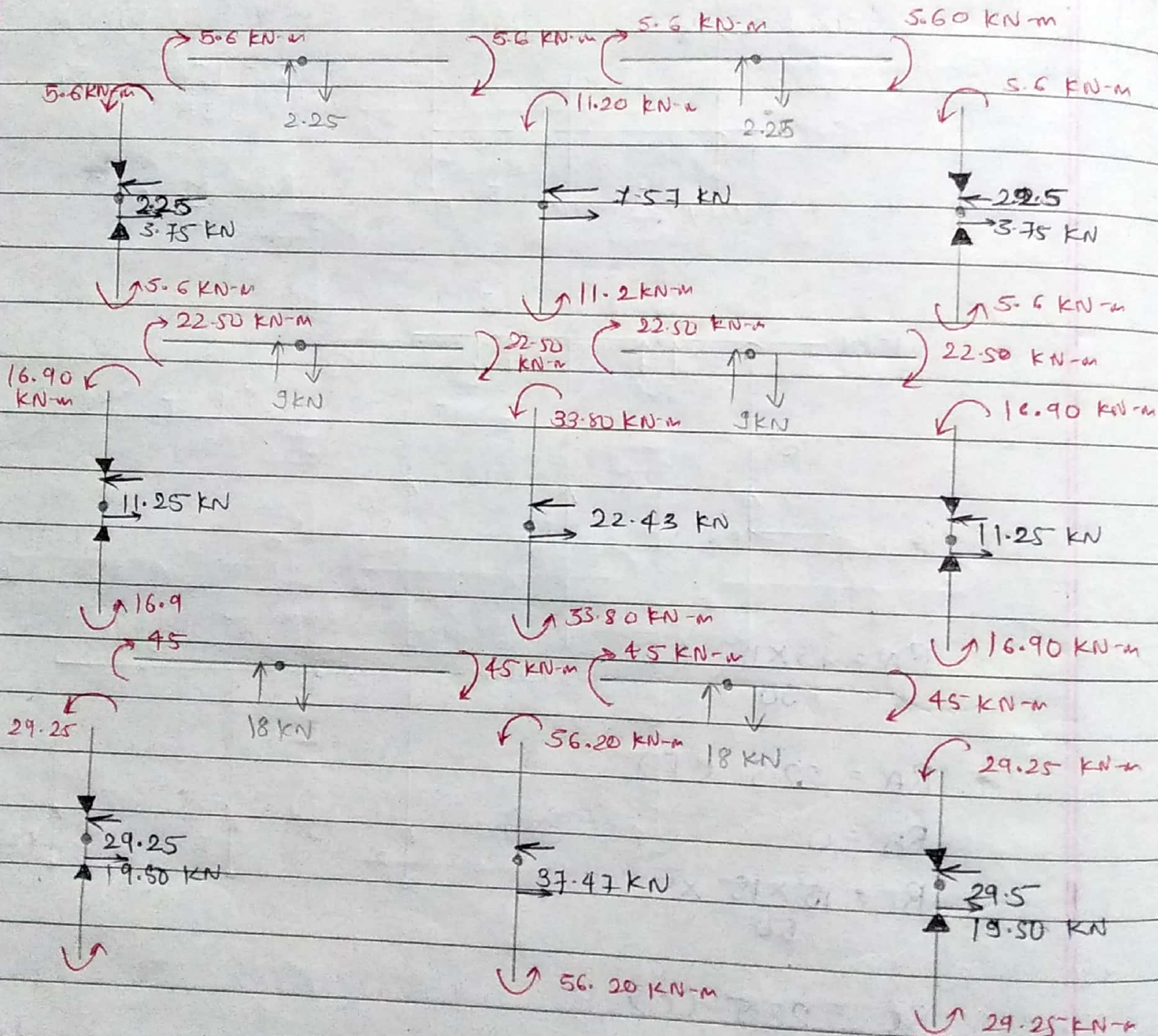
$R_G = 29.25$ (T)

$R_G = 29.25 - 11.25 = 18$ KN

$R_H = 0$ KN

29.25 (C)

$R_I = 18$ KN



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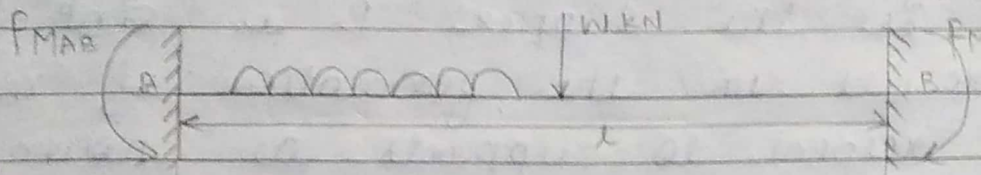
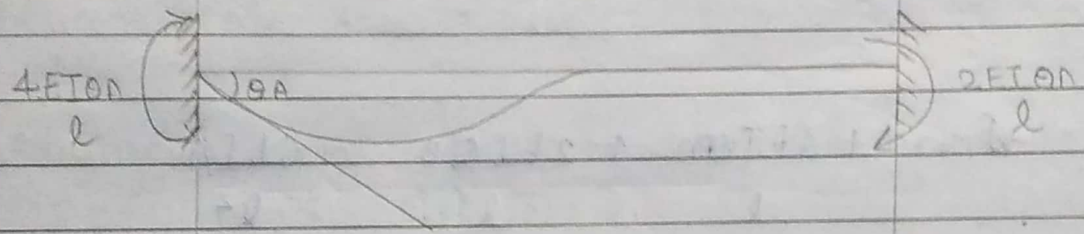
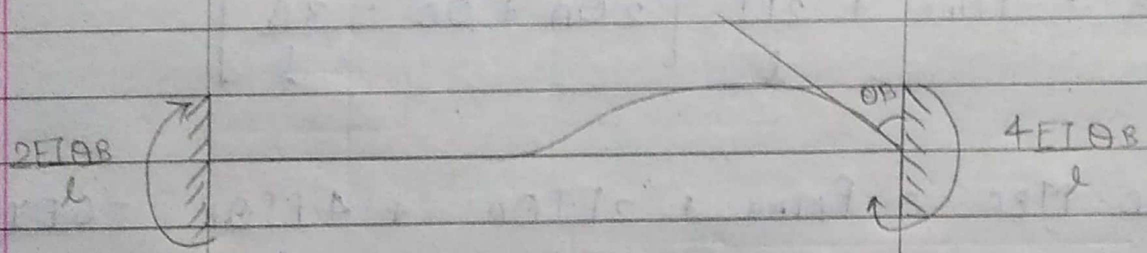
* Slope and Deflection method *

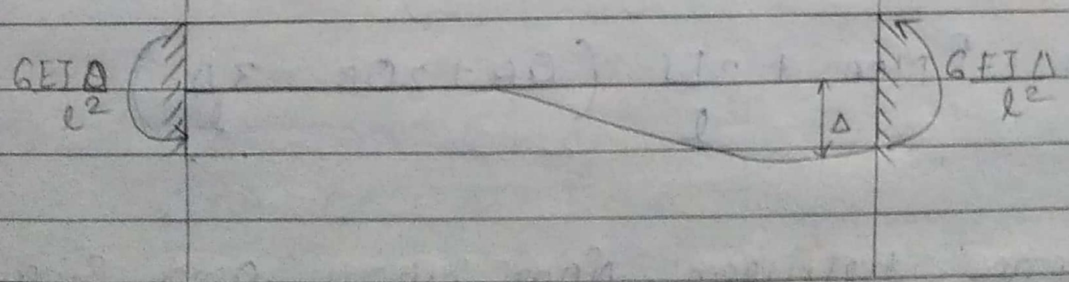
Q. Derivation for slope and deflection method



FEM of loaded beam







Let support A rotate clockwise by an amount θ_A result in the formation of moment at support A and B as shown in fig (1)

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Let the support B rotate clockwise by an amount θ_B results in the formation of moment at support A and B

Let the support B is sink by Δ which result in the formation of moment at the respect to supports as shown in fig(3)

Therefore final fixed end moment as per the given figure are calculated as B

F_{MAB}

$$M_{AB} = -F_{MAB} + \frac{4EI\theta_A}{l} + \frac{2EI\theta_B}{l} - \frac{6EI\Delta}{l^2}$$

$$M_{AB} = -F_{MAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\Delta}{l}\right]$$

Also $$M_{BA} = F_{MBA} + \frac{2EI\theta_A}{l} + \frac{4EI\theta_B}{l} - \frac{6EI\Delta}{l^2}$$

$$M_{BA} = F_{MBA} + \frac{2EI}{l}\left(\theta_A + 2\theta_B - \frac{3\Delta}{l}\right)$$

Difference between Non sway and Sway frame

Non Sway Frame	Sway Frame
1 In non sway frame the external loading are exactly ~~similar~~ symmetry	1 In sway frame the external loading are exactly unsymmetrical

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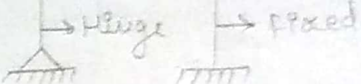

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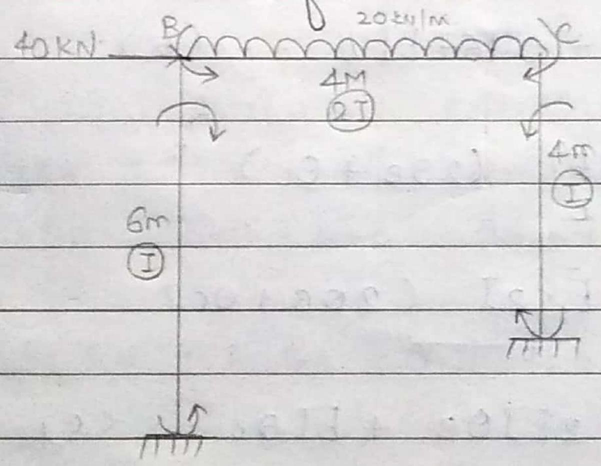
2 The frame having symmetrical EI	2 The frame having unsymmetrical EI
3 Length of the column will be same	3 Length of the column will be different
4 symmetrical support	4 unsymmetrical support

* Numericals *

Analyzed the given portal frame using slope and deflection



Given : θA, $\theta D = 0$

θA, θC, $\Delta = ?$

calculation of Fmoment at respective pt.

$F_{MAB} = F_{MBA} = F_{MCD} = F_{MDC} = 0$

$F_{MBC} = -F_{MAB} + \frac{4EI}{} \theta A$

$F_{MBC} = -\frac{wl^2}{12} = -\frac{20 \times 4^2}{12} = -26.67 \text{ KNm}$

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$(+)$ Sagging $\rightarrow \frac{wl^2}{8}$

$(-)$ Hogging $\rightarrow \frac{wl^2}{12}$

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$$fM_{CB} = \frac{wl^2}{12} = \frac{20 \times 4^2}{12} = 26.67 \text{ kNm}$$

$$M_{AB} = M^f_{AB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\Delta}{L}\right]$$

$$= 0 + \frac{2EI}{6}\left[0 + \theta_B - \frac{3\Delta}{6}\right]$$

$$M_{AB} = 0.33\, EI\theta_B - 0.167\, EI\Delta \quad \ldots (a)$$

$$M_{BA} = M^f_{BA} + \frac{2EI}{L}\left[2\theta_B + \theta_A - \frac{3\Delta}{L}\right]$$

$$= 0 + \frac{2EI}{6}\left[2\theta_B - \frac{3\Delta}{6}\right]$$

$$M_{BA} = 0.67\, EI\theta_B - 0.167\, EI\Delta \quad \ldots (b)$$

$$M_{BC} = -M^f_{BC} + \frac{2EI}{L}(2\theta_B + \theta_C)$$

$$= -26.67 + \frac{2E \times 2I}{4}(2\theta_B + \theta_C)$$

$$M_{BC} = -26.67 + 2EI\theta_B + EI\theta_C \quad \ldots (c)$$

$$M_{CB} = M^f_{CB} + \frac{2EI}{L}(2\theta_C + \theta_B)$$

$$= 26.67 + \frac{2E \times 2I}{4}(2\theta_C + \theta_B)$$

$$= 26.67 + 2EI\theta_C + EI\theta_B$$

$$M_{CB} = 26.67 + EI\theta_B + 2EI\theta_C \quad \ldots (d)$$

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$M_{CD} = M^F_{CD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\Delta}{L}\right)$

$= 0 + \frac{2EI}{4}\left[2\theta_C + \theta_D - \frac{3\Delta}{4}\right)$

$M_{CD} = EI\theta_C - 0.375\, EI\Delta \quad \cdots (e)$

$M_{DC} = M^f_{DC} + \frac{2EI}{L}\left(2\theta_D + \theta_C - \frac{3\Delta}{L}\right)$

$= 0 + \frac{2EI}{4}\left(0 + \theta_C - \frac{3\Delta}{4}\right)$

$M_{DC} = 0.5 EI\theta_C - 0.37\, EI\Delta \quad \cdots (f)$

Apply equilibrium conditions at joint B

$M_{BA} + M_{BC} = 0$

$0.67\, EI\theta_B - 0.167\, EI\Delta - 26.67 + 2EI\theta_B + EI\theta_C = 0$

$2.67\, EI\theta_B + EI\theta_C - 0.167\, EI\Delta = 26.67 \quad \cdots (1)$

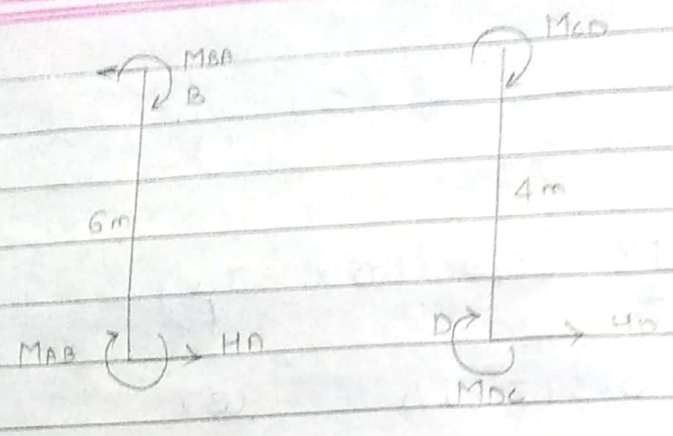
At joint C

$M_{CB} + M_{CD} = 0$

$26.67 + 2EI\theta_C + EI\theta_B + EI\theta_C - 0.375\, EI\Delta$

$EI\theta_B + 3EI\theta_C - 0.375\, EI\Delta = -26.67 \quad \cdots (2)$

Sway conditions



$\Sigma M @ B = 0$

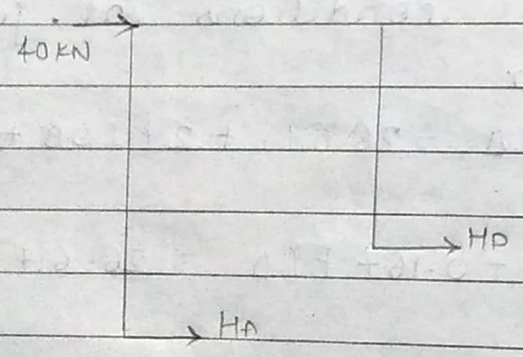
$M_{AB} + M_{BA} - H_A \times 6 = 0$

$\frac{M_{AB} + M_{BA}}{6} = H_A$

$\Sigma M @ C = 0$

$M_{BC} + M_{CD} - 4 \times H_D = 0$

$\frac{M_{DC} + M_{CD}}{4} = H_D$



$\Sigma F_x = 0$

$H_A + H_D + 40 = 0$

$H_A + H_D = -40$

$\left(\frac{M_{AB} + M_{BA}}{6}\right) + \left(\frac{M_{DC} + M_{CD}}{4}\right) = -40$

$\frac{M_{AB} + M_{BA} + 1.5 M_{DC} + 1.5 M_{CD}}{6} = -40$

$M_{AB} + M_{BA} + 1.5 M_{DC} + 1.5 M_{CD} = -240$

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$0.33\,EI\theta_B - 0.167\,EI\Delta + 0.67\,EI\theta_B - 0.167\,EI\Delta +$
$1.5\,[0.5\,EI\theta_C - 0.37\,EI\Delta] + 1.5\,[EI\theta_C - 0.37\,EI\Delta] = -240$

$0.33\,EI\theta_B - 0.167\,EI\Delta + 0.67\,EI\theta_B - 0.167\,EI\Delta + 0.75\,EI\theta_C$
$-0.55\,EI\Delta + 1.5\,EI\theta_C - 0.55\,EI\Delta = -240$

$EI\theta_B - 1.44\,EI\Delta + 2.25\,EI\theta_C = -240 \quad \cdots (3)$

from eq^n (1) (2) and (3)

$\theta_B = 18.83/EI$

$\theta_C = 8.67/EI$

$\Delta = 193.29/EI$

eq^n (a) $\Rightarrow$

$$M_{AB} = 0.33EI \times \frac{18.83}{EI} - 0.167\,EI \times \frac{193.29}{EI}$$

$M_{AB} = -26.06$ KN-m

eq^n (b) $\Rightarrow$

$$M_{BA} = 0.67\,EI \times \frac{18.83}{EI} - 0.167\,EI \times \frac{193.29}{EI}$$

$M_{BA} = -19.66$ KN-m

eq^n (c) $\Rightarrow$

$$M_{BC} = -26.67 + 2EI \times \frac{18.83}{EI} + EI \times \frac{8.67}{EI}$$

$M_{BC} = 19.66$

eq^n (d) $\Rightarrow$

$$M_{CB} = 26.67 + EI \times \frac{18.83}{EI} + 2EI \times \frac{8.67}{EI}$$

$M_{CB} = 62.84$ KN-m

eq^n (e) ⇒

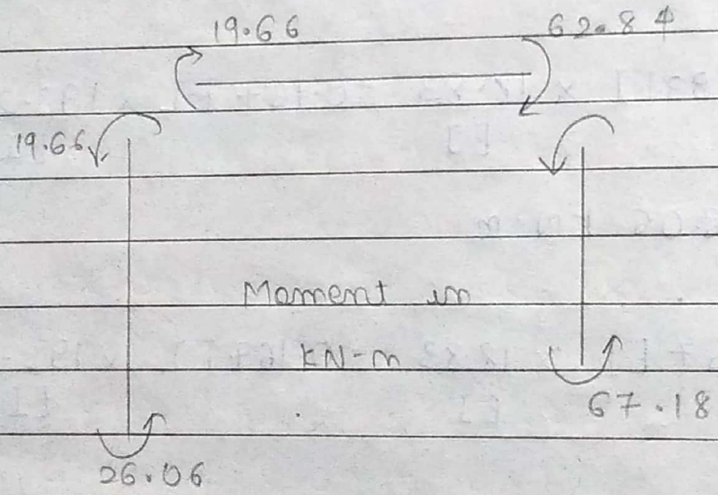
$M_{CD} = EI \times \frac{8.67}{EI} - 0.37\,EI \times \frac{193.29}{EI}$

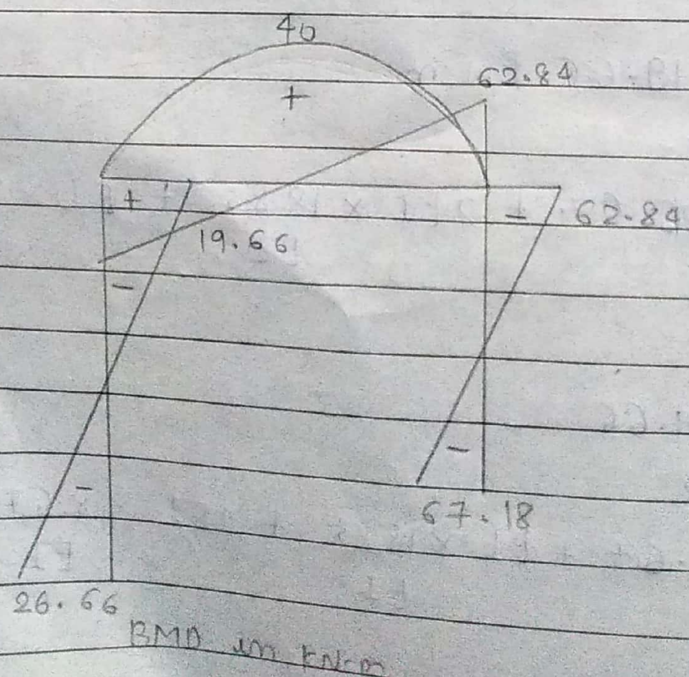
$M_{CD} = -62.84$ KN-m

eq^n (f) ⇒

$M_{DC} = 0.5\,EI \times \frac{8.67}{EI} - 0.37\,EI \times \frac{193.29}{EI}$

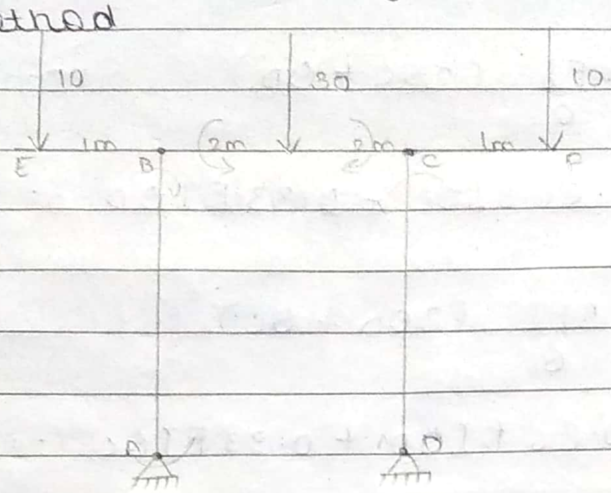
$M_{DC} = -67.18$ KN-m





BMD in KN-m

5.18 Q Analysed the portal frame using slope deflection method



$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0$

$M_{FBC} = \frac{-30 \times 4}{8} = -15$ KNm

$M_{FCB} = \frac{30 \times 4}{8} = 15$ KNm

$M_{AB} = \frac{2EI}{6}(2\theta A + \theta B)$

$M_{AB} = 0.66 EI \theta A + 0.33 EI \theta B$ .... (a)

$M_{BA} = \frac{2}{6} EI (2\theta B + \theta A)$

$= 0.66 EI \theta B + 0.33 EI \theta A$ .... (b)

$M_{BC} = -15 + \frac{2EI}{4}(2\theta B + \theta C)$

$= -15 + EI \theta B + 0.5 EI \theta C$ .... (c)

$M_{CB} = 15 + \frac{2EI}{4}(2\theta C + \theta B)$

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$M_{CB} = 15 + EI\theta_C + 0.5\,EI\theta_B$ .... (d)

$M_{CD} = \frac{2EI}{6}(2\theta_C + \theta_D)$

$M_{CD} = 0.66\,EI\theta_C + 0.33\,EI\theta_D$ .... (e)

$M_{DC} = \frac{2EI}{6}(2\theta_D + \theta_C)$

$M_{DC} = 0.66\,EI\theta_D + 0.33\,EI\theta_C$ .... (f)

Apply conditions of equilibrium at B

$0.66\,EI\theta_A + 0.33\,EI\theta_B = 0$ ....(1) As support A is hinge and moment at support A = 0

$M_{BA} + M_{BC} + 10 = 0$

$0.66\,EI\theta_B + 0.33\,EI\theta_A + (-15) + EI\theta_B + 0.5\,EI\theta_C + 10 = 0$

$0.33\,EI\theta_A + 1.66\,EI\theta_B + 0.5\,EI\theta_C = 5$ ...(2)

Apply condition of equilibrium at C

$M_{CB} + M_{CD} - 10 = 0$

$15 + EI\theta_C + 0.5\,EI\theta_B + 0.66\,EI\theta_C + 0.33\,EI\theta_D - 10 = 0$

$0.5\,EI\theta_B + 0.33\,EI\theta_D + 1.66\,EI\theta_C = -5$ ....(3)

$0.66\,EI\theta_D + 0.33\,EI\theta_C = 0$ .....(4)

$\theta_A = \frac{-2.47}{EI}$

$\theta_B = \frac{5.01}{EI}$

$\theta_C = \frac{-5.01}{EI}$

$\theta_D = \frac{2.51}{EI}$

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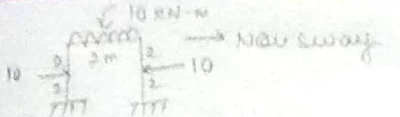

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$M_{BA} = 0.66\,EI\left(\frac{5.01}{EI}\right) + 0.33\,EI\left(\frac{-2.47}{EI}\right)$

$M_{BA} = 2.492$ KNm

$M_{BCD} = -15 + EI\left(\frac{5.01}{EI}\right) + 0.5\,EI\left(\frac{-5.01}{EI}\right)$

$M_{BC} = -12.495$ KN-m

$M_{CB} = 15 + EI\left(\frac{-5.01}{EI}\right) + 0.5\,EI\left(\frac{5.01}{EI}\right)$

$M_{CB} = 12.495$ KN-m

$M_{CD} = 0.66\,EI\left(\frac{-5.01}{EI}\right) + 0.33\,EI\left(\frac{2.51}{EI}\right)$

$= -2.47$ KN-m